"GROUND WATER YIELDS IN THE RALEIGH QUADRANGLE"

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GROUNDWATER YIELDS IN THE RALEIGH QUADRANGLE

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SUMMARY

Development of statistical techniques to be used for estimating ground-water yields in crystalline rocks such as underlie the Raleigh, North Carolina, area was the purpose of the investigation. Formulation of such statistical techniques required in this instance a suitable model of the water flow in the crystalline rocks. Because water moves through the crystalline rocks of the Raleigh area in irregularly spaced joints and fractures, theoretical analyses based upon assumption of a homogeneous aquifer do not seem to be appropriate. Also, the unavailability of observation wells required the evaluation of each well solely by measurements made on it alone. Hence a model was devised which is analogous to a tank with an intake pipe near the base and with a pump discharging over the rim of the tank. The report describes the theory, the techniques used, and the results obtained.

A term designated the well factor, CA_f, was derived and used in developing the bases for comparison of well yields. The well factor represents an attempt to express the hydraulic conditions around each individual well and in the fracture system supplying the water to the well in a form suitable for conversion to a set of standard conditions. The standard conditions can then be used to compare yields of the several wells.

From theoretical considerations as well as from experience gained during the investigation. it is concluded that for the approach used the critical point on the drawdown curve is the point at which the system stressed by pumping reaches a semi-steady state as shown by the approach of the draw-down curve to a straight line on a semilog plot of drawdown vs time.

Computations converting the data obtained during a pumping test to a

standard drawdown condition are shown. The standard conditions should be the bases of comparison of wells and of the statistical treatment of the data. However, because of the limited number of wells investigated, the application of the technique to a statistical study of the geologic parameters and well yields was not adequately established.

The report describes one approach to the problem of predicting well yields in crystalline rocks. It is believed that the basic concept underlying the approach described is useful, and even in the idealized form presented here, it represents a closer approximation to actual conditions than do the assumptions of homogeneous aquifers for crystalline rocks.

INTRODUCTION

Formulation of statistical methods for estimating groundwater yields from crystalline rocks in various geologic situations was the purpose of the investigation. Rock type, topography, soil thickness, and other geologic parameters were to be related to the actual yields of the wells and the relationships were to be treated in a statistical fashion.

No published data exist on the well yields of the Raleigh area and the geologic parameters affecting their yields. The small amount of unpublished information that the author could find is largely hearsay in nature and unsuitable for the type of study proposed. Generally, there was no information available on the geologic factors affecting the well yields, and it was planned that fracture systems, topographic position, thickness of weathered zone, and rock type would be investigated with respect to well yields determined during the course of the investigation. Geophysical techniques were to be utilized in measuring thicknesses of the weathered zone. Thus the plan of attack on the problem was to determine well yields and to combine these data with the geologic parameters to develop a technique for estimating yields in the Raleigh area as well as in other areas where crystalline rocks form the bedrock.

The project was initiated by Professor John M. Parker III in the spring of 1965 with the selection of equipment. The equipment was assembled and tested in the late fall of 1965, and actual field investigations were begun under direction of Charles W. Welby in the early spring of 1966. Because wells are drilled where they are needed, the search for them expanded beyond the border of the Raleigh Quadrangle into adjacent parts of Wake County.

Two factors influencing the course of the investigation became evident in the late spring of 1966. The first was that the bulk of the well yields reported by local drilling contractors as the result of pumping tests seldom are accompanied by a drawdown level or any indication of the length of the time the test was run. However, most tests are of short duration, and although some tests are probably reliable, others are not. There is no way to distinguish the good from the bad. Consequently, yield data based upon the information obtained from a well driller or well owner must be considered unsuited for use in statistical evaluations.

The second factor that became apparent was the difficulty of obtaining in the short time alloted to the project and with part-time student assistance an adequate amount of data for a meaningful statistical study of well yields. Accumulation of sufficient data for such a project requires almost constant daily contact with well drillers. On the other hand, it was believed possible to show the theoretical feasibility of such a project and to gather data and to make interpretations that would provide a basis for further work. On this basis the project was continued from July 1966 through June 1967.

One basic problem associated with a statistical study is the need to standardize certain data. In the case at hand, each well is tested at a slightly different pumping rate and at a slightly different drawdown from other wells. For example, it is necessary to compare a well with a yield of 20 gallons per minute at 35 feet of drawdown with a well whose yield is 5 gallons per minute at 75 feet of drawdown. Each well could have different drawdowns at other pumping rates. Thus the stress imposed upon each hydrologic system by the pumping of each individual well differs from well to well and from pump test to pump test. Yield data from individual pump tests should not be used in statistical treatments without standardizing it in some manner.

Although specific capacity (yield per foot of drawdown) is a convenient concept for discussion of well yields, it varies with time and discharge rates (Todd, 1958, p. 11; LeGrand, 1967, p. 4). Step-drawdown tests, in which pumping rates and drawdown are deliberately changed, could not in general be performed because of the limited time that wells were available for testing. Computations utilizing the several equilibrium and non-equilibrium formulae developed for flow through granular media (Todd, 1958; Ferris, et al, 1962) did not seem to provide meaningful answers.

Many publications discussing well yields report only a yield figure, making no mention of the drawdown, the rate of pumping, or other factors influencing the hydraulic system. A definite specific capacity may be noted, but this well parameter is controlled by the pumping rate. Without some standard means of comparison from well to well (hydraulic system to hydraulic system) statistical interpretation of the data is of little value. Even where yields and drawdown are given, this information can not be related directly to another set of data unless the drawdown is similar or porosity and permeability data are available.

Calculations made using the Theis and the Jacob non-equilibrium forumlae and their modifications (Ferris, et al, 1962) seem to lead to confusing and erroneous results in crystalline rocks. The theory on which these approaches are based assumed a confined, elastic, homogeneous aquifer. The rocks of the Raleigh area do not fit this description, and in general the water does not appear to occur under artesian conditions. Furthermore, no observation wells were available to help in defining the formation constants.

The water found in the crystalline rocks of the Raleigh area occurs in fractures, irregular both in size and spacing. Also, there appears to be a certain minimal fracture size which will yield water to a well. Thus a hole drilled into the crystalline rocks may not yield water even though it has been drilled below the water table. Only when the bit encounters a fracture of sufficiently large size will the well produce water. Once water enters the well, it will rise to the local water table. Hence, both the nature and the distribution of the fracture system control the entrance position and the flow of water into a well. Lewis and Burgy (1964) have pointed out that data obtained from pumping tests of wells drilled in fractured rocks generally are not amenable to analysis by means of non-equilibrium equations developed for conditions in granular aquifers.

Another problem involved in the use of artesian, non-equilibrium equations to determine well yields in crystalline rocks and under water table conditions is the fact that seldom does one have information concerning the thickness of the channel or interval through which water flows from the rock mass into the well. Thus the drawdown-averaging techniques used in adapting the equilibrium and non-equilibrium equations to water table conditions can not be used to approximate the shape of the water table. In the absence of one or more observation wells it is impossible to determine the change in shape of the water table surface as water is removed from the system. Modifications of the non-equilibrium equations which were derived to express flow in granular media under artesian conditions to express flow under water table conditions do not reflect the geologic relationships associated with groundwater occurrence in crystalline rocks either.

Since transmissibility is defined as the coefficient of permeability multiplied by the thickness of the saturated interval, transmissibility values will vary with drawdown in an unconfined aquifer and therefore with the pumping rate.

The permeability as expressed by the coefficient of permeability (hydraulic conductivity) that fractures impart to crystalline rocks seems more meaningful than does the transmissibility since the fractures occupy only a small fraction of the total surface area of the well bore. Without knowledge of the total area of the openings into the well bore, it is impossible to convert the transmissibility computed from the Theis and Jacob formulae to permeability. Furthermore, so long as the intersection of the fracture and the well bore is covered with the water in the well, the velocity of flow into the well is controlled by the height of the water above the fracture. Hence, field-measured rate of flow into the well can vary with the level of the water in the well, increasing as the water level drops. Absolute values for flow velocity into the well then depend upon the geometric relationships within the fracture system, the depth in the well bore below the water table to the intersection with the fracture system, the height of the water table, the water level in the well at any time during the pumping cycle, as well as the resistance to flow within the fracture system.

A means of comparing well yields has been devised. The method takes into account the geologic facts of groundwater occurrence in crystalline rocks as well as the need for comparison of the yields under a set of standard conditions. A standard porosity and a standard drawdown form the basis for the standard conditions. The procedures outlined in this report were designed to overcome some of the difficulties already discussed and to provide a basis for comparison of wells in different geologic settings.

Assumptions other than those described in a later section might have been made; however, it is believed that the approach chosen is appropriate to the problem at hand. The results are presented in terms of feasibility rather than in terms of a complete evaluation of the water yields for a particular group of crystalline rocks.

A recovery test run on the pumped well is another means of evaluating well behavior. However, a good recovery test on a pumped well requires the absence of geologic boundary conditions in the water-yielding rocks (Ferris, et al, 1962). In most of the crystalline rocks investigated the fracture geometry implies that boundary conditions are to be expected. Results from several of the tests supports this viewpoint.

A practical, applied aspect of the investigation was to devise a means of obtaining estimates of well yield and long-term behavior of individual water wells based on short-term pumping tests and in the absence of observation wells. Ability to provide a reasonably accurate prediction about the behavior of an individual well drilled in crystalline rocks would enhance the effectiveness of the water well industry. Long-term pumping tests are expensive, and observation wells are not available generally. Hence, a method by which a pumping test and/or a recovery test of a few hours duration could be used to predict accurately the potential productive capacity and future behavior of a well would have economic benefits for both the well driller and for the well owner. Information about the yield rate as well as total yield would aid the well owner in management of the groundwater resource.

The concepts and the mathematical manipulations described in this report, together with the associated curves, are believed to contribute to the solutions of the well comparison problem and to the practical problem also.

GEOLOGY OF THE RALEIGH QUADRANGLE

Figure 1 summarizes the major rock types within the Raleigh Quadrangle and adjacent areas. Locations of the wells investigated are given by symbols and Roman numerals. Mica and hornblende gneiss, quartz-microcline gneiss with its associated quartz-disc gneiss and muscovite garnet-schist, and granite comprise the rock types from which the data were collected. Variations in the nature of the foliation and lineation are present within each rock type. The gneisses and schists exhibit a well defined layering.

Structural trends are generally northeasterly; foliation is inclined to the west and northwest. Most joint sets are oriented in a west-northwest trend. The topography is gently rolling, and the soil and saprolite cover varies from a few inches to several tens of feet. The cover is generally thin (a few inches to a foot or two) on hill tops and ridge crests and somewhat thicker on the hillsides. Most of the soil is clay-rich. Pegmatite dikes and quartz veinlets are common in most of the rocks.

THEORY

Movement of water through the crystalline rocks is dominated by the fractures present in these rocks, their numbers in a given volume of rock, their size, the nature of the interconnection between the fractures, and their orientation with respect to a given well. While the path a given water particle follows to a well may be tortuous, the flow is along the fractures in a manner not unlike laminar flow through a complex series of pipes or laminar flow between closely spaced plates.

A model may be visualized in which the well bore is analogous to a tank.

GENERALIZED GEOLOGIC MAP
(after J.M. Parker, III)

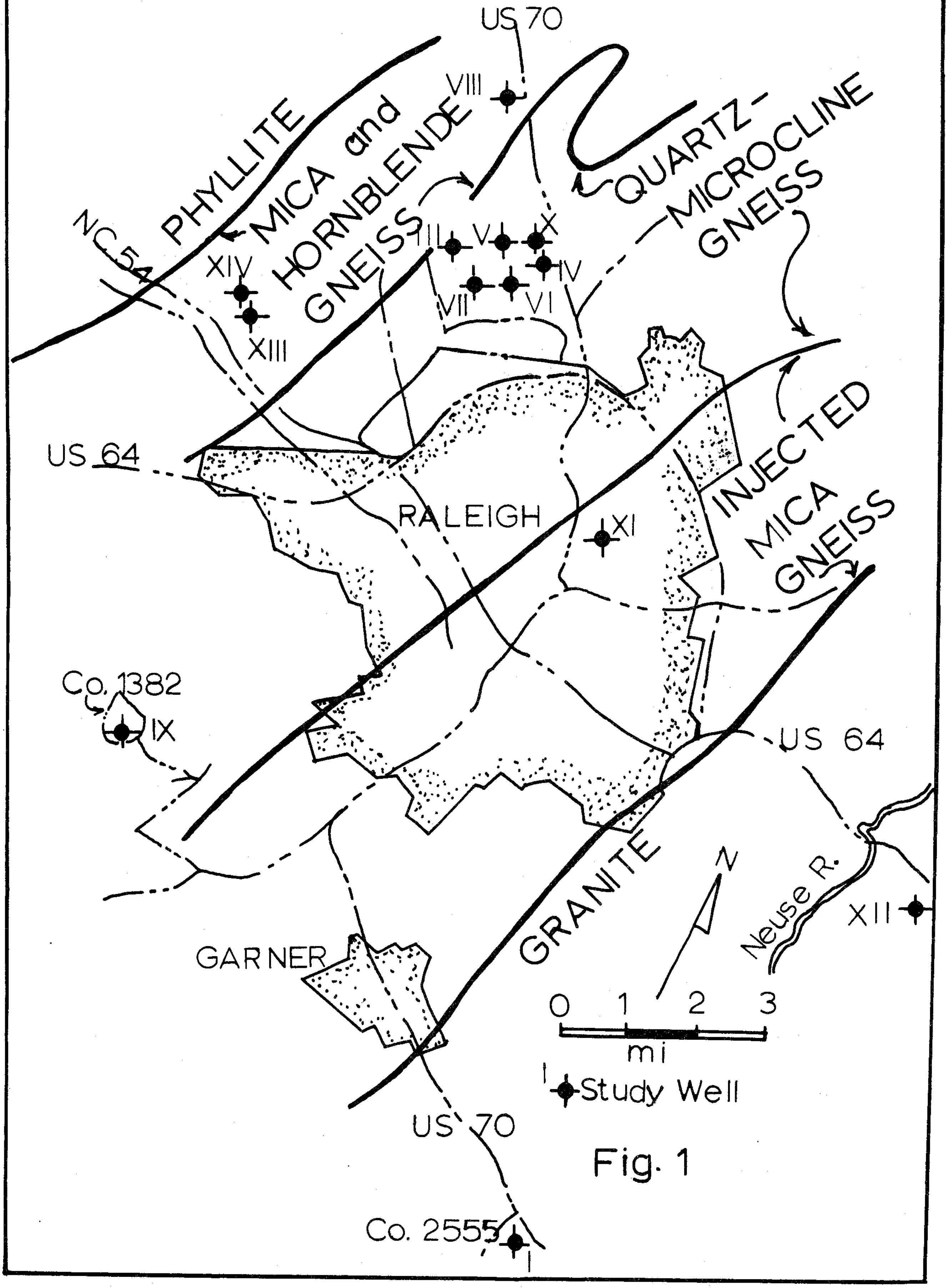


Fig. 1

In the model the water flows into the tank (well bore) through one or more pipes (fractures). Water is discharged from the tank (well bore) by a pump operating at a constant rate. It is assumed that water is supplied through the pipe (fracture) from an infinitely large reservoir.

