<u>THE THEORY OF THE LEAST COST COMBINATION</u>. C. W. Forster

<u>THE PROBLEM:</u> In developing the theory of least cost Professor Taylor starts with this question: "What results in respect to output when we try to increase the output from any instrument of industry by increasing the quantity of the auxiliary factor or factors combined with it?" As soon as any combination begins to give results the output will pass through the following phases:

1. The output will increase more than proportionally to the increase in the auxiliary factor.
2. The output will increase less than proportionally to the auxiliary factor and finally
3. The output will diminish.

This is called the principle of the three stages.

The problem illustrated: The effect of trying to increase output by increasing the auxiliary factor is illustrated by a set of imaginary combinations. These data are exhibited in the attached table. The fixed factor is designated as N and is applied in doses of 20 units each. The auxiliary factor is designated by L. These two factors are then associated in 27 different combinations. The output resulting from each of these combinations is shown in column IV. For example, the fifth combination consisting of 20 units of Ns and 6 units of Ls, gives an output of 84. The table shows also the proportional and the actual increase. The figures in parenthesis indicate the proportional increase and are derived, for any particular combination, by noting the increase in the number of Ls for that particular combination, ascertaining what percent of the number of Ls in the preceding combination this increase is, then computing what an equal percent would be. For example, in combination 13 there is an increase of 2 units over that of the 12th. This amounts to an increase of 12.5%. If the output had increased at the same rate as the Ls the 13th combination would have been 326 instead of 312.

The actual increase is also shown in the fifth column. It is calculated by simply subtracting any particular combination from the product of the next preceding combination. Thus the 13th combination shows a products of 312 from which is subtracted the output of the 12th combination. This gives 22 as the actual increase as compared with 36 as the proportional increase.

<u>The Average Product Measured in Terms of Each of the Factors</u>. The question is next raised "How does the average measured in terms of the fixed factor and the auxiliary factor behave?" According to the assumed data the output in terms of L (average) increases up to the 9th combination and then diminishes to the end. The reason for this is quite obvious. The output is increasing more rapidly (up to the 9th) than the number of units of the variable (Ls) factor. (Compare figures in the fifth column). From the relationship of the output to the L, it must be true that:

1. If for any series of combinations the output is increasing more rapidly than the variable or auxiliary factor, the quotient obtained (that is the average) must be increasing.
2. If for any series of combinations the output is increasing but

less rapidly than the number of units of variable factor, the average must be decreasing.

3. If for any series of combinations the output is diminishing absolutely while the number of units of the variable factor is increasing the average must be diminishing.

The output in terms of Ns is, of course, different than in terms of Ls because N is the fixed factor. In the case of the Ns the average increases up to the 19th combination and then decreases. From the data presented it is evident that:

1. Each average rises during a series of combinations, reaches a maximum in some one combination then diminishes for the rest of the combinations.
2. The maximum average combination is different for each factor.
3. The average measured in either factor (L or N) will be increasing for every combination prior to the 10th; and diminishing for every combination after the 19th. For the intervening combinations (between the 10th and the 19th) the average will be a diminishing one measured in Ls and an increasing one measured in Ns (see diagram)

<u>Change in the Marginal Product</u>. The attempt to increase output by increasing the variable factor or factors causes changes in the additions to the output as measured in the variable factor. For example, in the 13th combination there is an increase in the output of 22 units over that of the 12th combination. (312-290=22) This addition took place with the addition of 2 units of L. The marginal product of L then is 11 units of product. (22÷2=11). The course of the of the marginal product of L is important. It begins with 4 in the second combination and increases to 49 in the 5th. (See column VIII). From the course of the marginal product of L it can be stated that for any series of combinations the marginal product of the variable factor will increase up to a maximum coming somewhat earlier than combination at which the increase in the output becomes less (measured in Ns) than proportional, then diminishes to the end of the series.

