ABSTRACT

CHILAMKURTI, YESASWI NARENDRA. Experimental and Computational Studies of Gravity-Driven Dense Granular Flows. (Under the direction of Richard Gould).

The current work focusses on the characterization of gravity-driven dry granular flows in cylindrical tubes. With a motive of using dense particulate media as heat transfer fluids (HTF), the study was primarily focused to address the characteristics of flow regimes with a packing fraction of $\sim 60\%$. Experiments were conducted to understand the effects of different flow parameters, including: tube radius, tube inclination, tube length and exit diameter. These studies were conducted on two types of spherical particles – glass and ceramic – with mean diameters of $150\ \mu\text{m}$ and $300\ \mu\text{m}$ respectively. The experimental data was correlated with the semi-empirical equation based on Beverloo's law. In addition, the same flow configuration was studied through three-dimensional computer simulations by implementing the Discrete Element Method for the Lagrangian modelling of particles. A soft-particle formulation was used with Hertz-Mindilin contact models to resolve the interaction forces between particles. The simulation results were used to examine the velocity, shear rate and packing fraction profiles to study the detailed flow dynamics. Curve-fits were developed for the mean velocity profiles which could be used in developing hydrodynamic analogies for granular flows. In addition, the particle-wall contact behavior was also studied to characterize the heat transfer from the wall to the granular flow. Finally, the fluctuations inside the flow were also studied using computer simulations and their dependency on the tube length was characterized. Thus the basic features of gravity driven dense granular flows were identified to form a basis for defining their rheology.

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Experimental and Computational Studies of Gravity-Driven Dense Granular Flows

by
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DEDICATION

To my parents.

BIOGRAPHY

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Chapter 1

Introduction

With the increasing attention on using granular flows as a potential high temperature heat transfer fluid [1]–[3], a careful study of the flow physics is required before implementing them on an industrial scale. Use of flowing rigid particles, as opposed to the current heat transfer fluids like steam and molten salts, could increase the working temperature range of the heat transfer systems and open new opportunities for higher efficiency thermodynamic cycles. Thus, applications that rely on direct/indirect absorption of heat energy, for example concentrated solar power (CSP), may use dry particles for heat transfer and thermal storage [4].

Though direct absorption systems are relatively easy to operate (and have high heat flux density in case of CSPs), the convective losses and flow instabilities make it difficult to control [5]. On the other hand, indirect heat absorption systems offer better control of particulate flows making it preferable for industrial applications [6], [7]. Additionally, having dilute packing fractions could result in poor thermal behavior owing to the interstitial gasses. As a result, the current work primarily addresses the important characteristics of dense flow regimes in cylindrical tubes.

From the study of static granular beds to the modelling of particulate collisions, our understanding of granular physics has increased steadily [8]–[16]. With the help of high-speed imagery and other non-intrusive techniques like PIV, X-ray, MRI, and ECT, several intrinsic features were observed by the research community over the years. In addition, computational methodologies like Molecular Dynamics and Discrete Element Analyses enabled the investigation of these flows in an explicit manner. However, the flow characteristics vary from configuration to configuration. In fact, every geometrical configuration has different flow regimes, each of which having different governing physics. As a result, the most common approach in understanding the flow rheology of granular media is by studying individual configurations and extracting their intrinsic features.

The behavior of dense granular flows is highly sensitive to different geometrical parameters, loading conditions and particle properties. Based on the particulate velocities, granular flows are broadly divided into three types – quasi-static flows, dense flows and dilute collisional flows. In the quasi-static regime, the inertia of individual particles becomes negligible and the granular media behaves like a solid. This tightly packed regime ($\sim\phi_{\max}$), which is governed by particle properties like friction, elasticity, shape etc., can be modelled using plasticity models of soil mechanics [11]. The dilute collisional regime, on the other hand, is mainly governed by the inertia of the particles. This gas-like regime can be modelled using analogies of the kinetic theory of gases [17] as the particles are strongly agitated and the momentum transfer is mainly through collisions. Between these two, lies the dense collisional flow regime which has a fluid-like rheology with non-negligible particulate inertia. Despite the relatively high velocities, the particles maintain a continuous contact network [14], [18]

propagating the stresses layer by layer. All these regimes have different flow physics and as a result, no universal framework was developed to describe the whole range from solid-like quasi-static to gas-like collisional regimes. Hence, as mentioned earlier, the most convenient way of describing granular flows is by studying individual regimes and individual configurations and understanding their distinctive features separately.

Since we are currently interested in indirect heat absorption, the current work focuses on confined granular flows that are primarily characterized by shearing at the boundaries. These configurations can be represented by the non-dimensional inertial number I , which describes the relative importance of particulate inertia to the confining stresses in granular flows. It is the characteristic shear rate scaled by different time scales for different flow configurations [16], [18], [19]. From the definition, an increasing inertial number indicates an increasing deformation tendency of the particulate assemblies. In general, the granular flow lies in quasi-static regimes for $0 < I < 10^{-3}$ and in dense regimes for the ranges of $10^{-3} < I < 1$. On the other hand, the collisional regimes exist for $I > 1$. Though these limits are not universal and vary for different flow configurations, a qualitative understanding of the flow can be obtained using these limits.

Typically, the heat transfer coefficients of granular heat transfer fluids (HTFs) increase with increasing flow rates[1][20]. Similar trends are also observed with increasing packing fractions within certain limits of temperature [7], [21]. As a result, the current work primarily focusses on studies with higher packing fractions (60%) and relatively faster flows rates. Conveying of particle suspensions can be achieved with both pneumatically driven measures as well as body forces. But using high pressure air to move the dense suspensions may lead to

air voids and bubbles through the flow which would hinder the heat transfer phenomenon. As a result, the current work primarily relies on gravity-driven configurations for achieving dense flows.

Gravity driven flows have been studied over the years both experimentally and computationally [16], [22]. But a majority of these studies focused on quasi-static or dilute regimes. This is because most of the existing industrial applications and flow configurations fall in these regimes. Though several researchers worked in the transitional dense regime, the complexity associated with this regime has made its complete understanding a challenge [23]. Constitutive relations were developed to describe the stress-strain relations of this regime over the years. But a universal model was never developed and each of them were specifically tuned to their own flow/geometrical configurations. Moreover, observing dense configurations in cylindrical tubes is another challenge. The opaque nature of the flow would make high speed imaging techniques ineffective in studying the velocity and shear profiles. Unlike chute flows, the opaque and three-dimensional nature of tube flows makes it difficult to analyze the internal profile using flow visualization techniques. Though high speed visualization could be attempted in a transparent tube, it would still be insufficient in predicting the internal flow structure to analyze the velocity and shear profiles. In addition, non-intrusive techniques like ECT, X-ray Imaging, etc. prove ineffective owing to the high packing fractions of the flow [19]. Hence, the current research attempts to understand the different flow phenomena of the dense flow regime in cylindrical tubes using both experimental and computational techniques.

A bench-scale experimental setup was developed to measure the room temperature particulate flow rates in a vertical tube. The setup was designed to maintain a dense packing

fraction inside the tube. Preliminary studies were conducted to analyze the influence of several geometrical parameters of the flow. These results matched very well with those cited in the literature [9] and also enabled the observation of several flow phenomena. Later, the computational studies were conducted to observe the intrinsic features of the flow. The particles were modeled with a Lagrangian approach by considering Discrete Element Methods (DEM) [10]. Validated with the experimental results, the simulations studies helped us in gaining further understanding of the internal flow rheology. In addition to flow characteristics like velocity profiles, wall normal-shear stress profiles, flow rates and particle-wall contact behavior was also analyzed in the current study.

Chapter 2

Experimental Setup

The experimental setup was primarily designed to study the variation of flow rates with several geometrical parameters. Though it did not give a clear understanding of the internal flow configuration, the insights obtained from the experimental studies laid the foundation for the entire research work. The following sections describes the experimental design, the experimental methodology and the different cases studied.

2.1 Design

The experimental setup was designed to run in batches. Though conveyor setup was initially planned to be installed to maintain a continuous flow, the current setup, being a preliminary study was just designed in a simplistic manner. It includes a long vertical tube with a flowrate controlling orifice at the bottom end. To maintain the batch-run configuration, the setup was designed with a funnel at the top. The particles are fed into the funnel and they flow into the pipe. Instead of using a flat-bottomed opening, a conical funnel was designed with a divergence angle greater than the angle of repose of the particles (~26%). This was to make sure that there were no stagnant particles inside the funnel. The funnel was made of high

grade stainless steel and was erected vertically with four supports on the sides. The funnel setup can be observed in figure 2.1. Since the tube diameters in the experiments were smaller than the funnel exit, a flow reducer was attached.