It is this model that has been utilized in developing the theory. Specifically, the model was developed in response to a need for evaluation of short-term pumping tests combined with a need to evaluate these tests in the absence of an observation well or wells. The model has been chosen as a reasonable one because drillers often report water as being "hit" at one or two places in a given well in crystalline rocks (e.g., Well XI of this report), and often they report that no water was encountered until a certain depth was reached. In addition, examination of roadcuts through crystalline rocks often shows water seeping from one fracture alone despite the general distribution of water in the overlying saprolite and soil and the presence of numerous other fractures.

Analysis of the model leads to formulation of several constants which are believed to describe the characteristics of a given well and which are believed to be suitable for use in statistical studies of well yields in crystalline rocks. These constants are thought to represent the hydraulic system of each well to the extent that each was tested. Thus they are believed to be more representative of the water-yielding capability of each individual well than are other measures.

While the derivation is based upon a model of a well and one fracture, the basic concept can probably be applied to a well encountering several water-yielding fractures at different depths. Such an application would be by modifying the basic equation developed in ensuing paragraphs or by assuming that the flow through the several fractures could be expressed as flow through a single fracture. The cross-sectional area of the single hypothetical fracture and its position in the well bore would have to be such that the theoretical flow through the single fracture would be equivalent to the water flow through the several fractures.

Because of the limited amount of data collected in this investigation the author adopts the view that only the feasibility of this approach can be evaluated. It is recognized that refinements of the approach have to be made before it is generally applicable to statistical studies of well yields. The presentation made on the following pages describes the approach and its application to the data obtained in the study.

The diagram in Figure 2 illustrates the symbols used in the derivations, and they are further defined in Appendix F where all symbols used in this report are tabulated.

Under static conditions the total pressure relationship at the plane of intersection between the fracture and the well bore (Figure 2) is $p_f = p_w$, where p is the total pressure and the subscripts are defined in Figure 2. This relationship assumes that the flow from the fracture into the well bore is horizontal flow. At the plane of intersection the total pressure force in the fracture acts inward toward the well bore, and the total pressure force from the well acts outward into the fracture. This relationship exists because there are, theoretically, water columns of equal height above the fracture on either side of the plane and because atmospheric pressure is theoretically the same on both sides of the plane. Another way of viewing the relationship is to consider that the force potentials on either side of the plane are equal.

SYMBOLS USED IN DERIVATIONS

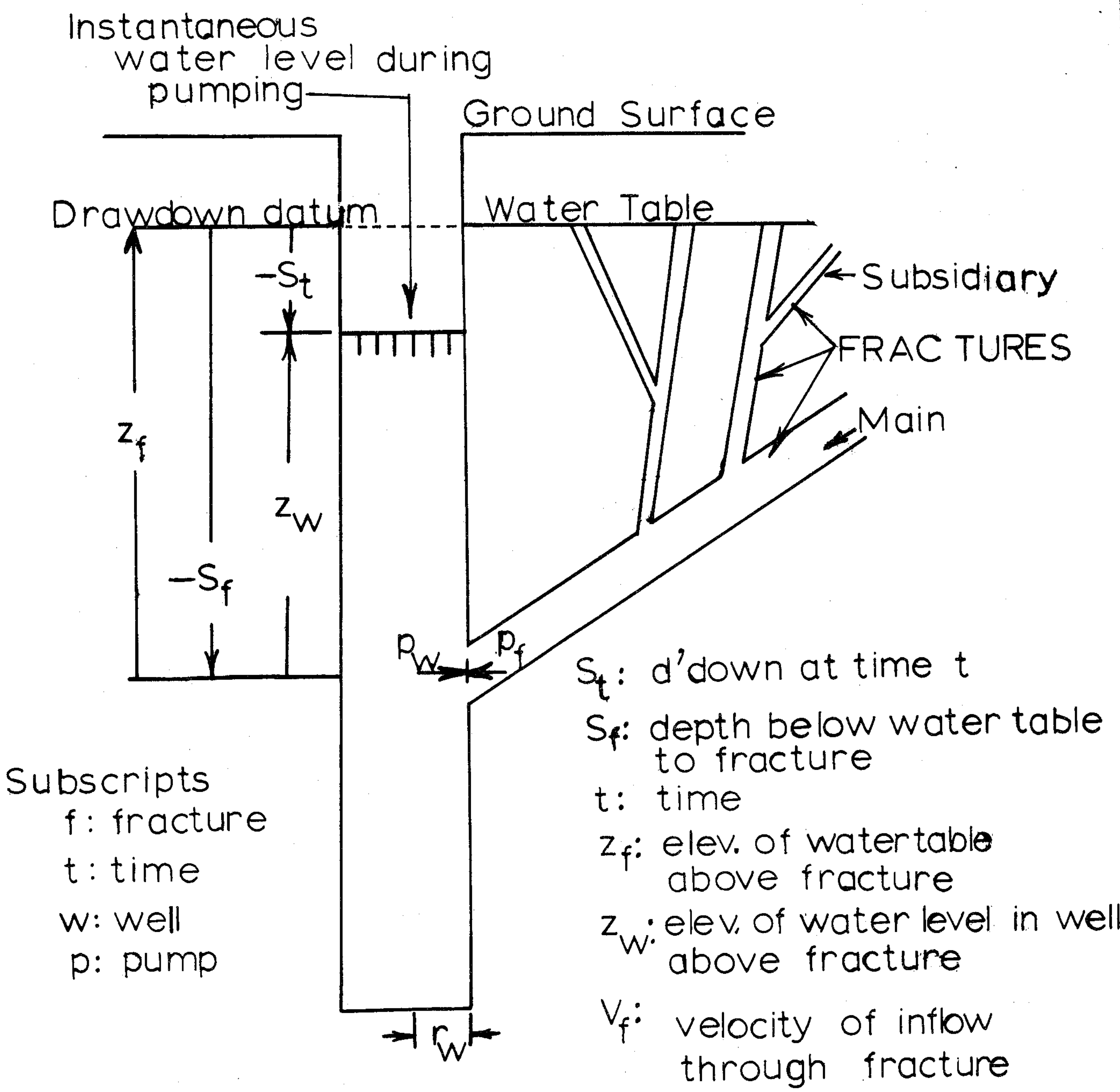


Fig. 2

Once pumping commences, the water level in the well is lowered, p_w drops, and water begins flowing into the well from the fracture since $p_f > p_w$ (for horizontal flow through the fracture opening). The volumetric rate of flow (e.g., gallons per minute, cubic feet per second) through the fracture depends upon a number of factors. Included among the factors are the rate at which the water level in the well is lowered, the effective cross-sectional area of the fracture, the rate at which the pressure differential between the well and the water in the rocks is transmitted through the system, and the resistance to flow provided by the fracture system.

When the system is changed from a static to a dynamic state by the start of pumping, the pressure relationship at the fracture can be analyzed in terms of the Bernoulli equation:

$$P_w + \frac{V_w^2}{2g} + z_w = P_f + \frac{V_f^2}{2g} + z_f \qquad (1a)$$

where P is the external pressure head on the system
V is the velocity of the water flow
z is the elevation above or below an arbitrarily chosen datum
g is the acceleration of gravity, ft/sec^2
w signifies the well
f signifies the fracture.

For water table conditions under normal atmospheric pressure $P_w = P_f$ as these two terms in the equation represent the external pressure head (from the atmospheric pressure) on the water in the well and on the water in the fracture system, respectively.

Elevation of the water in the well can be expressed as an elevation above or below some arbitrary datum. Examples of possible reference points include the bottom of the well, the next 100-foot contour below the local ground level, the fracture through which water is flowing into the well, or the water level in the well prior to commencement of pumping.

If the fracture is chosen as the datum for measurement of the height of the column of water in the well and the water table is picked as the zero point for measurement of drawdown, then under static conditions

$$z_f = -S_f \tag{1b}$$

where z_f is the height of the water table above the fracture (Figure 2)
and $-S_f$ is the depth below the water table to the fracture (Figure 2).

When the well is being pumped,

$$z_w = (-S_f) - (-S_t)$$

$$= S_t - S_f \tag{1c}$$

where z_w is the height of the water level in the well above the fracture; i.e., it is the height of the water column in the well above the fracture at time t (Figure 2)

$-S_t$ is the drawdown measured at time t (Figure 2)

$-S_f$ is the depth below the water table to the fracture; it is the drawdown in the well when the water level in the well reaches the fracture.

The position of the fracture can be described as $z_f = 0$ for the case in which the fracture is the datum. For the case in which the water table is the datum the position of the fracture can be expressed as $-S_f$, the negative sign being used to show that drawdown is measured from the water table downward. The absolute values of z_f and S_f are the same.

Since the position of the fracture will in general be unknown and since the initial water level in the well bore can be measured, it is useful to substitute the drawdown in the well for the elevation factor in the Bernoulli equation, making the appropriate sign change.

In general, the velocity head loss in the well can be ignored. For example, if the water surface in the well falls at the rate of 1 ft/min, $\frac{V_w^2}{2g} = 4.32 \times 10^{-5}$ ft. When the velocity head loss in the well is ignored and when the following substitutions are made into Equation (1a),

$$z_f = -S_f \tag{1b}$$

$$z_w = (S_t - S_f). \tag{1c}$$

Equation (1a) reduces to

$$S_t = \frac{V_f^2}{2g} \tag{1d}$$

for a frictionless system.

Thus

$$V_{ft}^2 = 2gS_t$$

or

$$V_f = \sqrt{2gS_t} \quad \text{(Torricelli's Theorem)} \tag{2}$$

where g is the acceleration of gravity, ft/sec^2

S_t is the drawdown at time t, in ft

V_f is the velocity (ft/sec) of water flow from the fracture into well at time t.

Evaluation of the relationship from a mass balance or total yield viewpoint for any time shows the following relationship:

$$\rho q_p = \rho q_w + \rho q_f \tag{3a}$$

where q_p is the total volume of water pumped up to time t

q_w is the volume of water removed from the well bore alone, resulting in the lowered water level in the well, up to time t

q_f is the volume of water that has flowed from the fracture into the well bore up to time t

ρ is the density of water.

The value of q_f for any given drawdown during a test may be determined by plotting the two curves: drawdown vs cumulative or total volume of water pumped and the drawdown vs the volume of water removed from the well bore alone (Figure 3). For any given drawdown the volume removed from the well bore is determined from knowledge of the well's diameter and the assumption that the well is a cylinder. Thus each foot of the drawdown represents a known volume of water removed from the well bore.

The drawdown values for the drawdown vs total volume of water pumped curve are obtained by use of two curves: the plot of total volume of water pumped (q_p) vs time (the slope of this curve gives Q_p) and the drawdown curve, the plot of the water level vs time (Figure 3). The time axes of these two curves provide the tie between them. Drawdown and total volume pumped for several selected times are determined and plotted as the drawdown vs cumulative or total volume curve.

The value of q_f for any given drawdown is the separation between the q_p vs drawdown curve and the q_w vs drawdown curve at the particular drawdown. Thus q_f at the given drawdown may be determined by subtracting the value shown on the q_w vs drawdown curve from the value shown on the q_p vs drawdown curve (Figure 3a). A graphical procedure is used as a matter of convenience. The volume removed from the fracture, q_f, could simply be computed from a tabulation of drawdown, q_p, and q_w . Similarly, q_f for any given time during the pumping of a well can be determined by plotting q_p vs time and q_w vs time. Appendix E describes in detail the techniques used in making the pumping tests and in interpreting the data obtained from them.