The interesting point, therefore, is the distinction which Professor Taylor makes between the increasing and decreasing marginal product of the variable factor L and the increasing and decreasing point where the marginal product of the changing factor begins to diminish is not the point where the output, measured in Ls, changes for a more to a less than proportional one. The point where the marginal product, in terms of ls begins to change, i. e., diminish is the 6th combination (column VIII) while the combination at which the output changes from a more to a less than proportional one is the 10th combination.

<u>The Least Cost Combination</u>. In opening this discussion Professor Taylor makes the following statement: "The cost per unit of product for any particular combination must be equal to the cost per unit measured in Ns plus the cost measured in Ls". Suppose, for example, that in any given combination the average output in Ns, the fixed factor, is 10 units and that each unit of the factor costs $1.00, then each unit of product will cost 10¢ ($1.00 ÷ 10) If the same output in measured in Ls, the variable factor, gives 20 units for each L, and if each L unit cost $1.00, then each unit of product measured in Ls will cost 5¢. The total cost of a unit of product will be 15¢ (5¢ plus 10¢)

It follows from this discussion that the cost per unit measured in either the two factors must increase as the average measured in either factr diminishes and the cost per unit will fall as the average measured in either factor increases. For example, if the average output measured in Ns increased from 10 to 20 units when each N costs $100 then the cost per unit, measured in Ns will fall from 10¢ to 5¢.

"Again, since the average measured in Ns is increasing from the second combination to the nineteenth, while that average diminishes from the 20th on, the cost, measured in Ns, must declime from the 2th to the 19th combination and must increase from the 20th on. On thé other hand, since the average, measured in Ls, increases up to the ninth combination and then diminishes to the end, the cost measured in Ls, must also diminish up the ninth combination and thereafter increase to the end. Further, since the decline in the average measured in Ls is slow during the first few combinations after the ninth, and increases rapidly as it approaches combination 19th, the cost in Ls rises slowly. during the earlier combinations after the ninth and rapidly during the later ones. In like manner the cost in Ns, though declining up to the 19th combination, does this rapidly only during the earlier combinations after 9, slowing up as it approaches the turning point at 19."

From the abovediscussion, it is easy to see how the total cost per unit, measured in both factors, must behave. Since the cost measured in either factor, diminishes up to the 9th combination, the total cost must also diminish. In like manner, the total cost will increase when the average cost measured in both factors increases. In short, for any particular pair of prices, for Ns and Ls, we are bound to have the following results:

1. During a shorter or longer series of combinations, cost will decline.
2. A least-cost combination will appear.
3. During a longer or shorter series, cost will increase.

"What, now, is to be said with respect to the location of the least-cost combination? In general, this must depend on the relative prices of Ns and Ls. As we have already seen, the influence of Ns must tend to lower cost with every movement toward combination 19, while the influence of Ls must tend to increase the cost with every movement from 19 toward 9: It follows that the least-cost point will tend to move toward 19 under the influence of Ns and toward 9 under the influence of Ls. Which of these opposing forces will outweigh the other depends upon their relative magnitude of the prices which the producer has to pay for Ns and Ls. If Ns are very costly, this will tend to push the least-cost point toward the 19th combination, and vice-versa. If, for example, Ns cost 20¢ each and Ls $1, the cheapest combination will be the 11th, while if Ns cost $1 each and Ls 40¢ each, the 17th combination will be the cheapest." (129)

CRITICISM: Nowhere does Professor Taylor say, in so many words, that his least-cost combination is the most profitable one. However, in several places the meaning is quite clear that he does believe it to be the most profitable combination. He states, on page 131, that any plant will naturally be planned and built on such a scale that when supplying its normal output, it will be working in the least-cost combination. As a matter of fact, however, the least-cost combination is not the most profitable one. From the data given the most profitable combination is the 19th assuming that each factor (Ns & Ls) cost $1.00 each and that the product will sell for $1.00 per unit. The 14th combination is indeed the least-cost one having a cost per unit of .1212¢. The net return, however, is only $290. The 19th combination has a cost of 15¢ but give the largest net re-