Figure 2.1 Funnel assembly in the experimental setup

The flow reducer at the end of the funnel was attached to a flow control valve. The cylindrical tube attached downstream of the valve with the help of compression fitting. The experimental setup was also designed to understand the influence of tube's inclination on the flow. For that case, a 45° elbow fitting was attached to the exit of the funnel followed by a flow reduced valve. The setup can be clearly observed in figure 2.1. The tubes are made of stainless steel. Flow rates through the tube were controlled by creating a constriction at the end of the tube. This was done by attaching orifices fixtures of different diameters, D_o , at the end

of the tube using compression fitting. They act as a flow reducer at the exit and control the particles flowing in the tube. They are also made of stainless steel. The overall experimental setup can be observed in the figure 2.2. To make it rigid and sturdy, the entire setup was mounted on a frame.

Figure 2.2 Overall Experimental setup for vertical tube and inclined tube (left to right)

2.2 Methodology

A single large quantity of granular material was loaded in the hopper (funnel) and the experiment is carried on until the batch was complete. Before the experiment was started, the bottom exit of the tube (orifice opening) was sealed and the particles are allowed to fill the

tube completely. This was done to make sure that the flow configuration remained dense from the initial stages of the experiment. Once the tube was filled, the orifice seal was removed and the granular material started flowing. The entire flow process can be clearly understood from the experimental schematic in figure 2.3.

Figure 2.3 Schematic of the experimental setup

Starting from the batch funnel/hopper the particles flow downward due to gravitational force acting on them. These particles, which get slowed down at the orifice exit and leave the tube, were collected in a basin that rests on an electronic mass scale (Adventurer™ Pro – AV2102). This was connected to a computer and a temporal evolution of the mass flow rate measurements were recorded throughout the experimental batch. Similar setup was also adopted for the inclined tube experiments

Since the entire setup was just designed to measure the flow rates, another mechanism/setup was needed to understand the flow pattern. The opaqueness of the stainless steel tubes makes it difficult to observe the same. Hence another setup was also made with a glass tube. The subsequent section gives details about the different experimental cases that were performed in the study.

2.3 Run cases

As mentioned earlier, the experimental setup was designed to understand the influence of different geometrical parameters on the flow rates. So, the different cases were run with different orifice diameters, different tube lengths, different tube diameters and different tube inclinations. Every experimental cases was run 5 times to avoid any errors in the obtained data. Once the data is obtained, the mean values for each reading were used for further analysis.

Two tube diameters were considered in the experimental study. With a thickness of 0.9525 mm, inner diameters (D_t) of 7.747 mm and 10.922 mm were used for the experimentation. Each tube diameter was tested at three different heights (L_t). They are 1.82 m (6 feet), 1.22 m (4 feet) and 0.61 m (3 feet). In addition to the vertical alignment, experiments were also

conducted with a tube inclination of 45° from the horizontal. Each geometrical configuration was tested with six orifice fixtures. The area ratios of these orifice fixtures, $A_{\text{exit}}/A_{\text{tube}} = (D_o/D_t)^2$, are 0.2, 0.25, 0.35, 0.5, 0.65 and 0.75. Preliminary experiments were conducted using glass particles of $150\ \mu\text{m}$ in diameters. But since, ceramic particles have good thermal properties and they are the main pHTF (particle heat transfer fluid) that needed to be studied in the research, further experimentation was done with them. The average diameters of these ceramic particles was $270\ \mu\text{m}$.

The subsequent chapter includes the discussions about different observations that were made in the experiments and the analyses that were done with the obtained data.

Chapter 3

Experimental Results

The following sections describe the different results obtained in the experiments and the observations made from them. The first and foremost discussion is about the influence of orifice diameter on the flow rates. Later observations about the influence of tube length, tube diameter and tube inclination were discussed.

3.1 Variation of flow rates with orifice diameters

The flow rates remained constant throughout the experimental batch with very minor fluctuations owing to the discrete nature of granular flows. Even though the height of the particle bed in the funnel was decreasing with time, the flow rates remained constant resembling a ‘choked’ flow at the tube exit. This contradicts to the observations one would make in continuum flows. As a result, though the experiments were done in batches, it remained like a steady state flow. From the observations made through the transparent glass tubes, the flow looked steady and the particles moved at a constant speed. There was a sense of intermittency in the flow that was negligible to observe. This could possibly be due to the discrete nature of granular flows. The smooth flow (white stream of particles) can be observed

in the figure 3.1. It can be observed that the flow resembles a continuous flow with steady stream of particle flowing out of the orifice.

Figure 3.1 Flow through the transparent tube resembling a continuum fluid

It was observed that with increasing orifice size, the flow rate increased. This was observed for all the tube diameters, tube lengths and particle types. In figure 3.2, the variation of mass flow rates with orifice diameter is plotted for different tube radii and particle types. It was observed that the experimental data followed the semi-empirical Beverloo correlation [24] very well. It is defined as:

$$\dot{M} = C\rho_b\sqrt{g}(D_o - kD_p)^{5/2} \quad (1)$$

It states that the mass flow rate $\dot{M}$ of a free flowing granular material through a circular orifice of diameter D_o is a function of orifice diameter, particle diameter D_p , bulk density of

the material ρ_b , and the acceleration due to gravity g . In equation 1, C and k are the empirical parameters that depend on the particle shape and friction[9]. This relation was obtained after extensive studies done by several research communities over the years. It was suggested that a free-fall dome is formed right about the exit of the orifice. It forms an arch shaped structure that encloses the exit. Above the arch, the grain are well-packed and the velocities are less. And on the other hand, below the arch, the particle accelerate freely under the influence of gravity. The characteristic size of this free-fall arch/dome proportional to the diameter of the orifice, D_o . As a result, the velocity of the particles at the exit would be proportional to $D_o^{1/2}$ (Velocity of a free fall object from height H is $\sqrt{2gH}$). Hence the flow rate of particles is proportional to $D_o^{5/2}$.

Figure 3.2 Variation of mass flow rates with orifice diameter

From Figure 3.2, it can also be observed that for larger orifice diameters, the flow rates suddenly increase to a high value and no longer follow the Beverloo Correlation curve. At this point, it was also observed that the granular flow loses the continuum nature inside the tube. The particle flowrates measured from the mass scales fluctuated with large magnitudes. In addition, when observed using the transparent tubes, the flow started to have air spaces and voids propagating through them. Thus, at $D_o = 7$ mm and 10 mm for the 8 mm and 11 mm tube diameters respectively, there is a transition from dense flows to dilute flows inside the tube with a “un-choking” phenomenon at the orifice. The take-off point at which the transition occurs is different for different tube diameters but similar for both the types of particles. Similar observations were made in other works [25], [26]. It was suggested by Huang et al. that the free-fall arch at the orifice usually dissipates the kinetic energy of oncoming particles and slow them down. It is then, these particles which undergo a free-fall and follow the Beverloo correlation. But if the kinetic energy of the granular media increases beyond a certain limit, its dissipation mechanism becomes weak and the free-fall dome disintegrates[22].

It can be easily understood that a dense flow configuration can be maintained inside the tube if the incoming flow rate is equal or greater than the outgoing flow rate. This means, the particle flow rate from the funnel exit at the top of the tube should be greater than or equal to the flow rate outside the orifice at all times. At this point, it should be noted that the funnel also behaves as an orifice. As a result, if the exit of the funnel is free (No particle accumulation), the Beverloo Correlation will also be valid for it with a free-fall dome at the exit. Since the diameter of the funnel is large, for a specific flow rate, the velocities of particles inside them would be low. As a result, their kinetic energy can easily be dissipated and the

flow will always be in a choked conditions if there is a free opening at the funnel exit. If there is no free opening at the funnel exit, the flow rate would simply be equal to the flow rate in the tube. Hence the maximum and minimum limits of particulate flow rates coming from the funnel are the Beverloo flow rate dictated by the exit orifice and tube flow rate. On the other hand, since the diameter of the tube is small, for a specific flow rate, the particles have high velocities. And if the flow rates are high, the higher particulate velocities and their kinetic energies may not be dissipated at the orifice exit. This might lead to the disintegration of the free-fall dome/arch at the orifice and thus the un-choking phenomenon. Now, if the flow is choked at the orifice exit, there will always be a dense flow configuration inside the tube as the orifice flow rate will always be equal to the flow rate from the funnel (can be understood from the limits mentioned above). As a result, it can be understood that to have a dense configuration inside the tube, the flow should always be choked at the orifice exit. This primarily lets us understand that the flow rates of granular flows, when used as HTFs, cannot be increased continuously despite of the increasing heat transfer coefficients[20] as it would lead to dilute flow configurations.

3.2 Influence of tube length, diameter and inclination on the flow

Figure 3.2 also allows us to understand that the particle size has no influence on the overall flow dynamics for our geometry as both ceramic and glass, which have different sizes, resulted in similar behavior. It can also be observed that the tube radius has no effect on the mass flow rate when in choked/dense regime, but influences the trends in the un-choked/dilute regime. In Figure 3.3, the variation of mass flow rates with orifice diameters are plotted for different tube

lengths and also for an inclined configuration. As suggested by the Beverloo correlation, it can be seen that the tube length has no effect on the mass flow rate when in the choked regime. On the other hand, the flow rates increased with increasing tube length in the dilute regime. For the inclined tubes, though the overall trend agreed with the Beverloo correlation, however the mass flow rates are less than with the vertical tubes. In this configuration, the effective gravity along the tube would be less. In addition, there would be a skewed free-fall arch at the orifice exit that could result in different values for C and k . More study needs to be done to obtain a clearer understanding of this phenomenon.