For the present approach the relationship of the volumetric flow rates from the well bore (Q_w), from the fracture (Q_f), and through the pump (Q_p)

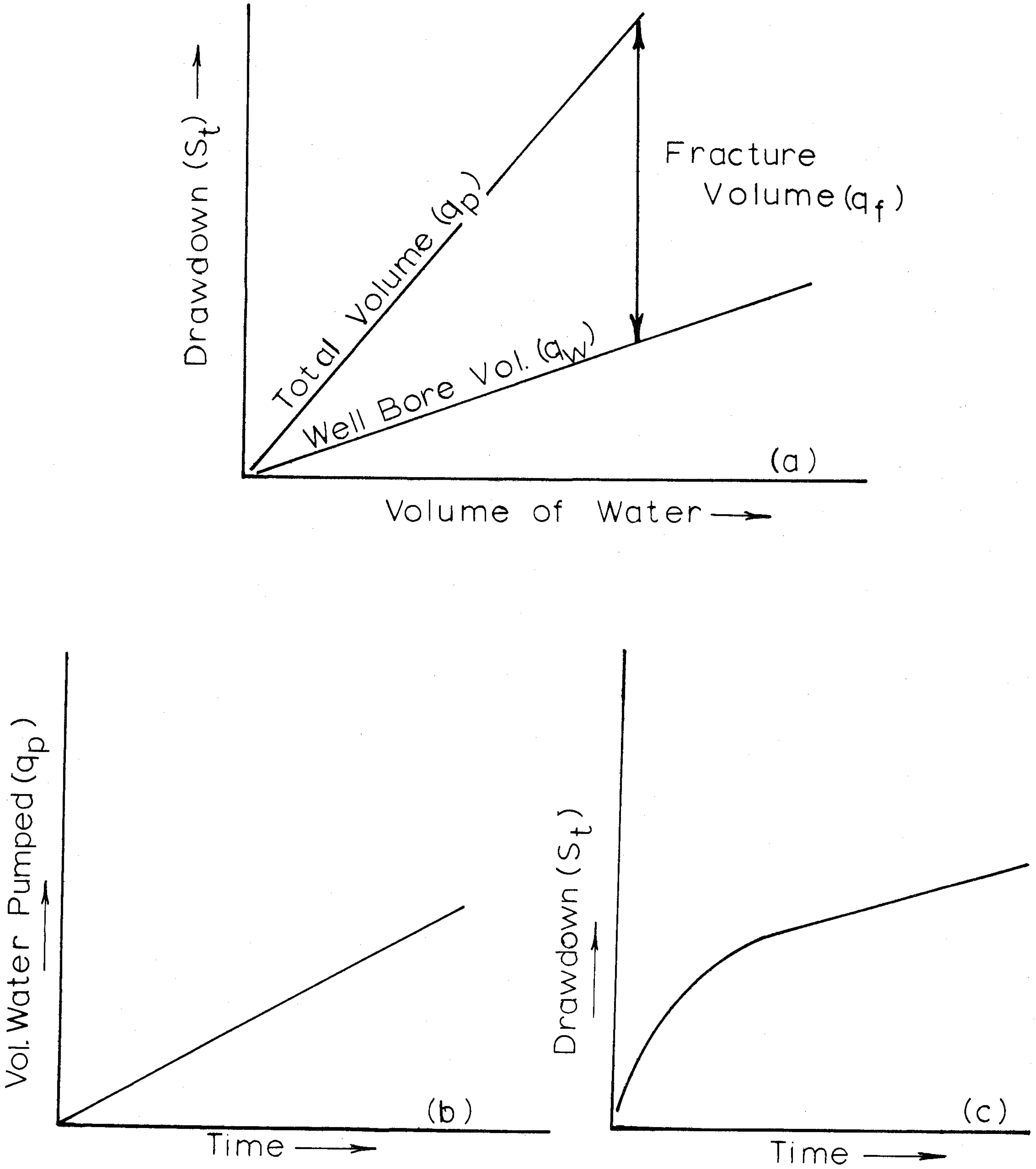


Fig. 3

CURVES USED IN ANALYSIS

(See Appendix E also)

are of importance. The following equation summarizes the relationship:

$$Q_p = Q_f + Q_w. \quad (3b)$$

(Units for Q used in this investigation were gallons per minute, gpm; other volumetric units per unit time could be used.)

The volumetric flow rate from the well bore alone (Q_w) over an interval of time is $\frac{A_w \Delta S}{\Delta t}$, where S is the drawdown, A_w is the cross-sectional area of the well, and t is time. The factor $\frac{\Delta S}{\Delta t}$ is the slope of the drawdown curve. In differential form:

$$Q_w = A_w \frac{dS_t}{dt}. \quad (3c)$$

Q_p is measured by timing the volume of water pumped through a flow meter or other suitable volume-measuring device. Q_f is the volumetric flow rate into the well from the fracture. If the well is pumped at a steady rate, then from Equation (3b) above, Q_f and Q_w must vary inversely until a steady state relationship has been established; at this time they have a constant relationship.

From the definition of volumetric flow rate,

$$Q_f = A_f \, V_f \quad (4)$$

where A_f is the effective cross-sectional area of the fracture
and V_f is the velocity of water flow through the fracture.

Substituting Equation (2) for V_f, Equation (4) becomes a frictionless system:

$$Q_f = A_f \sqrt{2gS_t} \quad (5)$$

where Q_f is the volumetric flow rate through the fracture at time t.

At this point a factor, C, can be introduced into Equation (5). Its purpose is to serve as a correction factor or coefficient for the effects of friction, the length of flow of water, and the inertial effects in the complete system for the conditions under which a well is pumped. The factor C is a variable and is dependent upon the conditions under which the well is pumped and upon the conditions in the whole hydraulic system at any time during the pumping test. It is particularly responsive to the amount of the drawdown and to the rate at which the water level in the well is lowered. With the insertion of the factor, C, the argument moves from the discussion of a frictionless system to a discussion of a system in which friction and other factors play a role. Equation (5) then becomes:

$$Q_f = CA_f\sqrt{2gS_t} \qquad (6)$$

Once the level of the water in the well has reached the fracture, that is $-S_t = -S_f$ (Figure 2), the velocity of the water moving from the fracture to the well bore is constant in a frictionless system and is equal to $\sqrt{2gS_f}$. Up to the moment at which S_t becomes equal to S_f, the velocity of the water moving from the fracture into the well bore is changing with drawdown, and hence time. In a real system the rate of velocity change should slow down as water is drawn into the main fracture from subsidiary fractures and the well factor C should be affected primarily by the length of flow of the water and the associated friction losses. From a practical point of view, C approaches a constant value when the water level in the well reaches the position of the fracture.

Up to the time at which the water level reaches the fracture, assuming a constant withdrawal from the well, the volumetric flow rate from the fracture into the well (Q_f) has been changing. Thus the time (t) vs drawdown (S) curve is exponential in form. Once the fracture is reached, the curve should approach a straight line since the rate of pumping is constant and the flow through the fracture is constant ($Q_f = A_f\sqrt{2gS_t}$ in a frictionless system and C approaches a constant value).

The volumetric rate of flow from the well bore alone (Q_w), shown physically by the downward movement of the water surface in the well bore, is the product of the cross-sectional area of the well and the downward velocity of the water surface:

$$A_w \frac{dS_t}{dt} = Q_w \,. \qquad (7)$$

Total Yield From Fracture

The total volume of water pumped from the well during a pumping test is the sum of the water volume yielded up to the time that the water level reaches the fracture and the water volume yielded between the time the water level reaches the fracture and the end of the pumping test. The following discussion shows the origin of the equation expressing the volume of water pumped from the well from the commencement of pumping until the pump is turned off.

Above the Fracture: The total volume of water entering the well bore from the fracture between the time pumping commences and the instant the water level in the well reaches the fracture may be expressed by Equation (8a)

$$q_{fa} = \frac{Q_{fa} - Q_o}{2}(t_f - t_o) \qquad (8a)$$

where q_{fa} = total volume of water that has flowed from the fracture from the beginning of the pumping to the time at which the water level reaches the fracture

Q_{fa} = volumetric flow rate from the fracture at the instant the water level reaches the fracture

Q_o = the volumetric flow rate from the fracture at the time pumping starts (equals zero for static conditions)

t_f = the time at which the water level reaches the fracture

t_o = time at which pumping begins.

The equation assumes that the volumetric rate of water inflow through the fracture from the instant pumping starts to the time the water level reaches the fracture increases at a constant rate. Thus the average volumetric rate of flow from the fracture into the well is one-half of the rate at the time the water level reaches the fracture. The total volume of water which enters the well from the fracture from the time pumping begins until the water level reaches the fracture (drawdown = $-S_f$) is

$$q_{fa} = \frac{Q_{fa}}{2}(t_f - t_o) \quad (8b)$$

$$= \frac{CA_f\sqrt{2gS_f}}{2}(t_f - t_o) \quad (8c)$$

where $Q_{fa} = CA_f\sqrt{2gS_f}$ from Equation (6) at $-S_t = -S_f$.

Should the water level in the well not drop to the fracture during the pumping test, the total volume of water that has entered the well from the fracture at any given time can be determined by solving the differential equation

$$\frac{d^2q_f}{dt^2} = CA_f\sqrt{2g}\ \frac{S_t^{-1/2}}{2}\ \frac{dS_t}{dt} \quad (8d)$$

which expresses the total amount of water from the fracture as a function of the drawdown and time. The equation is derived by differentiating the volumetric rate of flow through the fracture, Q_f, with respect to time, where $Q_f = CA_f\sqrt{2gS_t}$. (6)

$$\frac{dQ_f}{dt} = \frac{d}{dt}\frac{(dq_f)}{(dt\ )} = CA_f\sqrt{2g}\,\frac{(dS_t^{1/2})}{dt}$$

$$\frac{d^2q_f}{dt^2} = CA_f\sqrt{2g}\,\frac{S_t^{-1/2}}{2}\,\frac{dS_t}{dt} \qquad (8d)$$

where q_f = total volume that has flowed into the well bore from the fracture up to the moment of the drawdown (S_t) determination

Q_f = the volumetric flow rate from the fracture into the well

t = time

and the other symbols are as previously defined.

Below the fracture: If Equation (3a) is applied to water flow into the well after the water level in the well reaches the fracture, then q_w is the volume of water removed from the well bore alone between the instant the water level reaches the position of the fracture (t_f) and some later time (t_b). This volume may be obtained by subtracting the drawdown at time t_b from the drawdown at the instant the water reaches the fracture (t_f) and multiplying the difference by the cross-sectional area of the well. If previously defined symbols are used, the relationship can be expressed mathematically:

$$\rho q_{pb} = \rho q_{wb} + \rho q_{fb} \qquad (3a)$$

(Subscript b indicates values are measured below the fracture, and the other symbols are as previously defined.)

Height of water column removed from the well

$$= (-S_f) - (-S_t)$$

$$= (S_t - S_f)$$

and $$q_{wb} = A_w(S_t - S_f) \qquad (9)$$

Since $-S_t = -S_f$ when the water level reaches the position of the fracture and since $-S_f$ is a constant,

$$Q_{fb} = CA_f \quad 2gS_f \text{ .} \quad \text{(Equation (6) modified for } -S_f = -S_t\text{)}$$

Once the water level in the well drops below the fracture, Q_f theoretically approaches a constant value. Thus the total volume of water entering through the fracture once the water level in the well bore passes the fracture, q_{fb}, may be expressed as the product of the constant volumetric flow rate through the fracture (Q_f) and the time elapsed between the instant at which the water level reaches the fracture (t_f) and the time of a later water level measurement (t_b). Hence,

$$q_{fb} = (CA_f \sqrt{2gS_f})\ (t_b - t_f) \qquad (10)$$

where t_f is the time at which the water level in the well reaches the fracture

t_b is the time at which the water level in the well reaches the level of S_t below the fracture (measured from the initial water level)

q_{fb} is the total volume of water that has entered the well bore through the fracture from the time the water level passes the fracture to the time the water level reaches a drawdown $-S_t$ below the fracture.

The total volume of water pumped from the well after the water level passes the fracture, q_{pb}, is the sum of the volume of water removed from the well bore, q_{wb}, and the volume that enters the well through the fracture in the time interval $(t_b - t_f)$, q_{fb}:

$$q_{pb} = q_{wb} + q_{fb}$$ (Equation 3a; density of water, a common factor to all terms of the equation, has been removed.)

But $q_{wb} = A_w(S_t - S_f)$ (9)

$$\text{and } q_{fb} = (CA_f\sqrt{2gS_f})\ (t_b - t_f) \tag{10}$$

$$\text{or } q_{pb} = A_w(S_t - S_f) + (CA_f\sqrt{2gS_f})\ (t_b - t_f). \tag{11}$$

Converting to volumetric flow rate, Q_{pb},

$$Q_{pb} = \frac{q_{pb}}{(t_b - t_f)} = A_w\frac{(S_t - S_f)}{(t_b - t_f)} + (CA_f\sqrt{2gS_f})\ \frac{(t_b - t_f)}{(t_b - t_f)} \tag{12}$$

$$\text{or } Q_{pb} = A_w\frac{(S_t - S_f)}{(t_b - t_f)} + (CA_f\sqrt{2gS_f}). \tag{13}$$

Equations (12) and (13) express the relationships among the pumping rate, the rate of water removal from the well bore, and the rate of water flow into the well bore from the fracture during the time interval in which the water level in the well bore is below the fracture.

Combined Relationship: When equations (3c), (8c), and (11) are expressed in terms of the volume of water pumped (q) and then are combined to give the equation for the total volume of water pumped, the equation is

$$q_{pT} = \underset{[I]}{A_wS_{tT}} + \underset{[II]}{CA_f\sqrt{\frac{2gS_f}{2}}\ (t_f - t_o)} + \underset{[III]}{CA_f(\sqrt{2gS_f})\ (t_b - t_f)} \tag{14}$$

where q_{pT} is the total volume of water pumped from the well during the pumping test

S_{tT} is the total drawdown of the water level in the well during the pumping test,

and the other symbols are as previously defined.

The Roman numerals beneath the equation indicate the meaning of each term:

I: volume of water removed from the well bore alone

II: volume of water flowing into the well from the fracture up to the time the water level reaches the level of the fracture

III: volume of water flowing into the well from the fracture between the time the water level reaches the fracture and the time the pump is turned off.

Equation (14) expresses the total volume of water produced in terms of the drawdown, the total elapsed time of the test, and the varying velocity of water flow into the well during the test.

TECHNIQUE

If the change in velocity at which the surface of the water drops and the change in velocity of the water flowing into the well from the fracture stabilizes, then the drawdown curve becomes a straight line, and $\frac{dS_t}{dt}$ = a constant. The time at which the stabilization occurs is a function of the pumping rate, Q_p, the drawdown, and the nature of the rock fractures and of the water flow through them. The drawdown at which the balance is achieved is S_{tx}, and the time at which the balance is reached can be determined from the drawdown curve. The volumetric rate of flow from the pump can be determined. Hence the nature of the rock fracture and the hydraulics of the system can be characterized even though it may not be possible to describe the fractures themselves.

Utilizing the relationships described above and assuming that the water level does not reach the fracture prior to stabilization between Q_w and Q_f, the following equation can be written:

$$Q_p = A_w \frac{dS_t}{dt} + CA_f\sqrt{2gS_{tx}} \, . \qquad (15)$$

If the drawdown does reach the fracture, then the drawdown for stabilization, $S_{tx} = S_f$. When the hydraulic system reaches the state in which the drawdown curve is a straight line, the rate of drawdown, $\frac{dS_t}{dt}$, is its slope. Q_p is determined from metering of the water pumped; the well diameter can be determined, and S_{tx} is measured from the drawdown curve.

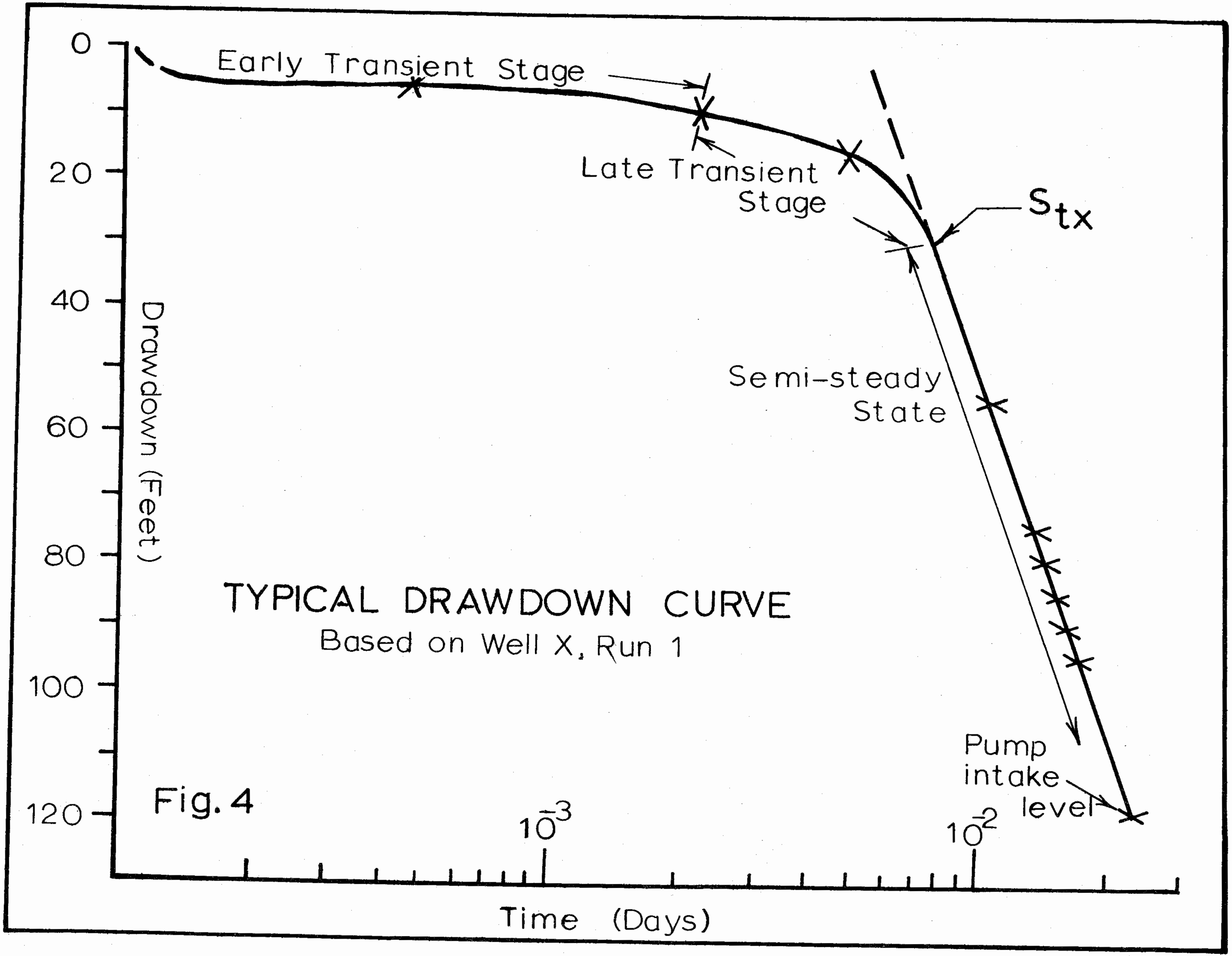
S_{tx} represents the point at which the drawdown curve passes from a transient stage to a semi-steady state stage. Figure 4 illustrates a typical drawdown curve, and these two stages are indicated on it. S_{tx} is determined graphically as the point at which the drawdown curve becomes a straight line on the semilog plot.

Knowledge of the position of S_{tx} on the curve permits solution for the well factor CA_f. A sample calculation is given in Appendix A. CA_f is not a unique characteristic of each well, for it is influenced by the hydraulics of the system and the cross-sectional area of the fracture. With experience one may evaluate individual wells and compare their yields semiquantitatively using the factor.

The curves of Figure 5 show the relationship between CA_f, drawdown S_t, and the volumetric flow rate from the fracture based on Equation (6). These curves can be used to understand the behavior of a given well under various pumping rates and conditions of drawdown and to determine approximate limits to the quantity of water that can be produced from the well at various pumping rates.

Model for Comparison Between Wells

A statistical study requires that the things being compared should be placed on some standard basis. During the course of this study it became apparent that a model would have to be devised by which volumetric flow rates, total volumes of water, slopes of hydraulic grade lines, and drawdowns could all be related to one another, perferably in the form of ratios. As the model for the various calculations a conical pattern to the hydraulic potential surface about the pumping well was chosen. Additionally, the assumption was



TYPICAL DRAWDOWN CURVE
Based on Well X, Run 1
Early Transient Stage
Late Transient Stage
Semi-steady State
S_{tx}
Pump intake level
Drawdown (Feet)
0
20
40
60
80
100
120
10^{-3}
10^{-2}
Time (Days)

Fig. 4

RELATIONSHIP BETWEEN Q_p, S_t, and CA_f

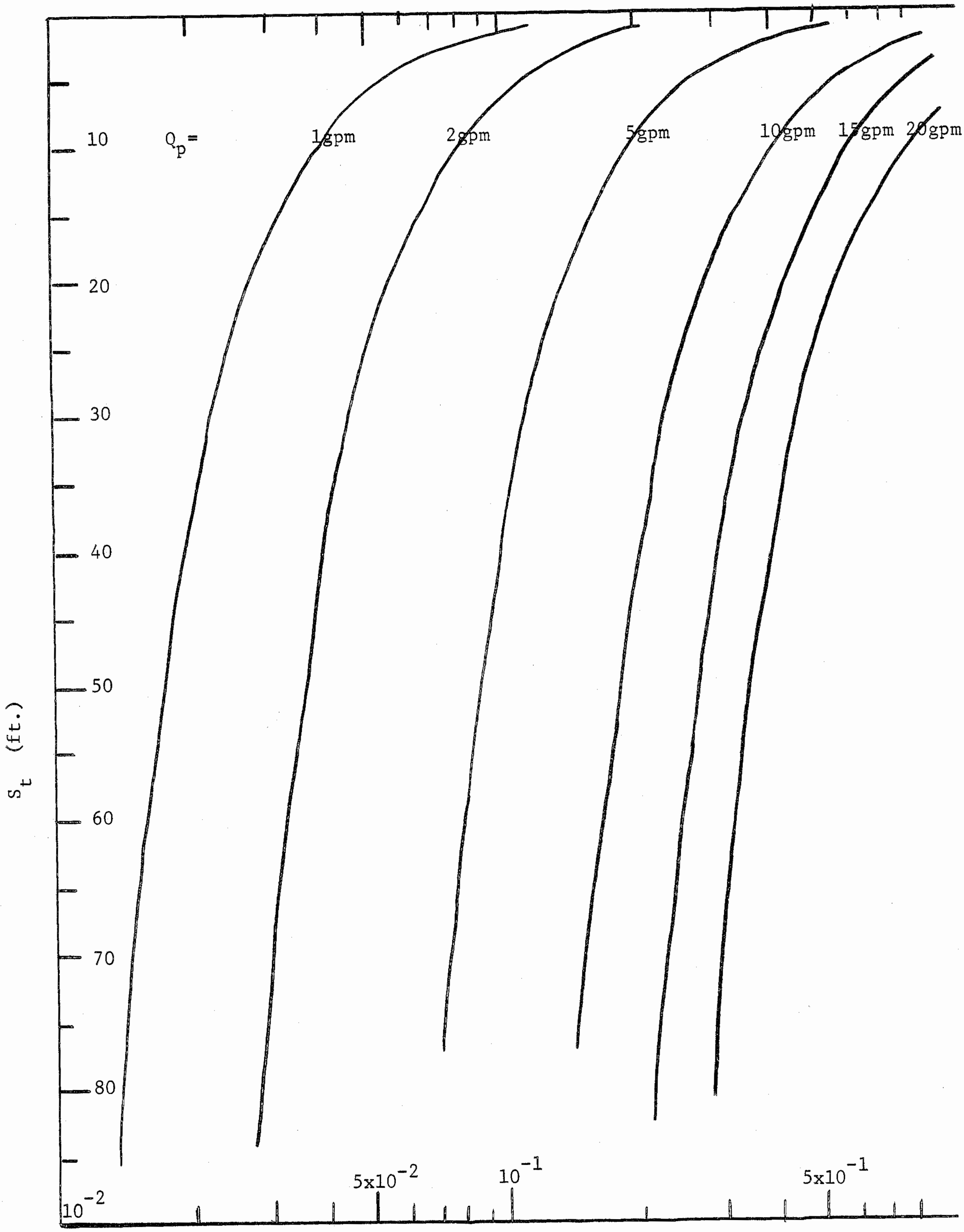


FIG. 5

made that the position of this surface at any moment corresponds to the position the water table would assume if the rocks into which the well is drilled were homogeneous. Entrance of water into the well bore from the rock is assumed to take place through one fracture which is connected to subsidiary fractures in the adjacent rock mass. Use of the model obviates the need for an exact description of the fracture pattern.

Because no porosity measurements were available for the rocks studied, a porosity of 1 per cent was assumed to apply to the model. Although such a porosity may be high for some cases and low for others, it appears to be a reasonable approximation. Additionally, the value has the advantage of ease of manipulation in the calculations. Other porosities could have been used.

The factor C, previously described, is a function of the effective head difference between the water level in the well and the water table, the flow rate of the water through the fracture, the inertial effects in the system, and the radius out from the well at which the hydraulic grade line intersects the water table. If it is assumed that the pumping of the well creates a conical pattern in the hydraulic potential surface about the well and that the volume of water removed from the fractures up to any given moment (q_f) is a measure of the total volume of open space within the dewatered zone, then the radius of an imaginary cone whose base lies at the water table can be computed. An additional necessary assumption is that governing the position of the apex of the cone. In the model used in this study the apex is placed at the drawdown S_{tx}, the point at which the hydraulic system assumes a semi-steady state condition. Other assumptions could have been made, their effect being to give a different configuration to the theoretical hydraulic surface about the well, but the position of S_{tx} is relatively easily determined from the drawdown curve. The conical relationship seems to fit best the average condition about a pumped well. A technique utilizing the assumption of a standard cross-sectional area for the fracture through which the water is assumed to enter the well could have been developed, but the approach seems fraught with more difficulties than the simple conical model assumed here.

From the assumption concerning the porosity of the rocks it is possible to compute the slope (tan Θ) of the sides of the cone, i.e., the hydraulic grade line. Sample calculations are given in Appendix B. The ratio of the slope to the well factor, CA_f, is a characteristic of the individual well and reflects the pumping rate, the response of the hydraulic system to the pumping stress, and the drawdown at which semi-steady state conditions set in. High yield wells with low S_{tx} values will exhibit very low ratios, and low yield wells with high S_{tx} values will have high values for the ratio tan Θ/CA_f.

Under some circumstances it is possible that wells of quite different drawdowns and yields may have similar values for CA_f as it is a function of both drawdown and pumping rate; so the factor is not necessarily mutually exclusive, but with judicious use it provides an insight to the relative yields of two or more wells and probably has a use in comparison of the geological factors. A similar statement can be made for the tan Θ/CA_f (Appendix B).

The slope of the hydraulic grade line can be used to bring the observed data to standard conditions. Under a particular set of conditions the hydraulic grade line slope reflects the response of the hydraulic system to the pumping stress. The response is reflected in the well factor CA_f. If it is assumed that CA_f remains constant with a change in the slope of the hydraulic grade line, and if some standard drawdown is assumed (100 feet for this study), the calculations for the radius of the cone can be reversed. By this calculation the radius of a "standard cone" is determined. The ratio between the yield from the fractures at the drawdown S_{tx} and the radius of the standard cone can be taken as a characteristic of the well and its associated hydraulic system, and this coefficient can be used for comparing wells.

For the purposes of well comparison the hydraulic grade line slope must be standardized. This process may be accomplished by the assumption of a slope of 45° for the hydraulic grade line and the multiplication of the various factors that can be calculated for the standard conditions by the ratio of the tangent of 45° to the tangent of the slope angle of the hydraulic grade line, $\frac{\tan 45}{\tan \Theta}$.

The most useful factor for comparison of wells appears to be the ratio $\frac{Q_{fs}}{r_s}$ for the standard condition where Q_{fs} is the calculated flow through the fractures for a 45° slope of the hydraulic grade line and a standard drawdown of 100 feet,and r_s is the calculated radius of the standard cone based on a standard drawdown of 100 feet.

The ratio $\frac{Q_{fs}}{r_s}$ takes into account the different rates of pumping, the different drawdowns, and expresses on a standard basis the yields of the individual wells. This ratio appears to be definitive of the character of the individual well and to be repeated in an individual well where several pumping tests at various pumping rates and drawdowns were made. In the one well where the technique could be checked against an S_{tx} of 100 feet, the $\frac{Q_{fs}}{r_s}$ ratio calculated from other pumping rates and drawdowns for the same well compared favorably with the value obtained from the pumping rate and drawdown of 100 feet obtained in one test (Well III).