Figure 3.3 Flow behavior for different tube lengths and inclinations

Chapter 4

Simulation Setup

The multiphase modelling of dry granular flows deals with the exchange of momentum between the continuum and dispersed phases – air and particles. This was resolved by implementing a Lagrangian method. In dense flow regimes, due to the high packing fractions of the dispersed phase (~60%), the inter-particulate interactions become frequent and accounting for their behavior is necessary to resolve the flow state. Though the continuum can be solved using the conventional Navier-Stokes equations, the collisional and contact interactions of the particles among themselves and with other boundaries need to be modelled for a precise simulation of the flow. This was achieved by considering the Discrete Element Method (DEM) which computes contact forces for each and every particle. The following sections give a complete picture of how the simulation modelling was done in this research.

4.1 Discrete Element Method

The Lagrangian Multiphase model solves the equation of motion for representative parcels of the dispersed phase as they pass through the solution domain. It is primarily suited for systems that consist mainly of a single continuous phase carrying a small volume of discrete

particles, droplets, or bubbles[27]. Using this approach is the best way to resolve interactions of the discrete phase with physical boundaries.

The Discrete Element Model, is an extension of the Lagrangian Multiphase approach. In this approach the inter-particulate contact forces are explicitly accounted. This numerical approach is typically suited for simulating motion of many interacting discrete objects that are usually solid particles. Though DEM modelling requires significant computing power, it provides a detailed resolution that other approaches cannot achieve. This model was primarily established by Cundall and Strack [10], where the inter-particle contact forces are included in the equation of motion.

The application of DEM can be broadly classified into two types – hard-particle and soft-particle approaches. While the former is suitable for rapid granular flows, the latter which allows multiple and enduring contacts between particles is appropriate for dense flows [28]. Thus the soft-particle approach was chosen in the current study. Here the term “soft-particle” refers to the fact that the particles can deform during a contact. But calculating the structural deformation of each and every particles in the simulation would be computationally intensive. To avoid that, each particle is considered to remain geometrically rigid but are allowed to overlap with contact boundaries. The amount of overlap is considered analogous to the deformation at the contact boundary and the interaction forces are computed from the amount of overlap using different contact models. Since the deformations of the individual particles are relatively small in comparison with the deformation of the granular assembly (which is primarily due to the movements of the particles as rigid bodies) as a whole, this assumption will not lead to un-realistic solution states. Hence the soft-particle approach can give a good

representation of the mechanical behavior of the system. The contact duration is finite and the multiple contacts may occur simultaneously in this approach. The soft-particle approach is a more flexible method as compared to the hard-particle method because of the variety of force models and particle shapes that can be accounted for. But on the other hand, it is typically a more time consuming approach due to necessary small integration time steps

As a whole, DEM includes three steps – 1) contact and overlap detection, 2) force calculation and 3) particulate motion. These individual computations are done continuously to update the state of the particles (position and velocity vector of its center) and the flow. The calculations performed in the discrete element method alternate between the application of contact-force laws and the Newtonian displacement laws.

The solution state of each particle in the DEM methodology includes the particle centroid location, particle radius, particle velocity and the forces acting on each particle. The particle-particle overlap is calculated using the particle centroid locations. The following sections describe the governing equations involved in calculating the contact forces and the resulting particulate motion.

4.1.1 Governing equations

In the Discrete Element Method, the contact forces are usually divided into two components – normal component and tangential component as shown in figure 4.1. The contact force formulation is considered as a variant of the spring-dashpot model. The spring model is used to generate the repulsive forces that act while the particles push against each other. On the other hand, the dashpot model represents the viscous damping forces that come into action when the particles have sustained contact. In this way, the spring model signifies the rigidity

of the particles in contact and the dashpot model signifies the inelasticity of the collisions among them. To resolve the forces in both the directions – normal and tangential, the particulate contacts are modeled as a pair of spring-dashpot oscillators with one for each component.

Figure 4.1 Particle contact force formulation on the contact plane

In the current simulation studies, the Hertz-Mindilin no-slip contact model is used for modelling the particle-particle and particle-wall contacts. This model is based on the Hertz-Mindilin contact theory[29]. As described above, the contact force is divided into normal and tangential components as follows:

$$F_{contact} = F_{normal} + F_{tangential} \quad (2)$$

Each force is calculated from the amount of overlap and the approach velocity at the point of contact; the former for repulsive and the latter for damping. Based on this, the repulsive normal contact forces are determined using the following equation:

$$F_N = -K_N O_N - N_N V_N \quad (3)$$

where K_N is the normal spring stiffness, O_N is the normal overlap, N_N is the normal damping coefficient and V_N is the normal approach velocity at the contact point. The subscript N in all these relations represent the normal direction. The normal spring stiffness is computed using the following equation

$$K_N = \frac{4}{3} E_{eq} \sqrt{O_N R_{eq}} \quad (4)$$

where E_{eq} and R_{eq} are equivalent Young's modulus and equivalent radius. They are computed as follows:

$$E_{eq} = \frac{1}{\frac{1 - \nu_A^2}{E_A} + \frac{1 - \nu_B^2}{E_B}} \quad (5)$$

$$R_{eq} = \frac{1}{\frac{1}{R_A} + \frac{1}{R_B}} \quad (6)$$

Where E_A and E_B are Young's moduli of the two particles in contact and ν_A and ν_B the Poisson coefficients. The terms R_A and R_B are the radii of the two particles. On the other hand, the normal damping coefficient is computed using the following equation:

$$N_n = \sqrt{(5K_N m_{eq})} n_N^{damp} \quad (7)$$

Where m_{eq} and n_N^{damp} are the equivalent mass and normal damping parameter. They are computed as

$$m_{eq} = \frac{1}{\frac{1}{m_A} + \frac{1}{m_B}} \quad (8)$$

$$n_N^{damp} = \frac{-\ln(C_N^{rest})}{\sqrt{\pi^2 + \ln(C_N^{rest})^2}} \quad (9)$$

where m_A and m_B are the masses of the two particles in contact. Here, C_N^{rest} is the normal coefficient of restitution between the two particles at the contact surface.

The contact forces in the tangential direction have similar relations only if the magnitudes are less than the maximum possible value – the static friction value from the Coulomb friction law. As a result, the tangential force representation takes the following form:

$$F_T = -K_T O_T - N_T V_T \quad \text{if } |K_T O_T| < |K_N O_N| C_{fs} \quad (10)$$

$$F_T = (|K_N O_N| C_{fs}) \frac{O_t}{|O_t|} \quad \text{if } |K_T O_T| > |K_N O_N| C_{fs} \quad (11)$$

where K_T is the tangential spring stiffness, O_T is the tangential overlap, N_T is the tangential damping coefficient and V_T is the tangential approach velocity at the contact point. Here, C_{fs} represents the static friction coefficient between the contact surfaces. With subscript T representing the tangential direction, all the properties are calculated in similar way as in equations (4)-(9) except for the tangential spring constant which is a function of equivalent shear modulus, G_{eq} . It is calculated as:

$$K_t = 8G_{eq} \sqrt{O_t R_{eq}} \quad (12)$$

Here, the equivalent shear modulus is calculated as follows:

$$G_{eq} = \frac{1}{\frac{2(2 - \nu_A)(1 + \nu_A)}{E_A} + \frac{2(2 - \nu_B)(1 + \nu_B)}{E_B}} \quad (13)$$

For particle-wall collisions, the formulas and the computations stay the same. But the wall radius and masses are assumed to be $R_{wall} = \infty$ and $M_{wall} = \infty$. In this case, the equivalent radius is reduced to $R_{eq} = R_{particle}$ and the equivalent mass would be $M_{wall} = M_{particle}$.

Once these forces are computed, the net force on a particle, resulting from both the normal and tangential components of particle-particle and particle-boundary contacts is calculated in the following manner:

$$F_{Contact} = \sum_{Neighboring\ Particles} F_c + \sum_{Neighboring\ boundaries} F_c \quad (14)$$

These are used in the momentum balance equation of the material particles as follows:

$$m_p \frac{dv_p}{dt} = F_S + F_B + F_{Contact} \quad (15)$$

where F_B represents the body forces on the particles including gravity, while F_S represents the surface forces that include the pressure gradient force and drag force. Here m_p and v_p are the mass and velocity of each material particle. Similarly, the DEM particle equations of motion incorporate angular momentum conservation equations as:

$$\frac{d}{dt}(I_p \omega_p) = \sum_{Neighboring\ Particles} T_c + \sum_{Neighboring\ boundaries} T_c \quad (16)$$

where I_p and ω_p are moment of inertial and angular velocity of the particle. The contact torque is computed from the contact forces on the particles as:

$$T_c = r_c \times F_{Contact} - \mu_r |r_c| |F_{Contact}| \frac{\omega_p}{|\omega_p|} \quad (17)$$

where r_c is the vector from the particle's center of gravity to the contact point and F_c is the contact force acting on the particle. Here μ_r is the coefficient of rolling friction.