The yield per foot of drawdown under standard conditions can also be computed as may the slope of the semi-steady state portion of the curve for standard conditions (Appendix C).

In some cases the values obtained from the several calculations may be unrealistic insofar as possible actual well yields are concerned. This fact is related to the assumptions made for the standard conditions. In this investigation the standard conditions were taken as 1 per cent porosity and 100 feet of drawdown (S_{ts}) for the change from transient to semi-steady state conditions. A sample calculation is given in Appendix C.

The volumetric rate of flow from the fracture into the well bore at the instant of drawdown S_{tx} is Q_{fx}. This flow rate, determined from the pumping rate and from the rate of water level drop in the well, is converted through use of the well factor , CA_f, to a comparable inflow rate under standard conditions (Q_{fs}). The pumping rate is converted on the assumption that the drawdown within the well and pumping rate are straight line functions of one another under semi-steady state conditions. This is the assumption made in the evaluation of a well in terms of its specific capacities at different pumping rates.

APPLICATION OF THE THEORY

Once the behavior of the hydraulic system under one condition of stress is determined, three routes to further investigation of the well can be followed: (1) The stress can be removed and the system allowed to return to its initial conditions before additional stresses are placed upon it. (2) If the initial pumping rate was set low enough, the pumping rate can be increased in a series of steps, and a step-drawdown test made. (3) The information from one stress application can be utilized to predict the behavior of the well and to compare it with other wells. The logistic problems connected with the present investigation required the third approach.

Fourteen wells were pumped one or more times during the course of the investigation. Their locations are given in the table of Appendix D together with their geologic setting; their locations are plotted on the map of Figure 1. Table I summarizes the pertinent information obtained from these wells. Appendix E summarizes the technique used in data collection and gives diagrammatic examples of the curves used.

To demonstrate the possible use of the well factor and the ratio of $\frac{Q_{fs}}{r_s}$ for standard conditions, Figure 6 and Figure 7 are presented. In each figure rock type is plotted against the parameters which are used in describing the wells. Other diagrams and curves could be used to illustrate the relations of the yields to geologic factors.

No statistical evaluation of rock types, topographic position, soil thickness or other geologic factors involved in water yields are presented as the number of wells investigated is inadequate for such calculations.

The theory has been developed on the basis of a well intersecting a single, more or less uniform, fracture. Complications arise when the water is contained in a cavity which is not recharged at the pumping rate or the rate at which the water enters the well. Also, presence of more than one fracture system supplying water influences the relationships. If one fracture is completely independent of the others, its presence may be recognized by changes in the slope of the semi-steady state portion of the drawdown curve. In any event, semi-steady state conditions probably obtain before a fracture is reached by the water level so that even if there

TABLE I

Pump Test Data

Well No.	Test No.	S_{tx} (feet)	Q_p (gpm)	q_{fx}* (gal)	Q_{fx}* (gpm)	CA_f $\left[\frac{gpm}{ft/sec}\right]$	r 1% porosity (ft)	tan Θ	$\frac{\tan \Theta}{CA_f}$ $\left[\frac{ft/sec}{gpm}\right]$
I	1	7.25	18.8	769	18.7	.86	36.8	.197	.239
	2	7.75	21.2	614	21.1	.94	31.9	.245	.26
II	1	91	11.5	276	4.9	.064	12.9	7.04	.011
	2	93	11.1	736	4.4	.057	10.1	9.2	.016
III	1	110	14.2	55	.1	.012	.8	137	116×10^3
	2	94	4.3	76	1.8	.023	3.2	29.4	128×10^3
	3	102	8.7	33	1.0	.039	2.0	50	128×10^3
	4	100	7.5	39	1.2	.032	2.4	41.5	128×10^3
IV	1	132	29.2	229	18.1	.20	4.7	28.0	143
	2	136	21.5	457	18.2	.19	6.6	20.7	107
	3	142	28.0	289	19.2	.20	5.1	27.8	139
	5	110	27.5	368	11.2	.13	6.6	16.8	127
V	1	40.5	24	680	14.8	.29	14.8	2.7	9.3
	2	40	25.1	790	16.6	.33	15.7	2.5	7.6
VI	1	140	7.9	6	.6	.0063	.73	192	30.4×10^3
VII	1	20	9.3	497	9.2	.26	17.8	1.12	4.4
VIII	1	78	7.4	17	1.0	.14	1.65	46.6	3.28×10^3
IX	1	86	21.0	60	7.5	.10	2.98	23.8	236
	3	70	9.4	212	7.4	.11	5.2	13.4	134
	4	90	15.7	96	7.5	.10	3.3	27.0	274
X	1	56	10.3	123	5.3	.086	5.3	10.5	122
	2	63	20.6	45	7.3	.11	3.1	20.3	178
	3	65	20.2	39	7.1	.11	7.8	23.4	213
XI	1	164	25.2	300	17.0	.17	1.5	108	654
	2	160	25.2	224	17.3	.17	1.4	117	685
XII	1	25	28.6	6	2.5	.062	1.75	14.3	230
	2	28	25.7	14	2.5	.059	2.53	11.1	189
XIII	1	20	10	12	9.8	.27	2.8	7.2	26.3
XIV	1	30	10	12	.5	.011	2.1	14.0	1.23×10^3

* q_{fx} = total volume of water that has flowed into the well at the instant the drawdown reaches S_{tx}.

Q_{fx} = the volumetric flow rate from the fracture at the same instant.

TABLE I (Cont'd)

Pump Test Data

Well No.	Test No.	Q_{fc}* (gpm)	r_c* (ft)	$\frac{Q_{fs}*}{r_s}$ [gpm/ft]	$\frac{Q_{fc}*}{S_{ts}}$ [gpm/vertical ft]	$\frac{Q_{fx}}{S_{tx}}$ [gpm/vertical ft]	$\frac{S_{tx}}{q_{fx}}$ [ft/gal]
I	1	69.2	506	.70	.69	2.58	38.5
	2	75.5	407	.76	.76	2.72	28.9
II	1	5.14	14.2	.052	.051	.54	3.0
	2	4.56	10.9	.045	.046	.48	7.9
III	1	.95	.73	.01	.0095	.009	20
	2	1.7	3.4	.02	.017	.019	12.4
	3	3.12	2.0	.03	.031	.01	20.9
	4	1.2	2.4	.01	.012	.012	25.6
IV	1	15.8	3.6	.16	.16	.14	.58
	2	15.6	4.8	.16	.16	.14	.30
	3	16.1	3.6	.16	.16	.14	.49
	5	10.6	5.96	.11	.11	.10	.30
V	1	23.4	38.0	.23	.234	.37	.060
	2	26.3	29.7	.26	.263	.42	.051
VI	1	.51	.52	.0051	.0051	.0043	23.4
VII	1	20.5	89.4	.20	.21	.46	.040
VIII	1	1.1	2.2	.011	.011	.13	4.6
IX	1	8.1	4.2	.081	.081	.087	1.43
	3	8.9	7.5	.088	.089	.11	.095
	4	7.9	3.7	.079	.079	.084	.094
X	1	6.9	9.5	.069	.069	.095	.46
	2	9.2	4.9	.092	.092	.12	1.45
	3	8.9	4.3	.087	.089	.11	1.62
XI	1	13.2	.93	.13	.13	.10	.55
	2	13.7	.86	.14	.14	.11	.71
XII	1	5.0	7.0	.05	.05	.1	4.2
	2	4.7	9.0	.047	.05	.09	2.0
XIII	1	21.9	13.9	.218	.22	.49	1.67
XIV	1	.92	7.14	.13	.009	.017	2.5

* Q_{fc}, r_c, calculated on basis of 1% porosity and S_{tx} = 100 ft. Q_{fs}, r_s, are Q_{fc} and r_c corrected to a standard hydraulic gradient of 1/1 by using the ratio $\frac{\tan 45}{\tan \Theta}$; S_{ts} is the standard drawdown, S_{tx} = 100 ft.

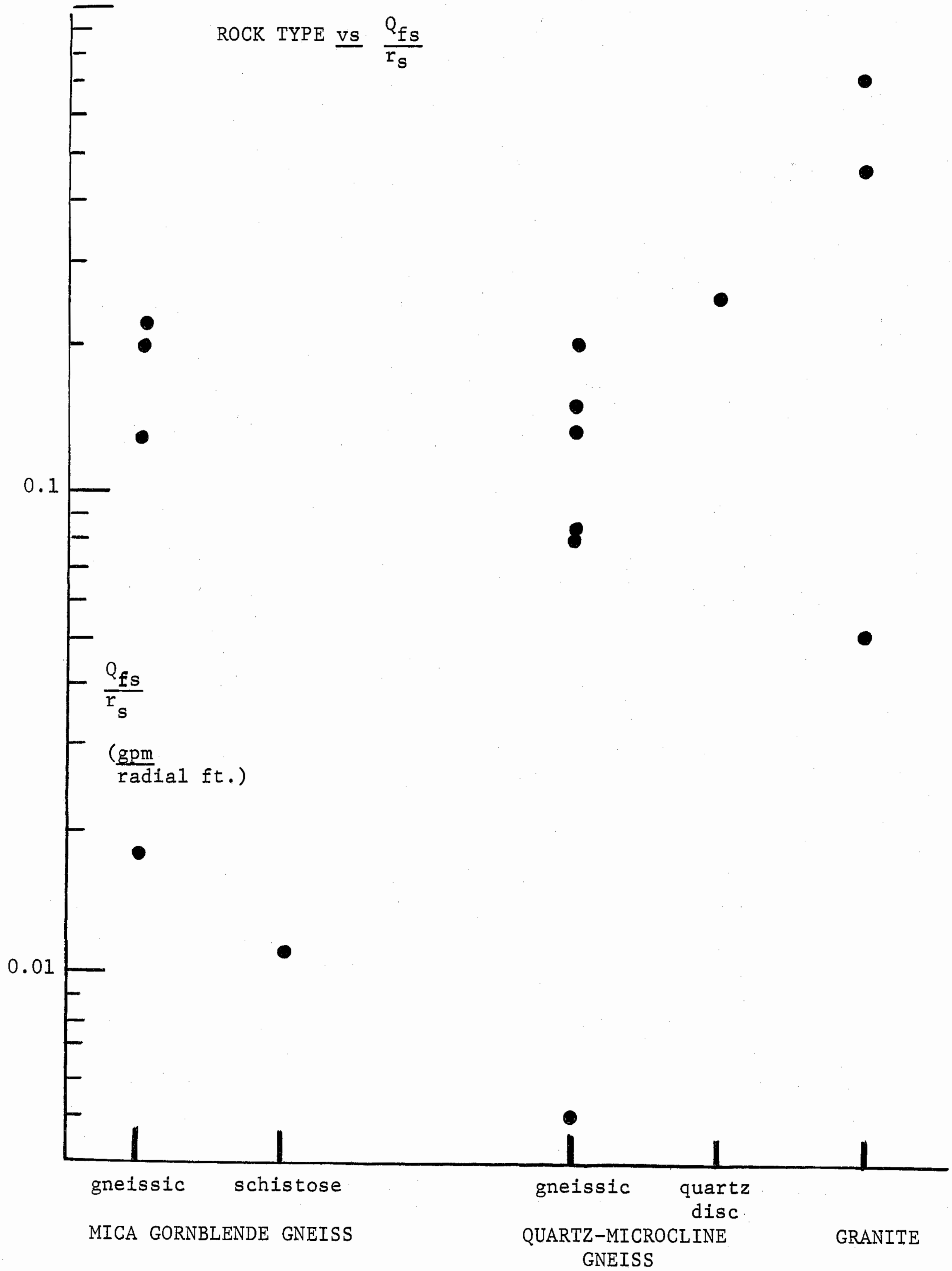

ROCK TYPE vs $\frac{Q_{fs}}{r_s}$
0.1
0.01
$\frac{Q_{fs}}{r_s}$
(gpm / radial ft.)
gneissic
schistose
gneissic
quartz disc
MICA GORNBLENDE GNEISS
QUARTZ-MICROCLINE GNEISS
GRANITE