From these momentum balance equations, the linear and the angular velocity at the updated time-steps are calculated. Using these values, the new position of the particulate network is calculated and this cycle is repeated. Similar to the contact forces, the lift/drag forces can also be modelled using different techniques available in the literature. But in the current dense particle simulation study, it was observed that there was no influence of the interstitial air on the particles. To validate this, two set of simulations were conducted, one by considering the particle-continuum coupling and one without the coupling. In both the studies, the particulate velocities, the particulate flowrates and all the other parameters of the granular flows remained the same. In addition, the computational time and intensity of the problem was drastically reduced when the coupling was removed. Hence, all the simulations studies in the current research were conducted by ignoring the lift/drag/torque forces on the particles from the continuum and the particle motion was only governed by the gravitational force and collision interactions with other particles and rigid boundaries.

4.1.2 Numerical method

The governing equations of the particles were solved numerically in the Cartesian coordinate system. Since the interstitial air and the particles were completely de-coupled, the momentum equation of the continuum was not solved in the current studies. As a result, a coarser mesh was used as the grid resolution had no direct influence on the flow. The general purpose CFD code STAR-CCM+ [27] was used as the numerical solver to explicitly integrate the governing equation. Its robust application of the aforementioned discrete element equations made it the primary choice for the simulation studies. Attempts were made to solve the problem on multiple processors to accelerate the computational speed. But since the number of particles

in each simulation are huge, the data transfer and communication among each processor for the computations increased and slowed down the solution even with the slightest increase of processors. Hence, all the studies were conducted on a single core using Intel® Xeon® E5-2687W processor.

A first order temporal discretization was used for the advancement of the solution state. A major assumption in DEM simulations is that the effect of contact between two particles is localized and does not propagate to the neighboring particles within a time-step[10]. Thus, the time step size needs to be restricted for these computations. This was done by limiting it to the time it takes the Rayleigh wave to propagate across the surface of the sphere to the opposite pole [29][30]. This time is calculated as:

$$\tau_1 = \pi \frac{R_{min}}{V_{Rayleigh}} \quad (18)$$

where R_{min} is the minimal particle radius. The Rayleigh wave velocity depends on the material properties of the particle. In addition, the time-step size is also limited by the duration of impact of two perfectly elastic spheres. This time is calculated as[31]:

$$\tau_2 = 2.94 \left(\frac{5\sqrt{2}\pi\rho}{4} \frac{1-v^2}{E} \right)^{\frac{2}{5}} \frac{R}{\sqrt[5]{v_{impact}}} \quad (19)$$

Finally, the third restriction on the time step size is based on the assumption that particles must not move too far within the time-step. This prevents missing contacts between DEM particles as well as particles and the wall. Thus, each particle is constrained such that it takes at least 10 time-steps for the particle to move the full-length of the radius. Thus the restriction is formulated as

$$\tau_3 = \frac{R}{v_{particle}} \quad (20)$$

4.2 Simulation Methodology

The current sections explains the geometries modelled for these simulation studies and also summarizes the different properties and parameters used. Similar to the experiments, the simulations were also conducted in a batch-wise fashion. As a result, the geometry was modelled exactly like the experimental setup with a batch funnel on the top of the tube and an orifice at the exit.

Figure 4.2 Particle injection into the solution domain using lattice structures

Before any flow was initiated, the entire solution domain was filled with particles by keeping the exit orifice closed. The particles were added at every time step into the solution domain and they settle down in the tube and the funnel due to the gravitational force acting on them. This was done by assigning a lattice structure in the solution domain and by adding

particles at every lattice point, if a free space was available. In this way, the amount of computational effort in setting up the simulation (adding particles alone) was avoided. Figure 4.2 gives a clear representation of the process. Once the entire solution domain is filled completely, the flow is initiated by removing the orifice block. Finally, the required solution scalars and vectors are recorded at every time step and further post-processing was done to obtain necessary and meaningful results.

4.2.1 Geometry

As mentioned earlier the simulation geometry includes a funnel at the top and an orifice at the bottom end of the tube. All the simulation studies were conducted with only a vertical configuration. Initially, the simulation geometry was set with a tube as long as the one used in the experiments. But as a result, the number of particles in the solution domain was huge slowing down the computational process. As discussed in the previous sections, the length of the tube, once longer than the saturation length (explained later), had no influence on the dense flow regime. Similar flow rates were obtained in all the cases with different tube lengths. Though the internal flow profile might be vary with tube length, to reduce the computational cost, the simulations were done with shorter tube lengths. Figure 4.3 shows the geometry used in the simulations.

In the current research, each of the simulation cases with a specific tube diameter, tube length, and particle diameter was performed with different orifice openings. This was done to observe the flow physics for different flow rates (as understood from the Beverloo studies performed in the preliminary experiments). It was mentioned earlier that every simulation takes a considerable amount of computational effort and solution time to fill the domain with

particles. Hence, it would not be wise to repeat the same for each and every orifice diameter. To avoid that, the simulation geometry was modelled in such a manner that the filling was performed only once and the orifice opening can be changed after being filled.

Figure 4.3 Simulation geometry without and with particles (left to right)

4.2.2 Properties and parameters

To observe the particle surface more clearly, Scanning Electron Microscopic images were taken on the particle sample. Each sample was coated with gold-palladium ions for the electron

microscope to get the images. It also enabled in identifying the composition of the sample using X-Ray energy spectrum.

Figure 4.4 (clockwise) (i) SEM image of particulate sample at 10X magnification (ii) SEM image of particulate sample at 50X magnification (iii) Energy spectrum of the particulate sample at position S1

It was identified that the particle diameter ranges from 275 microns to 330microns. The particles were found to be mostly spherical and have a smooth outer surface. From the spectral imaging at point S1 in, the particle chemical composition was found with Silica and Zirconia as the dominant constituents. Similar to these experimental samples, a distribution of particulate sizes can be used for the simulation studies. But this would make the computational process intensive as every particle should have its own radius instead of having a constant value for the entire batch. As a result, mono-sized spherical particles were used in all the simulation studies.

The different properties required for the simulation studies include, particle density, particle Young's modulus, particle Poisson's ratio, particle-particle/particle-wall static/rolling friction and particle-particle/particle-wall coefficient of restitution. The density of the particles is experimentally calculated by adding a known mass of particles in a beaker of water. Since the porosity of the particle is low, as they are added into water, the water level rises. The change in the volume of the water would be equal to the volume of the particles. In this way the volume of the known mass of particles is found and eventually the density was calculated using the relation $\rho = \frac{mass}{volume}$. The Young's modulus and the Poisson's ratio of the particles (Si-Zr) are obtained from the Catalogue provided by the suppliers. For both particle-particle and particle-wall, two coefficients of restitution are required in the simulations – one in the normal direction and one in the tangential direction. They signify the amount of kinetic energy (momentum) lost in the event of a collision. In the current scope of studies, the packing fractions inside the flow domain are close to 60%. As a result, the number of continuous contacts among particles and between particles and wall is significantly higher as compared to the number of collision

events leading to a weak or no influence of restitution on the flow[16]. This assertion was also verified from the simulations by running cases with different restitution coefficient while keeping all the other parameters constant. This was performed for all four values of restitution coefficients. It was observed that the mass flow rates inside the tube remained constant in all the cases, even though the restitution coefficient was varied. As a result, best guess values and data available from the material catalogues were considered for the coefficients of restitution.

To identify the particle-particle friction, the angle of repose was used. It is the steepest angle of descent or dip relative to the horizontal plane to which the material can be piled without slumping[32]. At this angle, the material on the slope face is on the verge of sliding. When bulk granular materials are poured onto a horizontal surface, a conical pile will form. It is the internal angle between the surface of the pile and the horizontal surface that is defined as the angle of repose. It is primarily dependent to the density of the particles, the surface area and the coefficient of friction of the material (particle-particle friction). Using this idea, simulations were conducted with different particle-particle friction values and the corresponding angle of repose was calculated in each case. This was done by initially adding particles in a conical structure such that they are completely at rest. Once the particles are settled, the conical wall around them was removed and the particles slowly settle at their normal angle of repose. In this way, for different particle-particle frictions, the angle of repose was computed. The process is clearly illustrated in figure 4.5. It was experimentally identified that for the granular material used in the experimental studies had an angle of repose of 26° . Thus, the particle-particle friction for which the same angle of repose was computationally observed was considered as the value in the simulation studies.

Figure 4.5 Methodology used to calculate the angle of repose using simulations

The particle-wall friction on the other hand is difficult to measure experimentally. Hence it was identified indirectly from the simulations. From the previous experimental studies, the flowrates for different orifice diameters were identified. Similar orifice geometry was modelled for the simulation and the particle-wall friction was tuned such that the flow rate matches those from experiments. This friction values was also verified with orifice diameters and the experimental flow rates. Hence, the particle-wall friction was found to have identified. Rolling friction is another property that needs to be assigned for the simulations. Similar to the restitution coefficients, the rolling friction also had absolutely no influence on the overall flow

rates. This might again be due to the high packing fraction in the flow domain. As a result a best guess value was assigned to these parameters. The following table describes all the properties used for the simulation studies.