FIG. 6

ROCK TYPE vs WELL FACTOR, CA_f

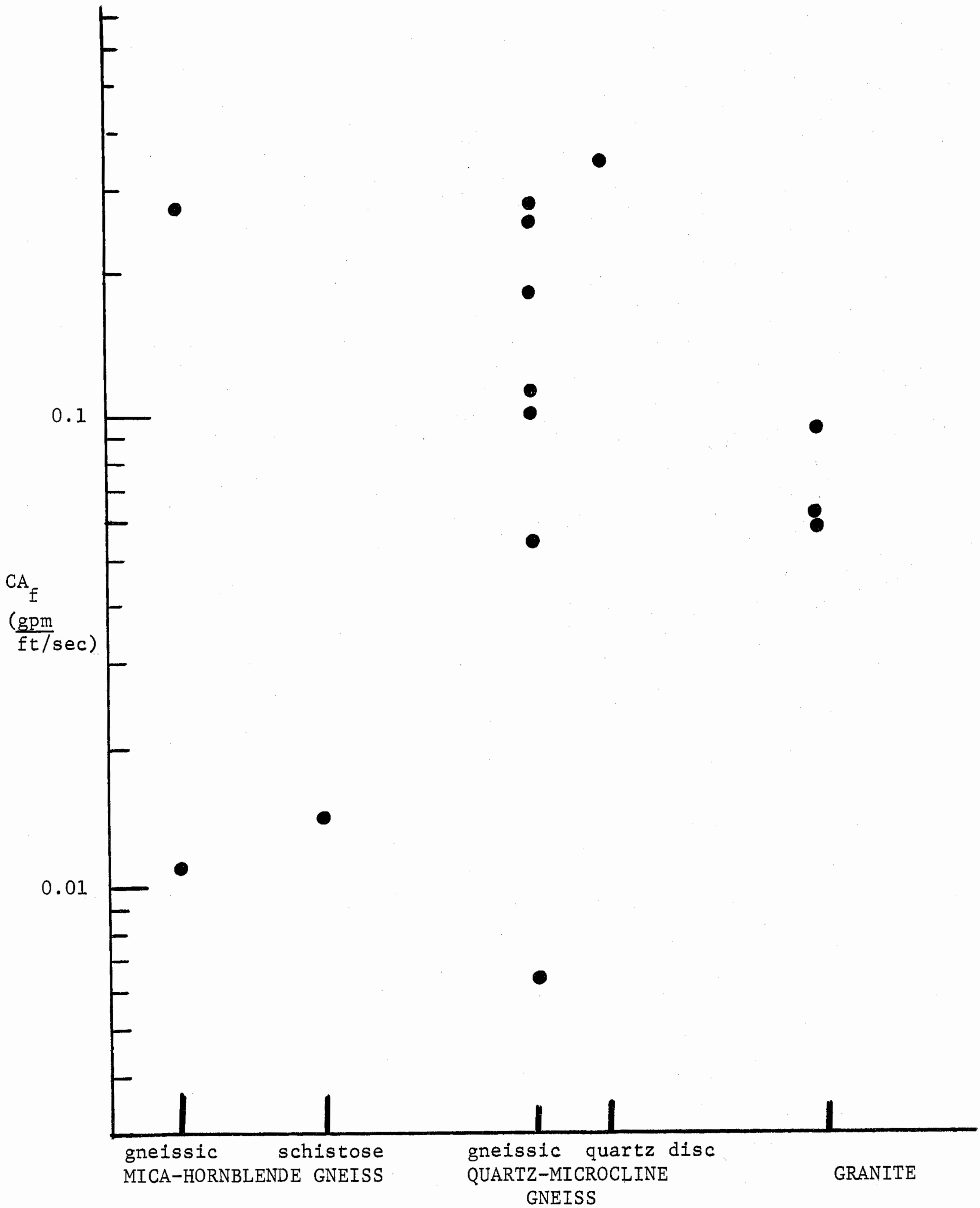


FIG. 7

is more than one fracture providing water, the changes of pressure, velocity, and slope of the hydraulic gradients are gradual and of a semi-steady state nature. For each well the evidence must be judged upon its own merits. In an idealized system without friction or other complications the drawdown for inception of semi-steady state conditions, S_{tx}, would be the drawdown to the single fracture of the model; however, in a real well S_{tx} probably represents a balancing of several influences on the system. Accordingly, it can be best viewed as an "effective S_f."

The graphs of the drawdown vs CA_f can be used to estimate the behavior of a given well under a variety of stresses. The curves of Figure 5 can be entered using the CA_f and the Q_{fx} determined from a pumping test. Estimates of other drawdowns required for a similar Q_{fx} can be made and the pumping rate required to achieve semi-steady state conditions of these drawdowns can be estimated from the relationships:

$$Q_p = Q_w + Q_f$$

$$Q_p = A_w \frac{dS_t}{dt} + CA_f \sqrt{2gS_{tx}} \ .$$

By the assumption of various values for $\frac{dS_t}{dt}$ one can calculate the slope of the drawdown curve at semi-steady state conditions, and the value of the pumping rate necessary to achieve the semi-steady state conditions for a given drawdown. Also, this calculation leads to estimates of the amount of water that can be withdrawn from the rocks at a given pumping rate, drawdown, and yield from the fractures. High pumping rates which give rise to steep drawdown curve slopes will not necessarily yield large total volumes of water.

The following calculation is presented to demonstrate the use of the curves of Figure 5. Assume a CA_f value of 0.09 $\frac{\text{gpm}}{\text{(ft/sec)}}$ and a Q_{fx} value of 5 gpm with a semi-steady state drawdown, S_{tx}, of 50 feet. Following the 5 gpm curve to a drawdown of 25 feet, one finds a CA_f value of $0.125 \ \frac{gpm}{ft/sec}$. At what rate should this well be pumped if a drawdown curve slope of 0.05 ft/min is desired? How much water can be expected from the fractures at this pumping rate assuming an infinite reservoir without geologic boundary conditions? Assume a 6-inch well.

If the well is to yield 5 gpm on a sustained basis (several hours), the pumping rate is determined as follows:

$$A_W \frac{dS_t}{dt} = 0.196 \ ft^2 \times 0.05 \ ft/m \times 7.48 \ gal/ft^3$$

$$= 0.073 \ gpm$$

$$Q_p = 0.073 + 5 = 5.07 \ gpm.$$

The amount of water to be obtained from the fracture is

$$5 \ gpm \times 60 \ min/hr = 300 \ gal/hr.$$

In one hour the drawdown would be three feet from 25 feet to 28 feet.

The exact behavior of the well and its associated hydraulic system during the transient stages can not be predicted using the CA_f factor. Thus the time at which the semi-steady state would be reached after pumping begins can not be predicted with great accuracy, yet knowledge of the pumping rate and the limits of the straight-line portion of the curve permit a reasonable estimate.

Obviously a well has a maximum yield from the fractures, depending upon the hydraulic relationships within the system. The curves of Figure 5 can be utilized to estimate the drawdown, slope, pumping rate, and maximum volumetric flow rate from the fractures.

RECOVERY CURVES

Recovery curves are an important means of evaluating the behavior of a well. In this study the theoretical analysis has been concentrated on the development of evaluation techniques utilizing the drawdown curves. Similar analyses have been made in a preliminary manner for the recovery curves. It appears that the point of maximum curvature on the recovery curve may be of significance. Taken together with the slope of the tangent to the curve at this point and the quantity of water removed from the system, the drawdown and time which define this point may provide useful information. This data probably can be used for comparing the yields of wells and their hydraulic systems on a standard basis.

The early part of the recovery curve represents transient conditions; the sharp curvature found on most curves represents a late transient stage. Although many recovery curves approach a straight line in the latter part of the recovery period, others are exponential in form. Hence it appears that an analysis of the recovery curve in the later transient part may provide important information about the behavior of the hydraulic system. An analogy between the well and a surge tank in a pipe system seems appropriate to the problem.

SUGGESTED FURTHER WORK

The approach to well yields described herein needs to be tested more fully than was possible in the present investigation. In particular, the various ratios and constants derived from the analysis of the pumping tests need to be related to the geologic factors.

Uses of recovery data in the evaluation of well yields with respect to the various geologic factors need to be studied further. The exact meaning of the recovery curves and their various parts in terms of local geologic factors as well as the hydraulic system should be investigated.

Carefully planned and executed studies of models might aid in interpreting the various pumping and recovery data.

ACKNOWLEDGMENTS

Appreciation for the cooperation and assistance in the project must be extended to Mr. Robert Marshburn and Mr. Clarence Ryals of Rawl's Pump Company, who made arrangements for access to several of the wells used in the study and discussed at considerable length some of the problems involved in it.

Professor John M. Parker III began the project, and his intimate knowledge of the geology of the Raleigh area has provided insight into the geological relationships.

Mr. P. M. Brown and Mr. Harry LeGrand of the U. S. Geological Survey have provided wise counsel and general assistance to the project. Their continuing interest in it have assisted greatly. Professor A. I. Kashef kindly discussed some of the ideas set forth in this report.

Finally, appreciation should be expressed to the approximately ten undergraduate and graduate students who assisted at various times in the project. They ran pumping tests and did much of the routine calculations and curve plotting.

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APPENDIX A

Calculation of Well Factor, CA_f

Based on data from Well IV

S_{tx} = 132 ft. (from drawdown curve)

$\frac{dS_t}{dt}$ = 7.4 ft/min (from drawdown curve)

Well diameter = 6 inches

$A_w = .196\ ft^2$

$$Q_w = A_w \frac{dS_t}{dt}$$

$$= .196 \times 7.4 = 1.44\ ft^3/min$$

$$= 11.1\ gal/min$$

Q_p = 29.2 gpm (from pumpimg rate)

$$Q_{fx} = Q_p - Q_w$$

$$= 29.2 - 11.1$$

$$= 18.1\ gpm$$

$$Q_{fx} = CA_f \sqrt{2gS_{tx}}$$

$$CA_f = \frac{18.1}{\sqrt{64.4 \times 132}}$$

$$CA_f = .196\ \frac{gpm}{ft/sec}$$

APPENDIX B

Sample Calculation of Theoretical Cone

Based on data from Well IV

Assumptions:

Effective Porosity = 1%

q_{fx} = total volume of water removed from fractures up to the time at which the drawdown is equal to S_{tx}; a measure of the effective porosity.

S_{tx} = drawdown at moment at which semi-steady state conditions obtain = height of the cone = 132 ft.

q_{fx} = 229 gallons. Determined from plots of drawdown vs q_p and drawdown vs q_w at drawdown S_{tx}.

1 ft^3 = 7.48 gallons.

Vol. of rock in standard cone = $q_{fx} \times 100 = 22.9 \times 10^3$ gallons.

V = 1/3 Ah (volume of cone)

$$A = \pi r^2$$

$$r^2 = \frac{22.9 \times 10^3 \times 3}{7.48 \times \pi \times S_{tx}}$$

$$r = \sqrt{\frac{22.9 \times 10^3 \times 3}{7.48 \times \pi \times 132}}$$

$$r = 4.71 \text{ ft.}$$

Θ = slope angle for side of cone and slope of hydraulic grade line

$$\tan \Theta = \frac{S_{tx}}{r} = \frac{132}{4.7}$$

$$\tan \Theta = 28.0$$

$$CA_f = 0.196 \ \frac{\text{gpm}}{\text{ft/sec}}$$ (from calculations given in Appendix A)

$$\frac{\tan \Theta}{CA_f} = \frac{28.0}{0.196}$$

$$= 143 \ \frac{\text{ft/sec}}{\text{gpm}}$$

APPENDIX C

Calculations for the Standard Cone

Based on data from Well IV. See footnote, Table I for meaning of subscripts of the symbols.

Standard drawdown, S_{ts} = 100 ft.

Standard effective porosity = 1%

S_{tx} = 132 ft. $CA_f = 0.196 \frac{gpm}{ft/sec}$ (Appendix A)

Q_p = 29.2 gpm

Q_{fx} = 18.1 gpm (Appendix A)

tan Θ = 28.0 (Appendix B)

$\tan \Theta = \frac{S_{tx}}{r}$; for standard cone; $S_{tx} = S_{ts}$ = 100 ft.

$$r_s = \frac{100}{28.0}$$

r_s = 3.56 ft.

Vol. of rock in Standard Cone:

$$V = \frac{\pi}{3} (3.56)^2 \times 100$$

$$= 1320 \text{ ft}^3$$

1% Porosity = 13.2 ft^3

= 98.7 gallons = q_{fx}

$$Q_f = CA_f \sqrt{2gS_t}$$

$$Q_{fs} = 0.196 \sqrt{2g \times 100}$$

= 15.75 gpm

$$Q_{ps} = 29.2 \times \frac{100}{132} = 22.2 \text{ gpm}$$

- continued -

Appendix C (cont'd)

$$Q_{ws} = 22.2 - 15.8$$

$$= \underline{6.4} \text{ gpm}$$

$$Q_{ws} = A_w \frac{dS_t}{dt}$$

$$\frac{dS_t}{dt} = \frac{6.4 \text{ gpm}}{1.5 \text{ g/ft}} = 4.26 \text{ ft/min}$$

This value is the theoretical semi-steady state slope of the drawdown curve under 100 ft. of drawdown and with a well factor, CA_f , of 0.196.

$$\frac{Q_{fs100}}{r_{s100}} = \frac{15.8}{3.56}$$

$$= 4.44 \frac{\text{gpm}}{\text{radial ft.}}$$

where $Q_{fs100} = Q_f$ corrected only for S_{tx} = 100 ft.

and r_{s100} = r corrected only for S_{tx} = 100 ft.

$$\frac{Q_{fs}}{r_s} = 4.44 \times \frac{\tan 45}{\tan \Theta}$$

$$= 0.159 \frac{\text{gpm}}{\text{radial ft.}}$$

$$\frac{Q_{fs}}{S_{ts}} = \frac{15.8}{100} \times \frac{\tan 45}{\tan \Theta}$$

$$= \frac{15.8}{100} \times \frac{1}{28.0} = 0.0565 \frac{\text{gpm}}{\text{vertical ft.}}$$

where $Q_{fs} = Q_f$ corrected for both standard drawdown and standard hydraulic grade line

and $S_{ts} = S_{tx}$ corrected for both standard drawdown and standard hydraulic grade line.