Table 4.1 Particulate properties for the simulation studies

Diameter		0.0003 m
Density		3934.06 kg/m ³
Young's Modulus		75800 MPa
Poisson's Coefficient		0.235
Static Friction Coefficient	Particle-Particle	0.8
	Particle-Wall	0.5
Rolling Friction Coefficient	Particle-Particle	0.001
	Particle-Wall	0.001
Normal Restitution Coefficient	Particle-Particle	0.6
	Particle-Wall	0.5
Tangential Restitution Coefficient	Particle-Particle	0.8
	Particle-Wall	0.5

4.2.3 Run cases

The entire simulation study was divided into two broad groups – Short tube measurements and Long tube measurements. The short tube measurements are primarily used to understand the radial profiles of velocities, packing fractions and flow parameters like flow rates and particle-wall contact behavior, while the long tube measurements are used to understand the axial profiles of these quantities and observe the temporal internal flow characteristics along the tube axis.

For the short-tube simulations, three tube diameters of 5mm, 7.747mm and 10.922mm were studied. The tube lengths in these cases were 0.05m, 0.075m and 0.1m respectively. These lengths were calculated such that the velocity profiles are measured at a location where

the wall-pressure saturates (explained in detail in next sections). All these cases were computed with 10-15 different orifice sizes to observe the variation of flow rheology at different mass fluxes through the tube. The main parameters that are recorded in these simulations are; particulate velocities, particle-wall contact forces, particulate flow rate through the orifice, particle-wall contact points, and packing fraction profiles. To understand the influence of particle diameter on the flow physics, simulations were done in a 7.747m diameter tube with a length of 0.075m. Particles of 300 μ m, 475 μ m and 600 μ m were studied in these simulations. Each case is again computed for different orifice sizes (flow rates). The short-tube simulations involved in solving around 500,000 particles in the computational domain.

The main motivation for the conducting long-tube measurements is to study the flow fluctuations. In short-tube measurements, it was observed that the flow fluctuations amplify at higher axial locations above the orifice (explained in detail in next sections). To observe and quantify this behavior, the simulations were conducted with a tube of 0.3m in length. The tube diameter in these studies was 7.747 mm and particles of 300 μ m in diameter were used. These studies were conducted on 4 different orifice sizes to understand the influence of mass flux on these fluctuations. In addition, the axial profiles of aforementioned quantities, profiles of mass flow rates at different axial locations inside the tube were also obtained through these studies. Since the long-tube simulations involved solving around a million particles in the solution domain, they took relatively longer computational time than the short-tube simulation. The subsequent sections present the various results and observations made in the simulations studies.

Chapter 5

Simulation Results and Discussions

The simulation studies, which were conducted in batches were validated by comparing them with the experimental data. Since the flow rates are the only measurements in the experiments, they were used for this verification. Hence tube-orifice geometries were matched to the exact dimensions as in the experiments. The flow rates were measured by calculating the number of particles crossing the orifice exit per unit time. This tracking was done over a period of time and a thus temporal evolution of the flow rates was found.

Similar to the experiments, the flow rates remained constant throughout the granular batch. In figure 5.1, the flow rates obtained for different orifice sizes were plotted for both the experiments and the simulations. This was done for both the tubes of diameter 7.747m and 10.922m. It can be observed that the data matched perfectly and have similar trends. The Beverloo trend is also followed by the simulation results. Similar to the experiments, the simulation mass flow rates also deviated from the Beverloo Law after a critical mass flow rate was exceeded. All these observations will be further explained in the future sections, but for now, this validation study suggests that the simulation methodology and parameters give good and accurate results.

Figure 5.1 Validation of flow rates obtained in the Discrete Element simulations

The following sections briefly describe the important observations made for dense granular flows in cylindrical tubes. Though the simulations were performed on a Cartesian coordinate system, a cylindrical system was used to discuss the results with z as the axial, r as the radial and θ as the angular directions, respectively.

5.1 Particulate velocity profiles

As mentioned earlier, the solution state for each particle includes the position and velocity. To obtain the velocity profiles, the velocity of particles were recorded and the required data-processing was done. But since each simulation involves a large number of particles, it would be computationally intensive to track/record the state of every particle. As a result, the velocity profiles were obtained indirectly using the method of Boundary Sampling[27] (discussed further in this section).

Figure 5.2 Contour plots of particulate velocities (a) through the orifice (b) through a flow cross-section

The particulate velocities were observed over time using contour plots. They are plotted for a specific case at an intermediate time-step in figure 5.2. Based on the color legend and the distribution, it can be observed that the particulate velocities remain almost uniform in the core.

It can also be observed that the particles near the walls have a lower velocity as compared to the ones at the core. It is important to observe that the flow remains uniform at all polar angles in the cross-section. As the particles exit the tube, due to the orifice they accelerate. This can be seen in the contour plot from the red shade at the orifice exit. Finally it can be seen that the particles have a slip velocity at the walls similar to all granular flow configurations.

Though the contour plots give a rough overview of the velocities and the animations made from them depict the flow fluctuations, the time averaged velocities and the radial profiles are necessary to gain a clear understanding of the flow. As mentioned earlier, the boundary sampling method is used to obtain velocity profile information. In this technique, the data recording occurs only at certain instances in the simulation. The recording event can be triggered by setting different criterion on the particles. In the present case, the data recording event was set to trigger whenever a particle crosses a specific plane or a specific sampling region. For example, if there are 1000 particles in a cylindrical solution domain, but only 50 particles in contact with the bottom face of the cylinder, the solution state of the 50 particles alone can be accessed by using the bottom face as the sampling boundary. Similarly, as another example, if the particles are crossing a plane in a solution domain, the data recording event can be specifically triggered for particles that are on or just crossed the plane at a specific time step. In this way, the data recording will occur throughout the solution time at every time-step but only for those particles that are on or just crossed the plane of concern (sampling boundary). The boundary sampling method thus stands as a robust and efficient way of accessing the data from the granular geometries as a whole.

In the current scenario, to obtain the radial profiles of velocity, the flow cross-section is divided into annular rings of different radii as shown in figure 5.3. Every annular ring is a single plane that is located at a specific radial location. Using boundary sampling, the velocity data is recorded for all the particles that come in contact or just cross a specific annular plane. This is analogous to measuring the velocity of all the particles that are at that specific radial location. Finally taking the average of the velocity data for that annular plane gives the particulate velocity at that specific radial location at that time-step. This is primarily based on the observation that the particles have similar flow behavior at different polar angles, i.e. axisymmetric flow. In this way, the same process is repeated on all the annular sections and is continued at every time step. Thus, the temporal evolution of particulate velocities at different radial locations is obtained using the method of boundary sampling. Since a boundary layer was expected near the tube walls, to capture it precisely, the annular sections were not made of uniform thickness but are thinner near the wall. This is analogous to using a prism mesh to resolve the boundary layer in continuum flow problems.

Figure 5.3 Sectioning of flow cross-section into annular rings for boundary sampling

It was observed that the magnitudes of the radial and angular velocities were negligible as compared to the axial velocities. As a result, from this point on, the term “velocity” refers to the axial velocity of the flow. From the time evolution of velocity at different radial locations, it was observed that the velocities fluctuate about a mean value. This can be observed in figure 5.4 which was plotted for an orifice with tube diameter of 7.747mm and particles of 300 micron in diameter. The different colors in the plots represent a different radial location at a specific axial location. Though fluctuations make it look erratic, every radial location had a mean velocity. The velocity fluctuations describe the intermittent and rearranging nature of the particulate flows. Though not plotted in the current discussion, it was observed that the velocity fluctuations were higher at the tube wall as compared at the core of the tube. This signifies that the particles are undergoing relatively higher re-arrangement in the wall region.

Figure 5.4 Time evolution of particulate velocities at different radial locations

To understand the internal flow profile more thoroughly, time averaged velocity plots were obtained from the temporal data. It is important to understand that the velocity data is obtained only when a particle crosses through or stays on the annular plane. For slower flow rates, there would be a scenario where no particle crosses the sampled boundary and as a result no data would be recorded at those time steps. This is a direct result of the discrete nature of the granular flow. As a result, the time-averaging for this data set should be ensemble averaging. Since we are dealing with vertical tubes and the flow is a gravity-driven configuration, the axial components of the particulate velocities would be of negative magnitudes as the gravity acts in negative z-direction of the flow. In figure 5.5, the time averaged velocity was plotted versus the radius of the cross-section for a flow rate of 3.5g/s.