APPENDIX D

Well Locations and General Geology

Well No.	Location	Date Tested	Static Water Level, Ft. Below Casing Top	Well Depth (ft.)	Approx. Casing Depth (ft.)	Well Dia. (in.)	Approx. Pump Depth (ft.)
I	Camelot Development Lane of Sir Galahad near intersection with Trail of King Arthur; 0.5 mi. East of Wake Co. Rd. 2555, near Auburn	Apr-May, 1967	5	45	-	6	26
II	Camelot Development near intersection of Lane of Sir Galahad and Trail of Merlin; off Co. Road 2555, near Auburn	June, 1967	5	225±	-	6	140±
III	End of Rockwood Drive off Duraleigh Road, near Crabtree Creek	Jan., 1967	5	235	±50	6	220
IV	Oak Park, Holly Ridge Road, Lot No. 103, Supplies Community Water System	May-June,1966	2	200±	-	6-1/4	190
V	Oscar Miller Paving Co., Duraleigh Hwy. at Crabtree Creek	Nov., 1966	27	250	-	6	231
VI	Arbutus Drive Laurel Hills; House No. 4225 on Co. Road 1700	July, 1966	22	247	-	5-3/4	225
VII	Azalea Drive, Laurel Hills, second lot from intersection with Huckleberry (Lot No. 88)	June, 1966	84	260	6	6-1/4	255
VIII	Lake Ann Development, West of Raleigh, 1st House on lake side of Co. Road 1710	July, 1966	12	205	60	6-1/4	203

APPENDIX D (Cont'd)

Well Locations and General Geology

Well No.	Location	Date Tested	Static Water Level, Ft. Below Casing Top	Well Depth (ft.)	Approx. Casing Depth (ft.)	Well Dia. (in.)	Approx. Pump Depth (ft.)
IX	Heritage Acres Mobile Home Park, near junction of Co. Rd. 1382 and 1380	June, 1966	14	128	-	5-7/8	118
X	Oak Park, Cor. Weaver St. and Oak Park Rd., opposite Dundee Place; Supplies Community Water System	May, 1966	5	138	8	6	130
XI	Residence at 1019 Cowper Rd., Raleigh	June, 1966	6 (Water first reported at 210')	365	22	6-1/4	360
XII	Approx. 1 mi. west of Knightdale on Daniel Circle (Co. Rd. 2576), off Hodge Rd. (Co. Rd. 2516), open lot adjacent to Hocutt residence	Feb., 1966	8	100±	75	6	90
XIII	Northwest Corner Co. Rd. 1655 and 1757, near Medfield Estates, near northeast corner Cary	March, 1966	24	95	50	6	90±
XIV	County Road 1757, Second House west of creek (Lot No. 55)	March, 1966	43	165	45	6	150±

APPENDIX D (Cont'd)

Well No.	Rock Type	Topographic Relations	Soil and Saprolite Cover
I	Granite; cuttings show quartz vein	Moderate easterly slope from Rd. 2555, on side of small ravine	Clay-rich weathered granite; soil 5-10 ft. maximum thickness
II	Granite	Gentle broad southeasterly slope to major stream draining area; topographically about 30 ft. below I.	Sandy Loam; soil and saprolite probably 10-20 ft.
III	Quartz microcline gneiss	Steep westerly slope to tributary of Crabtree Creek; well located at base of slope	Thin clay-rich, probably over 5-10 ft. at base of slope; less than 5 ft. halfway up slope.
IV	Quartz microcline gneiss, irregularly fractured; quartz veins.	Well adjacent to small stream; local topography slopes to well; regionally located on major slope to tributary of Crabtree Creek	Soil and saprolite cover about 4 ft. Clay-rich with unweathered quartz and feldspar. Weathered bedrock crops out within 15 ft. of well site.
V	Quartz-microcline gneiss; quartz disc gneiss well-jointed	Moderately steep slope southeasterly to Crabtree Creek, about one-third way down slope. Topographic slope in general strike direction of foliation and lineation.	Soil and saprolite 1-5 ft. in places; at well site ±10 ft.
VI	Quartz microcline gneiss; possibly drilled into muscovite-garnet schist zone within the gneiss	Steep slope northward to Crabtree Creek. About one-third distance from crest of hill at Galax Dr. (Co. Rd. 1699) to Crabtree Creek, on east side of small ravine.	Less than 10 ft. Bedrock outcrops on hillside below house.
VII	Quartz-microcline gneiss, possibly in quartz disc variety	Steep northerly slope to Crabtree Creek. Lot located near top of hill. High regionally also.	Less than 10 ft. Weathered bedrock outcrops nearby clay-rich, but with relatively unweathered fragments of minerals.

APPENDIX D (Cont'd)

Well No.	Rock Type	Topographic Relations	Soil and Saprolite Cover
VIII	Mica-hornblende gneiss; schistose texture	Moderately steep hillside, about half way down slope to lake, and about 20 ft. above lake level.	Less than 10 ft.
IX	Quartz micro-cline gneiss, high mica content	Broad, gentle slope, near its base	Clay-rich, 5 feet estimated, saprolite 5 feet on average in area.
X	Quartz micro-cline gneiss.	Broad gentle slope to tributary to Crabtree Creek near base of north-westerly slope. Located on northerly slope of ridge on which Well _V_ is located on southerly slope.	Cover less than 10 ft. Bedrock outcrops nearby.
XI	Quartz micro-cline gneiss; highly micaceous, common quartz veins.	Built up area in Raleigh City limits; in low area adjacent to small stream; well about midway up gentle slope to creek.	Not estimated.
XII	Granite with quartz veinlets.	Hillside about 40 ft. below crest of ridge and about 300 yds. horizontally on broad gentle slope; adjacent to minor draw in slope.	Well-weathered medium grained granite; clay-rich; soil cover 5-10 ft. thick ridge crest.
XIII	Mica-hornblende gneiss	Westerly slope to stream in generally regionally high area. About 30 ft. above stream and half-way down slope.	Soil and saprolite cover 20 ft. although less on ridge to east; soil clay-rich.
XIV	Mica-hornblende gneiss; cuttings show gneissic texture	Near crest of ridge; drainage to E. and S. to stream separating this well from Well XIII.	Soil clay-rich; estimated to be 10 ft. with 10 ft. or more of saprolite.

APPENDIX E

The following paragraphs describe the technique used in making the pumping tests and in interpreting the data therefrom. Also included in the description are the typical curves plotted from the data and a brief explanation of their use.

Pumping Test

Some of the pumping tests were run with the project's pump, a one-horsepower Reda submersible pump. Other tests were run using pumps already installed.

Water levels were measured by an electric tape. During a test the electrodes of the tape were lowered below the water surface, and the time at which the circuit was broken was recorded. One-foot intervals were marked on the tape with paint, the five-and ten-foot intervals being marked in a color different from that of the other markings.

In the early stages of the test the measurements were made at one- and two-foot intervals. Once a semi-steady state of drawdown had been reached, the intervals were increased. In some wells in which the semi-steady state portion of the drawdown curve had a very low slope measurements were made at lesser intervals.

Volumes of water extracted from the well were metered with a Neptune Triseal one-inch split case meter reading in gallons. The flow rate from the well was controlled by a gate valve located between the wellhead and the meter. Since the technique requires a steady pumping rate, the control valve was set at a particular position and left unchanged. Readings of the meter were made at two-minute intervals in the early stages of the test

Appendix E (cont'd)

and at longer intervals in the later stages.

Data for the recovery curves were gathered in a manner similar to that for the pumping test. The electric tape was raised above the water level in one- or two-foot intervals, and the time at which the water level reached the tape was recorded.

The elapsed time from the beginning of the pumping test to the water level measurement, whether in the pumping test or in the recovery portion of the investigation, was designated as $\underline{t}$. The recovery time measured from the instant the pump was turned off was designated as $\underline{t'}$.

Computations

Computations involved the plotting of various curves. Chief among these are the drawdown curve and the recovery curve (Fig. E-1) plots of the difference between the static water level and the water level at a given time (drawdown) vs time. Plots of the drawdown and recovery curves were made using both arithmetic scales and semi-log paper. The most useful plot for these curves appears to be the semi-log plot of drawdown (S) vs time (t) measured in days. The pumping rate was determined graphically from the meter readings and the time of their measurement (Fig. E-2). The slope of this curve is Q_p.

Determination of S_{tx} requires study of the drawdown curve and the choice of the point at which the curvilinear portion of the curve changes to the straight portion representing the semi-steady state condition.

Other curves used in the interpretation of the data are plots of drawdown vs total gallons pumped (Fig. E-3) and drawdown vs gallons removed from the well bore alone (Fig. E-3). The drawdown for a given gallonage is determined by using the drawdown (Fig. E-1) and pumping rate curves (Fig. E-2).

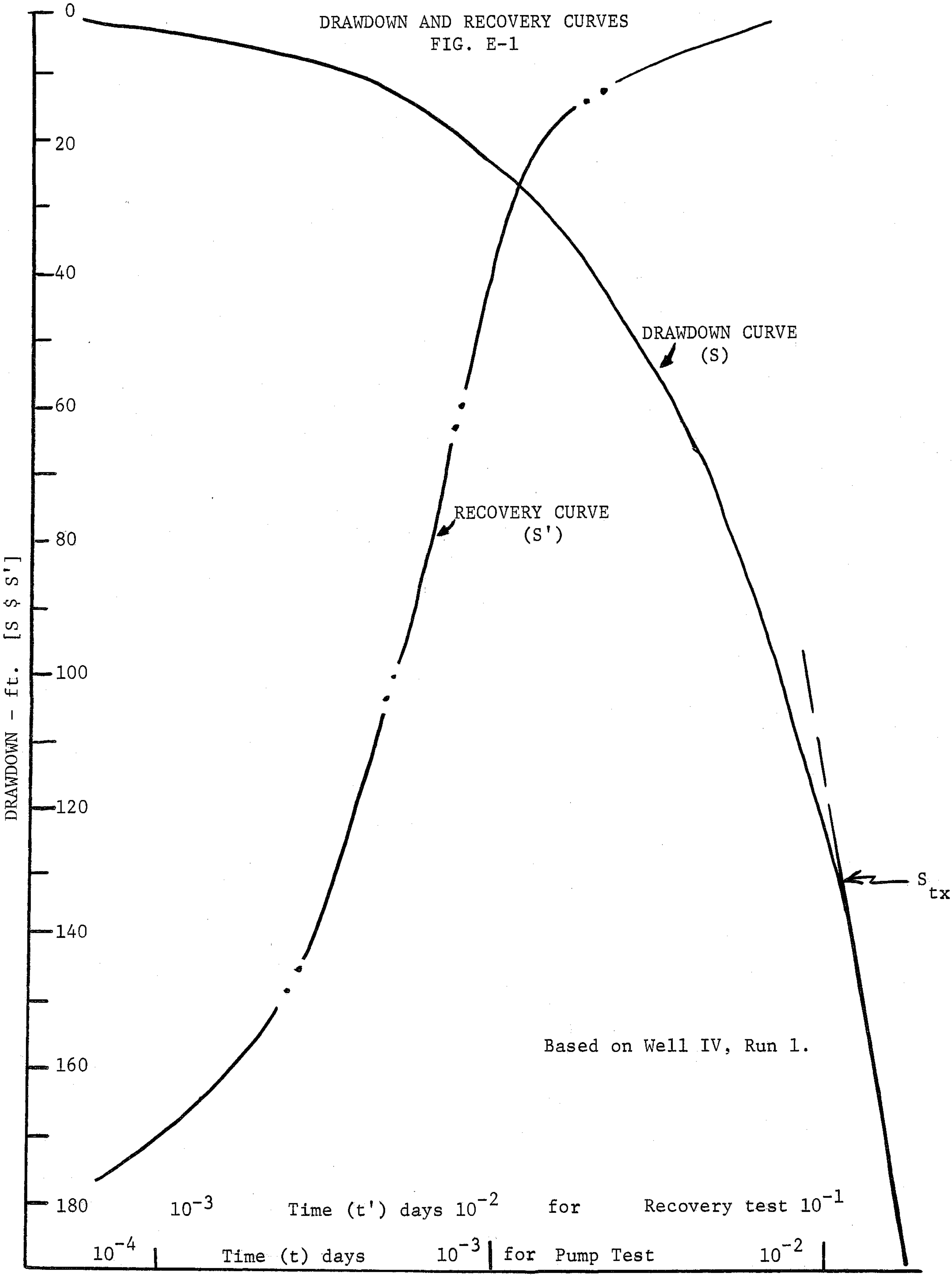
Appendix E (cont'd)

The difference at the drawdown S_{tx} between the two curves plotted in Fig. E-3 gives the number of gallons of water removed from the rocks (q_{fx}) up to the time of drawdown S_{tx}; q_{fx} is used in computing the radius of the standard cone, and it, of course, supplies information about the total number of gallons that the well can supply under a given set of pumping conditions.

A plot of gallons removed from the well <u>vs</u> time also proved helpful in understanding the hydraulic system of individual wells.

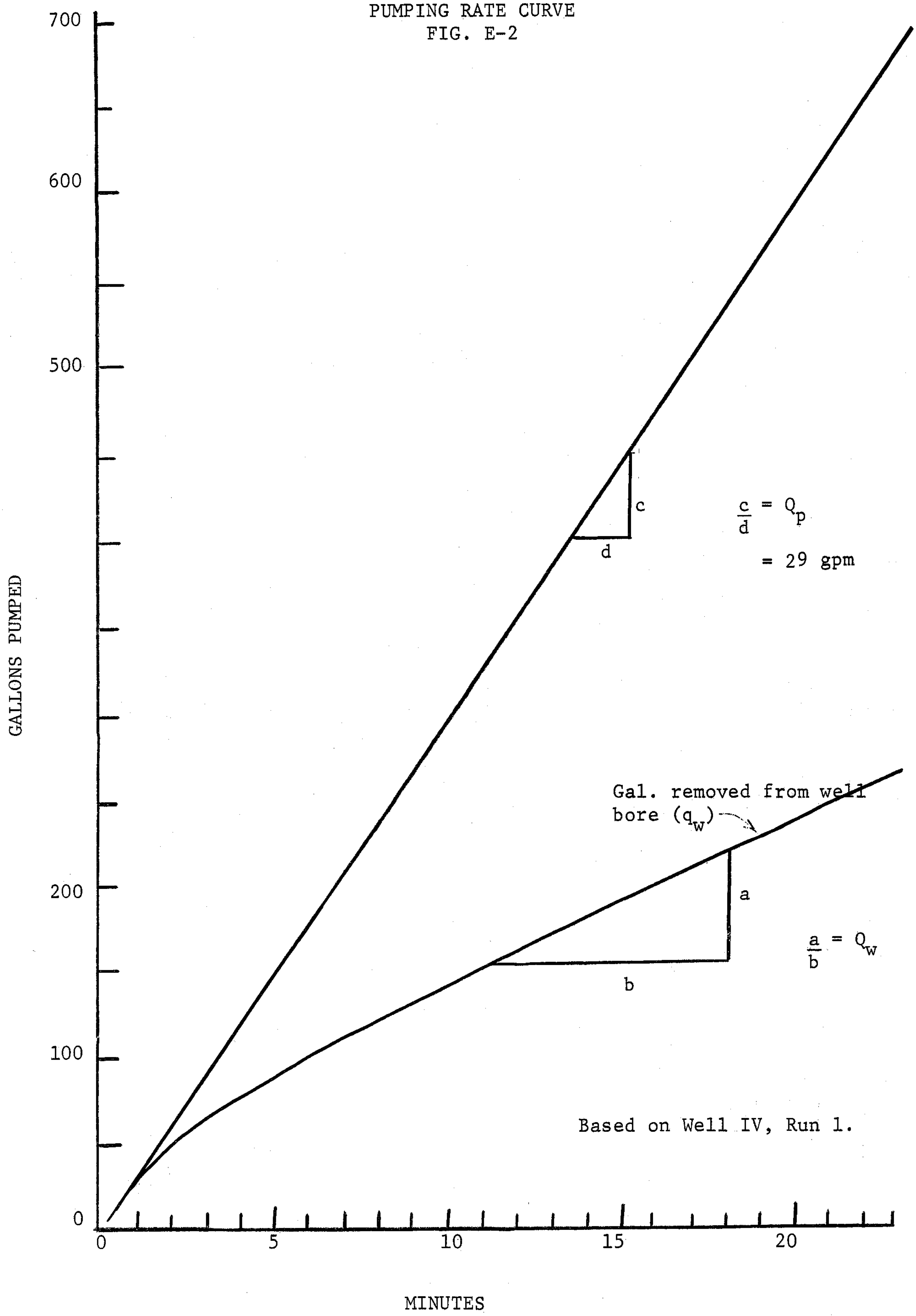
The recovery curve was plotted as residual drawdown (S') <u>vs</u> the time elapsed from the pump cutoff (t'). Also a plot of t/t' <u>vs</u> residual drawdown was prepared on semi-log paper (Fig. E-4). In all cases the drawdown was plotted on an arithmetic scale.