Figure 5.5 Time averaged particulate velocity at 3.5g/s mass flow rate (tube diameter: 0.007747m and particle diameter: 300 microns)

From figure 5.5, it can be observed that the velocity profile comprises two distinct regions – a) core region and b) shear region. In the core region, the velocity profile has near zero velocity gradient. Though the time average velocity values are not exactly uniform, the smooth nature can clearly be observed. Since the velocity data was obtained using the technique of boundary sampling, this figure depicts discrete time averaged data points. The core region occupies a significant portion of the overall flow cross-section. It extends from the center of the tube to almost the tube wall. The shear region on the other hand had significant magnitudes of velocity gradient, thus the name “shear” region. It roughly occupies 1-2 particle diameters from the wall. It can be observed that the velocity magnitudes are less in the shear region as compared to the core region. This is similar to what was observed from the contour plots. As a whole, both the regions together resemble a “plug-flow” configuration to the flow. It is also important to observe that the flow has a slip velocity at the wall. This was similar to all the previous measurements done on granular flows [16]. The dotted-line in figure 5.5 represents the average velocity of the flow measured from the flow rate. It is remarkable to observe that the core velocity is very much close to the average velocity.

For two-dimensional gravity driven granular flows, several constitutive relations were proposed by different research groups[18], [33]–[36]. Since the time averaged velocity profiles resembled turbulent flows with a plug flow configuration and continuous fluctuations, numerous efforts were put for the development of hydro-dynamic analogies for these granular flows. Even several models, that resemble mixing length models, were developed and worked on. One of the most famous one among them is:

$$\eta_{eff} = \rho_p L^2 |\dot{\gamma}| \quad (21)$$

where η_{eff} , ρ_p , L and $\dot{\gamma}$ are the effective flow viscosity, particulate density, characteristic length scale and shear rate respectively. The characteristic length is taken as a function of local packing fraction (ϕ) in these works. Several formulations of these functions were made over the years [12][18][33][35][36]. Each of these studies suggested the existence of different velocity profiles, some of them including logarithmic and exponential ones. The main basis and validation data for these formulations were some experimental studies and the observations made through them. Some of them relied on replicating the shear layer behaviors while others relied on obtaining an accurate velocity profile. Though all these works were performed for the development of constitutive models, the flow configurations were different in each. While some relied on the experimental data obtained from shear cells, others relied on the flow visualization studies in two-dimensional chutes. Moreover, each of these studies came up with different conclusions and observations. At this point, it is important to remember that the flow rheology of granular material is too complicated to be modelled with a universal relation. This is because the intrinsic characteristics of these flows differ drastically from regime to regime and configuration to configuration. As a result, the most common way of studying granular flows is by observing individual configurations and extracting the rheological details from them. The current research also works on similar lines and thus primarily focusses on observing the features specific to gravity-driven dense granular flows through cylindrical tubes instead of using the constitutive models developed for entirely different regimes/configurations.

In the current DEM studies, the time averaged velocities were found to follow a Gaussian profile of the form:

$$v(r) = A \exp\left(-\left(\frac{r-B}{Y}\right)^2\right) + Z \quad (22)$$

where A , B , Y and Z are parameters that depend on the flowrates. This is plotted in figure 5.6. This is for the same discrete velocity points in figure 5.6. The X-axis of this plot was scaled with the particle diameter of 300 microns for clear depiction. It should also be noted that the positive y-axis values in this cases, as opposed to the negative values doesn't have any physical significance. This figure was deliberately inverted about x-axis for clear depiction of the Gaussian profile. The Gaussian profile perfectly estimated the asymptotic nature of the velocity as it approaches the center. While most of the velocity profile estimates required two separate functions to describe the flow, the Gaussian profile matched the discrete data very well from the shear region to the core. In the following paragraphs, the variation of velocity profiles with different flow and geometric parameters are discussed.

Figure 5.6 Gaussian fit for time-averaged particulate velocity at 3.5g/s flow rate (tube diameter: 0.007747m and particle diameter: 300 microns)

In Figures 5.7 and 5.8, the velocity profiles and Gaussian fits were plotted for different flow rates for tubes of 7.747 mm and 10.9922 mm with 300 microns particles. It can be observed that the thickness of the shear-bands does not remain constant but increases with flow rate suggesting that the shear band thickness is flowrate dependent. This observation contradicts previous two-dimensional granular flow experiments. Additionally, for similar flow rates, the shear band thickness remained constant for both tube diameters suggesting that the shear layer is independent of the tube diameter.

Figure 5.7 Radial profiles of axial velocity for different flow rates for 0.007747m tube diameter (mass fluxes vary from 56.2kg/m²-s g/s to 336.6kg/m²-s)

Figure 5.8 Radial profiles of axial velocity for different flow rates for 0.010922m tube diameter (mass fluxes vary from 74.2kg/m²-s g/s to 281.4kg/m²-s)

The difference between the previous works of Nedderman[36] and Pouliquen[35], and the current DEM studies can be attributed to the different operational regimes in these cases. While our work focuses on the transitional dense regime, the previous works specifically considered

the quasi-static regime. This also explains the invalidity of the hydro-dynamic analogies and the mixing-length models for this type of flow. Hence, the current observations are novel and can be considered as the fundamental characteristics of dense granular flows in tubes

Figure 5.9 Variation of slip and maximum velocity with mass fluxes

In Figure 5.9, the time-averaged slip velocity and maximum velocity are plotted with respect to mass flux for both the tubes. This was done to make a qualitative comparison of the shear bands formed in both the tube diameters. With increasing mass flux, the maximum velocity increased for both tubes. This variation is linear and is identical for both the tubes.

Similarly, the slip velocity increases linearly with increasing mass flux and also has an identical trend for both the tubes. Since the velocity has a Gaussian profile, the difference between the slip and maximum velocity represents the peak of the Gaussian curve. Since this peak varies with the flow rate, the shear rate characteristic, which depends on the shape of the Gaussian curve varies with the flow rate. This qualitatively explains the shear band behavior for different flow rates in the dense flow regime. These observations of velocity profiles could also prove useful in understanding the convection behavior of granular flows in the dense flow regime.

Figure 5.10 Normalized mean axial velocity profiles for different flow rates in 0.005m diameter tube (mass fluxes vary from 35.7kg/m²-s g/s to 336.6kg/m²-s)

In figure 5.10, the variation of velocity profiles across the radial locations were shown for different flow rates with a tube diameter of 5mm and 300 microns particles. The negative values in these plots refers to downward velocities of particles. The continuous lines are not the plots corresponding to discrete data points from the simulations but are the Gaussian fits with an R-squared value of 0.999. To compare all the flow rates together, the y-axis was plotted with normalized velocity profiles define as $(V - V_{slip}) / (V_{center} - V_{slip})$. The radial location (x-axis in these plots) was scaled with particle diameter D_p . In this figure, the mass fluxes varied from 35.7kg/m²-s to 253.7 kg/m²-s. It can be observed that with increasing mass-flux (flow rates), the shear band thickness increases suggesting that they are indeed flow-dependent. This observation, which is on similar lines with the previous plots, further affirms that increasing shear band thicknesses with mass fluxes is indeed a characteristic of dense flow regimes. It is also remarkable to see that the shear band thickness for all the flowrates is always between 1-2 particle diameters.

Figures 5.11 and 5.12 show the velocity profiles for particle diameters of 475microns and 600microns respectively in a tube of 7.747mm in diameter. For higher mass fluxes, it can be observed that the data points perfectly agree with the Gaussian fits in both the cases. The increasing shear band thickness and uniform core region were very much similar to that of other observations made with 300micron particles. But for lower mass flux values, the velocity profiles become erratic and the Gaussian velocity profile doesn't fit well. It can also be observed that a concave down trend exists for the velocities in these regimes. Though this may be a statistical remnant of the discrete data points and fluctuations, the intermittency and

particulate rearrangement could be a likely cause for this observations. More studies are currently being done to understand this phenomenon.

Figure 5.11 Radial profiles of axial velocity for different mass fluxes with 475 micron particles and 0.007747m tube diameter (mass fluxes vary from 6kg/m²-s g/s to 300kg/m²-s)

Though not reported in the current discussion, similar observations were also made with particles of 300microns diameter and with tubes of diameters 5mm and 10.922mm. The mass

flux range at which the transition occurs from Gaussian profiles is not the same in these cases. Thus, a different metric was required to observe if there was any transition in the operational regime at these mass flux values.

Figure 5.12 Radial profiles of axial velocity for different mass fluxes with 600 micron particles and 0.007747m tube diameter (mass fluxes vary from 14kg/m²-s g/s to 310kg/m²-s)

One way to compare the operating regimes of these flows is by studying the Inertial number of the flows. Defined as $I = \frac{|\dot{\gamma}_w|D_p}{\sqrt{P/\rho_p}}$, where $\dot{\gamma}_w$ is the wall shear rate, D_p is the particle diameter, P is the pressure and ρ_p is the particle density, an increasing inertial number indicates an increasing deformation tendency of the particulate assemblies. This non-dimensional number describes the relative importance of particulate inertia to the confining stresses in granular flows. In general, several limits were identified for this number in each flow regimes (quasi-static, dense and dilute). But these limits are not universal and vary for different flow configurations. As a result, in the current study, the variation of effective friction with inertial number was observed to identify the operating regimes for different flow rates.

The effective friction, μ_{eff} of the granular flow is described by Mohr-Coulomb law. The method of obtaining these values will be discussed in the next section and the current discussion will further be continued over there.

5.2 Wall normal and shear force profiles

The tube wall in the simulation was sectioned into several cylindrical segments to study the particle-wall contact forces at different axial locations. Each contact force was resolved into two components – a) normal component that is perpendicular to the tube wall surface and b) shear component that is parallel to the tube wall along negative z-direction. In each annular control volume, the total normal and shear forces were calculated and were divide by its area to find the time-averaged (mean) normal and shear stress distribution.

Figure 5.13 Wall normal stresses versus axial position in tube of diameter 0.007747m for different flow rates.

In Figure 5.13, the axial distribution of mean wall normal stress are plotted for different flow rates in tube of diameter 7.747mm. The positive direction of the x-axis in this plot represents the axial position measured downward from the neck of the inlet hopper. The coordinate $z=0$ represents the inlet of the tube and the increasing z -coordinate implies increasing depth inside the tube. Since there is no bulk flow in radial direction, the wall normal

stress is analogous to the thermodynamic pressure in the granular media. Hence, from this point on, the wall normal shear and pressure will be used interchangeably.

With increasing depth, the wall normal stress increases initially and saturates to a constant value at ~ 0.03 m from the tube inlet. At the end of the tube, the values deviate from the saturation values owing to the constriction created by the orifice. A similar behavior was also observed in ‘static’ granular beds from the experiments of Janssen and other researchers [8], where a saturation behavior of the hydrostatic pressure was observed with increasing bed depth. This was primarily attributed to friction forces between the particles and the tube wall. In liquids, a friction/shear force is acted upon the flow only when a velocity gradient exists at the wall. Hence, no shear force acts on static liquids and the increasing depth results in a continuously increasing static pressure. To the contrary, the frictional behavior between granular media and a rigid surface is analogous to rigid body contact mechanics. This behavior, described by Mohr-Coulomb law, results in the following relation:

$$\tau_w = \mu_{eff} \sigma_w \quad (23)$$

where τ_w and σ_w are the shear (friction) and normal (pressure) stresses on the contact surface and μ_{eff} is the effective friction coefficient between the particles and wall. Owing to this relation, the shear force developed on the tube wall balances the increasing particulate mass with depth. Hence, after a certain length, the pressure saturates with no further gradient. As a result, the pressure profile in static granular beds follows:

$$P(z) = \left(\frac{\rho_b g R_t}{2\mu_{eff}} \right) \left(1 - e^{-\frac{2\mu_{eff}}{R_t} z} \right) \quad (24)$$

where ρ_b , g and R_t are bulk granular density, gravitational acceleration and radius of the tube. Though similar pressure profiles exist for ‘flowing’ granular media, their magnitudes vary with

flow rate. By using equation (24) to fit the discrete data in Figure 5.13 (dotted lines), we identified μ_{eff} for each curve. This led to the observation that μ_{eff} is indeed a function of flow rate for the current operational regimes. In Figure 6.14, the minor deviation in the pressure profile at the top of the tube ($z = 0$) is due to the non-uniformity of granular flow at the neck of the funnel.

Figure 5.14 Wall shear stresses versus axial position in tube of diameter 0.007747m for different flow rates.

In Figure 5.14, the axial distribution of wall shear stress are plotted for different flow rates in the 10.922mm diameter tube. Similar to the pressure profiles, the wall shear stress increases

with increasing depth and saturates to a constant magnitude. Unlike the normal stresses, however, the flow rate has little influence on the saturation magnitude which is approximately 45 N.

As mentioned earlier, the velocity profiles were investigated at an axial location below the saturation lengths, where there is zero pressure gradient. Hence, from a basic force balance in axial direction, a linear profile for the internal shear stress can be obtained in the following form:

$$\tau(r) = \left(\frac{\rho_b g}{2}\right) r \quad (25)$$

where ρ_b is bulk granular density and g is the gravitational acceleration. Though a relation for effective viscosity can be obtained from the velocity and shear profiles in equations (25) and (22), more studies are needed to develop a function that is independent of the geometrical parameters of the flow. Similar trends and plots were also observed for all the cases with different tube diameters, different flow rates and different particle diameters.

From here onwards, the discussion in the previous section will be continued to analyze the variation of effective friction coefficient with Inertial number. As mentioned above the saturated normal and shear forces were obtained for all the cases. Using these values and equation (23), the effective friction coefficient for each flow configuration was calculated. Since the velocity profiles were also known for each of these configurations, the Inertial number, which depends on wall shear rates were also calculated. It was identified and verified in several other research works [16] that the quasi-static regime has constant effective friction with increasing inertial number, while dense regime has increasing effective friction with increasing inertial number. These were incorporated to identify the operational regimes of the different

simulation cases. In figure 5.15, the variation of these effective friction values with inertial number for all the configuration we studied were plotted.

Figure 5.15 Variation of effective friction with inertial number for different simulation cases

For particle diameters of 300microns, the effective friction increased with increasing inertial number. This observation confirms that these configurations, which produced Gaussian velocity profiles, are indeed in the dense inertial regimes. On the other hand, for particle diameters of 475microns and 600microns, the effective friction remains constant initially and then shows the increasing behavior with increasing inertial number. This may suggest that the first few data points of these plots, which correspond the lower mass fluxes, are in quasi-static regime. These are the same mass fluxes where the Gaussian velocity profiles does not fit well.

As a result it can be suggested that the existence of a Gaussian velocity profile is a characteristic of dense flow regimes.

5.3 Packing fraction profiles

The packing fraction profiles are important in estimating the effective thermal properties of granular flows when used for heat transfer applications. Similar to the velocity profiles, these were obtained by dividing the flow cross-section into different annular regions. From the locations of particle-centroids in the solution domain, the total number of particles in a sample volume were calculated and thus the packing fraction was computed. In Figures 5.16 and 5.17, radial profiles of packing fraction, for tube diameters of 7.747mm and 10.922mm were plotted for different flow rates.

Figure 5.16 Packing fraction profiles for different flow rates for 0.007747m tube diameter

In Figures 5.16 and 5.17, the solid horizontal lines are the area-averaged packing fraction values for different flow rates. Similar to the velocity profile plots, the x-axis is scaled with particle diameter. From the discrete data points, it can be observed that the packing fraction remains uniform at the tube center and varies as it approaches the tube wall. The oscillating nature of the packing fraction near the walls can be attributed to the annular sampling of flow cross-section.

Figure 5.17 Radial profiles of packing fraction for different flow rates for 0.010922m tube diameter

To precisely capture the shear-band velocity profiles, fine annular samples were considered near the wall (recall that the particle centroid locates the particle). In fact, some of these annuli near the walls are smaller than the particle diameters to give sub-particle resolution near the

wall. As a result, the packing fractions are over or under-estimated in these regions. Over-estimation anomalies are countered by under-estimation of packing fractions in the adjacent annuli. It should thus be remembered that the oscillating nature of mean packing fraction shown near the wall is just a remnant of sampling methodology and has no physical implication. As a result, trend lines (dotted lines) are plotted to qualitatively understand the true mean packing fractions profiles.

With an average value around 60%, the packing fractions are maximum at the center and minimum at the tube wall. Since mono-sized particles were used, this behavior is more clearly seen in the simulation studies. In addition, there is no variation in these profiles with different flow rates. Hence, the area-averaged packing fractions and the trend lines overlap for different flow rates. No conclusions can be drawn about the influence of tube diameter on these profiles owing to their erratic behavior near the wall. As a whole, the packing fraction profiles are important while using granular flows as heat transfer fluids.

5.4 Flow rates and Beverloo correlation

The variation of flow rates with orifice diameters was observed for tubes of three different diameters. Up until un-choking, the flow rates perfectly agreed with the semi-empirical Beverloo correlation. As observed in the previous experimental results, the un-choking occurs at different orifice diameters for different tube diameters. In figure 5.18, we can see that the flow rate data points for the three tube diameters follow a single Beverloo correlation. This suggests that the tube diameter has no influence on the flow rates when in the choked regime. The Beverloo constants for these data points was found to be 0.75 and 1.5 for C and k . These

are within the prescribed limits suggested by several experimental studies done by other research groups.

Studies were also conducted to see the influence of particle diameter on the flow rates and the Beverloo constants. For a constant tube diameter, it was observed that the values of C and k still remain the same, irrespective of the particle diameter. Though this is directly suggested from the Beverloo correlation, the obtaining same C and k suggests their independence from the particle size.