The other appendices show the calculations involved in using the several curves.



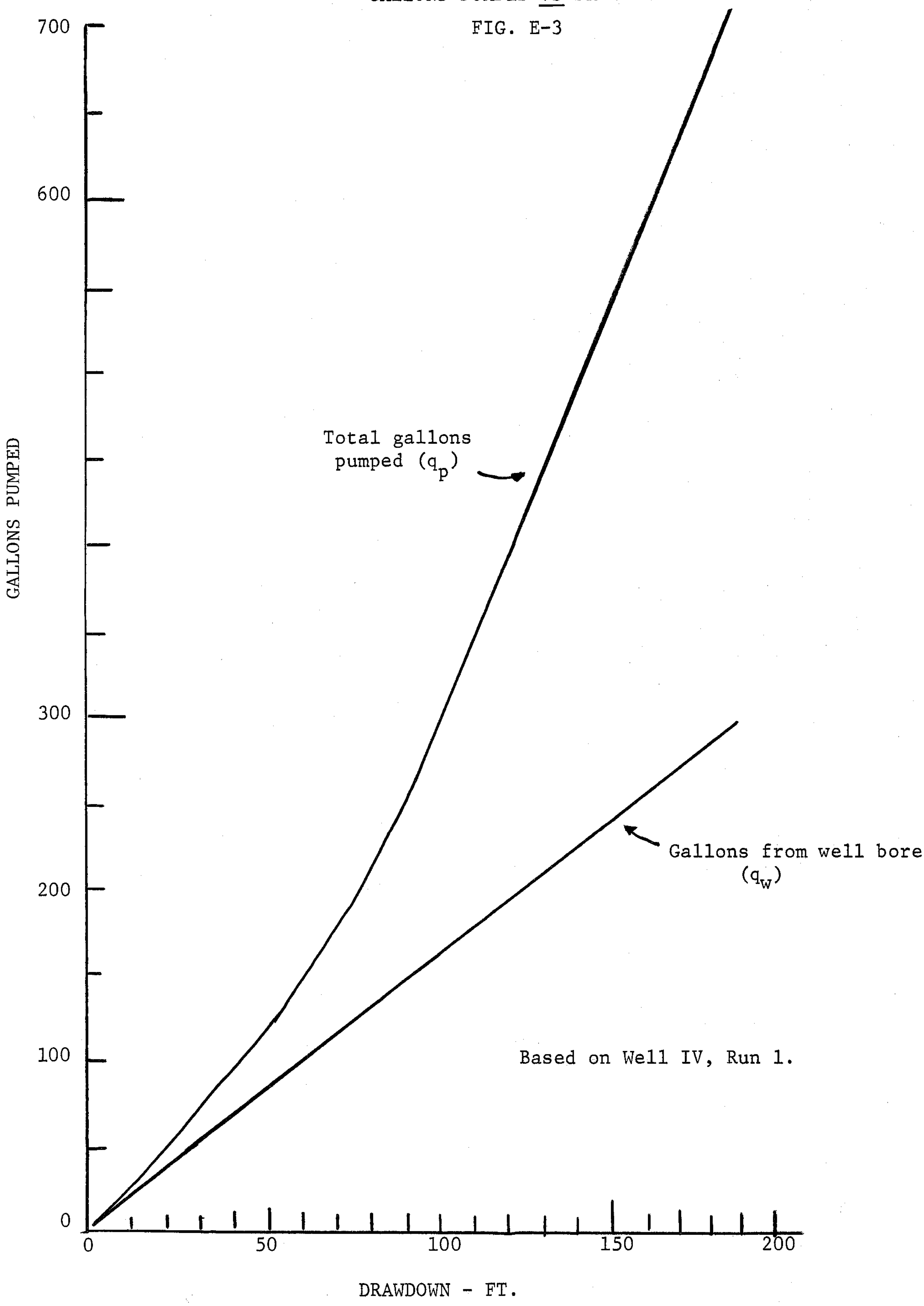
DRAWDOWN AND RECOVERY CURVES
FIG. E-1
DRAWDOWN CURVE (S)
RECOVERY CURVE (S')
S tx
Based on Well IV, Run 1.
DRAWDOWN - ft. [S $ S']
0
20
40
60
80
100
120
140
160
180
10-3 Time (t') days 10-2 for Recovery test 10-1
10-4 Time (t) days 10-3 for Pump Test 10-2

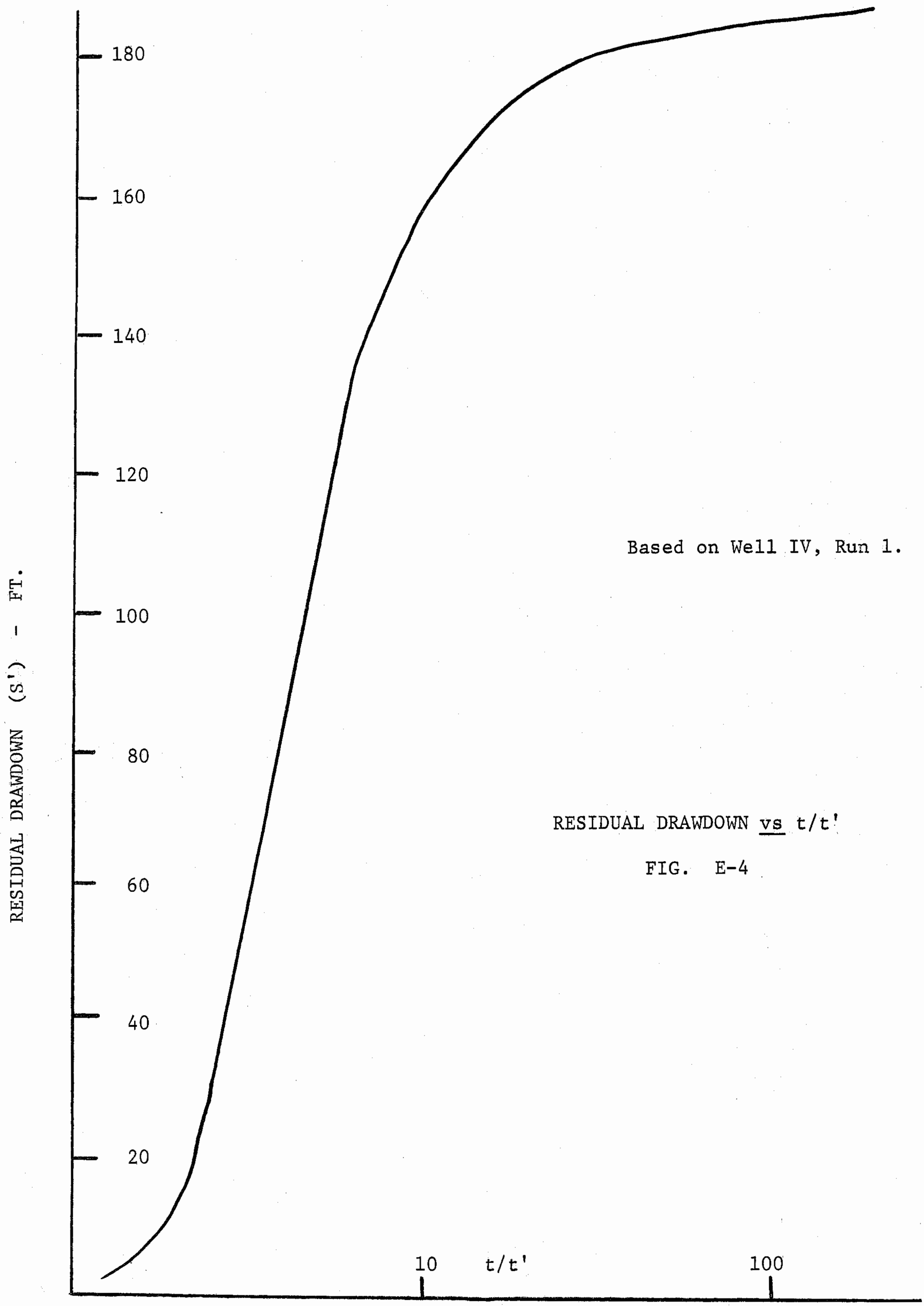
PUMPING RATE CURVE
FIG. E-2



GALLONS PUMPED *vs* DRAWDOWN

FIG. E-3





RESIDUAL DRAWDOWN vs t/t'

FIG. E-4

APPENDIX F

Symbols Used

A_f = Cross-sectional area of the fracture.

A_w = Cross-sectional area of the well bore.

C = Correction factor or coefficient applied to area of the fracture to compensate for various hydraulic factors in the hydraulic system.

g = Acceleration of gravity, ft/sec^2.

p = Total pressure.

P = External pressure head.

Q = Volumetric flow rate.

Q_f = Volumetric flow rate from fracture into well bore.

Q_p = Volumetric flow rate through the metering device; volume pumped per unit time.

Q_w = Volumetric flow rate from the well bore alone.

Q_o = Volumetric flow rate from the fracture at the time pumping starts.

Q_{fa} = Volumetric flow rate from the fracture at the instant the water level reaches the fracture during pumping test.

Q_{fb} = Volumetric flow rate from fracture into well with the water level in the well below the fracture.

Q_{fx} = Volumetric flow rate from the fracture into the well at the time corresponding to S_{tx} on the drawdown vs time plot.

q = Total volume of water

q_f = Total volume of water that has flowed from the fracture into the well up to the time t after the beginning of the pumping test.

q_p = Total volume of water pumped up to the time t after the beginning of the pumping test.

q_w = Total volume of water removed from well bore alone; the product of the cross-sectional area of the bore and the drawdown.

q_{fa} = Total volume of water that has flowed from the fracture between the beginning of pumping and the time at which the water level reaches the fracture.

Appendix F (cont'd)

q_{fb} = Total volume of water that has entered the well through the fracture between the time the water level passes the fracture and the time the water level reaches a drawdown S_t below the fracture.

q_{fx} = Total volume of water that has entered the well between the commencement of pumping and the time corresponding to S_{tx} on the drawdown vs time plot.

q_{pb} = Total volume of water pumped through the metering device from the instant the water level passes the fracture until the time of drawdown S_t below the fracture.

q_{wb} = Total volume of water removed from the well bore from the instant the water level passes the fracture until the time of the drawdown S_t below the fracture.

r = Radius of the base of the hypothetical cone.

r_c = Radius of the base of the hypothetical cone corrected to a standard drawdown of 100 feet.

r_s = Radius of the base of the hypothetical cone corrected to a standard of 100 feet and a standard 1/1 hydraulic gradient by use of the ratio $\frac{\tan 45}{\tan \Theta}$.

r_w = Radius of well bore.

S = The distance down the well bore from the static level of the water to the water level during pumping; the drawdown.

S' = Residual drawdown during a recovery test.

S_f = Drawdown to the fracture from the static level of the water in the well (water table).

S_t = Drawdown in the well at any given instant during a pumping test measured from the static level in the well (water table); drawdown at time t.

S_{tx} = Drawdown at which the plot of drawdown vs time reflects a change from transient conditions to semi-steady state conditions.

tan Θ = Slope of the sides of the theoretical cone used in the model.

t = Time; also elapsed time.

t' = Time elapsed between the time pump is shut off and the time a water level measurement is made during a recovery test.

t_o = Time at start of pumping test

t_b = Time at which the water level reaches a particular position, as defined by the drawdown S_t, below the fracture.

t_f = Time at which water level in the well reaches the fracture.

V = Velocity of water flow; distance per unit time.

V_f = Velocity of water flow from fracture into well.

V_{ft} = Velocity of water flow from fracture into well at time t.

V_w = Velocity at which water level in well drops during a pumping test.

z = Elevation above a datum.

SUBSCRIPTS

a signifies above the fracture.

b signifies below the fracture.

c signifies calculations corrected to a standard hydraulic gradient of 1/1 by using the ratio $\frac{\tan 45}{\tan \Theta}$.

f signifies fracture.

o signifies instant at which pump is turned on.

p signifies pump.

s signifies calculations at 1% porosity and S_{tx} = 100 feet.

T signifies totals for the whole pumping test.

t signifies a particular time or instant.

w signifies well.

x signifies the point on drawdown curve where the well and the associated hydraulic system passes from a transient condition to a semi-steady state.

UNITS

Units used in this investigation are as follows:

linear measurements in feet.

time measurement in minutes, seconds and on some graphs, days.

volumetric measurements in gallons.

volumetric flow rate measurements in gallons per minute.