Figure 5.18 Variation of mass flow rates with orifice diameters for different tube diameters

Since the Beverloo constants remained the same with particle and tube diameters, the influence of frictional properties on them was studied by keeping either the particle-particle

friction coefficient or the particle-wall friction coefficient as constant and changing the other. In all these studies, the k value remained constant at 1.5 with only C changing with the friction. This agrees with other literature suggesting that k is just a function of particle shape. For a constant particle-particle friction coefficient of 0.8 and an increasing particle-wall friction from 0.1 to 0.9, the C value decreased from 1.15 to 0.69. Similarly, for a constant particle-wall friction of 0.5 and increasing particle-particle friction coefficient from 0.3 to 0.8, the C value decreased from 0.95 to 0.77. This shows that the Beverloo constant C , is a direct function of friction coefficient. Currently more studies are being conducted to relate the value of C with the angle of repose of the particles.

Figure 5.19 Variation of mass flow rates with orifice diameter for different particle sizes

5.5 Particle-Wall contact behavior

Since the main objective of the current research is to understand the possibility of using dense granular flows as heat transfer fluids, the particle-wall contact behavior becomes an important parameter to observe. Similar to the case with continuous fluids, the overall heat transfer in granular flow includes two mechanism – radial conduction of heat from tube wall to the particles and axial convection of heat by the particles in the direction of the flow. Radial conduction involves transfer of heat from one particle to another due the contact between them, while axial convection includes transfer of heat energy by the bulk motion of particles moving in the flow direction. While the latter is primarily a function of the velocity profiles and flow rate magnitudes, the radial conduction is primarily governed by the temperature profile and the particle thermal conductivity which is governed by the particle to wall contact behavior. Hence it is tantamount to study this phenomenon for particulate flows.

The contact behavior was studied by counting the number of particles that come in contact with tube wall at any time instant. This was observed per unit tube wall area. The simulations geometries were similar to those described in previous sections with particle flow rate being controlled by different orifice size.

In figure 5.20, the variation of particle-wall contact behavior was observed with different particle sizes – 300microns, 450microns and 600microns. To maintain homogeneity, the orifice size in each case was adjusted in such a manner they all had equal mass flow rates. It can be observed that with increasing particle diameter, the number of contact points decreases.

Figure 5.20 Variation of particle-wall contact behavior with different particle diameters in 0.007747m diameter tube

At this point of discussion, it is important to identify that the contact stiffness and damping coefficients in the DEM contact models are a function of particle geometry too. As a result, the larger the diameter of each particle, the smaller the stiffness of the contact points. As a result, larger particles end up having more overlap as compared to the smaller for similar loading conditions. Since the amount of overlap between two particles is directly analogous to the contact area, it is natural to understand that larger particles have higher particle-wall/particle-particle contact area as compared to smaller particles. This was also observed in the simulations where the average particle-wall contact area for a single contact point increases

almost linearly from $1.10\text{E-}08 \text{ m}^2$ to $2.01\text{E-}08 \text{ m}^2$ for particle diameters varying from 300microns to 600microns. Though this may counter the effect of reducing number of contacts, the overall particle-wall contact area per unit tube wall area still decreases with increasing particle diameter. This can be observed from the secondary axis of figure 5.20. The conduction between a single particle and the tube wall, which is the first line of radial conduction, depends on the temperature difference between the two, the thermal properties and the magnitude of contact area. Since the overall particle-wall contact area is reducing with increasing particle diameter, it can thus be understood that the radial conduction from the tube wall to the particulate flow decreases with increasing particle sizes. Thus for a similar flow rate where the axial convection remains the same, increasing the particle diameter may result in weaker heat transfer owing to poorer contact behavior.

Another remarkable observation was that the packing fraction decreased with increasing particle diameter (~3% reduction). So finally, it can be concluded that the smaller the particle size, the better the heat transfer behavior. This statement would be valid only if the particle thermal properties remained the same and if there is no influence from the static charge forces for smaller particles.

Studies were also conducted to understand the influence of flow rates on the particle-wall contact behavior by keeping the particle size constant. From figure 5.21, it can be understood that the number of particle-wall contacts per unit area decreased with increasing flow rates. On the other hand, there was an increase in the individual particle-wall contact area with flow rates. This could be due to the increased agitation of the particles at higher flow rates. As a result, the higher approach velocity of each collision at higher flow rates may lead to higher

contact deformation and thus the contact area. In spite of this phenomenon counteracting the reduction of particle contacts, the overall particle-wall contact area still decreases with increasing flow rates. Hence it can be understood that with increasing flow rates, the radial conduction from the tube wall to the flow decreases.

Figure 5.21 Variation of particle-wall contact behavior with different flow rates in 0.007747m diameter tube

Comparing the secondary axes in figures 5.20 and 5.21, it can be observed that the variation in the overall particle-wall contact area with flow rates is relatively smaller compared with changing diameters. In a previous experimental study[20], it was observed that the heat transfer coefficients increases with increasing flow rates, despite of the currently observed reduction in radial conduction. This could possibly due to higher axial convection and more particulate

agitation (mixing) at higher flow rates. More computational studies are currently being done to gain insights into this phenomenon.

5.6 Flow fluctuations and density waves

As observed in the section 5.1, the particulate velocities have fluctuations throughout time. It was also observed that wave like structures propagated through the tube over time. This can be clearly seen in figure 5.22.

Figure 5.22 Propagation of wave-like structures in short-tube simulations (increasing solution time with a time-step size of 0.01s from left to right)

In figure 5.22, the 7 images of axial velocity are taken at 7 subsequent time steps with a time interval of 0.01s. The time increased from left to right. It can be observed that a high velocity train moves from bottom to top as time passes. As this wave propagates, the magnitude of this high velocity train increases. These images were obtained from short tube simulations

(tube length = 0.06m). Since the behavior looked like it's amplifying with the axial position, long-tube simulations were conducted to study this phenomenon.

Figure 5.23 Propagation of density waves in 0.3m long tube shown in contour plots of particulate velocities and scalar plots of center-line velocities (Solution time increases with a time-step size of 0.0025s from left to right)

In figure 5.23, contour plots were plotted along with the center-line particulate velocities for a tube of 0.3m in length with particles of 300 micron diameter and with a flow rate of 2.6 g/s. The solution time increases from left to right with 0.0025s as the time step size between them. Here the propagation of these structures can clearly be seen. From the velocity plots, it appears like a wave-like structure propagating in the upstream direction of the flow. It was observed that the packing fraction inside the tubes varies as these structures propagate causing the bulk density to change. As a result, these structures will be called “density waves” from this point on. It was observed that the waves continuously originate from the orifice into the flow. The velocity variations at the onset of the wave front are minimal (blue-ish shades in the velocity contours). This was clearly observed in animations created from the simulations. As these waves travel through the tube, they either dampen or overlap with other wave fronts to form a more stable wave with higher magnitudes of velocity fluctuations (red-ish shades in the velocity contours). These overlapped wave fronts continued to propagate upstream. This was clearly evident from the simulations results as only some waves turn red in color (in contour plots) and completely traverse the pipe to reach the free surface. From the velocity magnitude plots in figure 5.23, it can be observed that the particulate velocity increases within a wave and abruptly falls moving from top to bottom. This increase in velocity, which is of quadratic nature ($v \sim h^{0.5}$), can perhaps be explained as particles undergoing free-fall. So it can be understood that the density waves are actually minor free spaces created within the flow and the high velocity magnitudes of the particles are a result of the free-fall within these spaces. It is a result of these free spaces, the bulk properties also change when a wave propagates through the tube.

The phenomenon of density waves is not uncommon and has been experimentally identified by several research groups[25], [37], [38]. In general, a density wave consists of a densely packed particulate clog on the top and a low density bubble in the bottom[38]. Here the term “bubble” doesn’t represent any physical resemblances to a bubble but is just to explain the relatively less dense region. Raafat, et. al., suggested that for $6 < D_{\text{tube}}/D_{\text{particle}} < 30$, the length of the density waves and translational velocity remains constant with a (30-40) % variation in the packing fraction. Though the present case has $D_{\text{tube}}/D_{\text{particle}} \sim 25$, the density waves appear to be increasing their length as they travel along the tube height. Also, the packing fraction was not found to change by such drastic magnitudes. This may be because of the dense nature of the current operating regime.

To explicitly observe the features of these structures, a single wave front is observed and is plotted in figure 5.24. Here, the particle velocity and the contact force contour plots were made in a density wave structure. It can be observed that the particulate velocity increases from top to bottom. At the end, the particles become almost stagnant with zero velocity. This also supports the assertion that particles are undergoing a free fall state within the density wave. From the contact forces plot, the red region at the bottom of the wave signifies high magnitudes result from the sudden stopping of the particles after the free-fall. From the structure of the density wave, it can be understood that the longer the density wave spans along the tube, the higher the free-fall velocity is for the particles. As a result, the amplitude of fluctuations, which can be considered as a measure of the intensity of the density waves increases with the length of waves.

Figure 5.24 Density wave structure depicted using contour plots of particulate velocities and particulate contact forces

To quantitatively observe the flow fluctuations, the axial profiles were obtained from the simulation studies. In figure 5.25, the time averaged packing fractions and the particle flow rates were plotted at different axial locations for a mass flow rate of 2.6g/s in a tube 0.3m long and 7.747 mm in diameter. Particles of 300 microns were used in these simulations. The red-colored horizontal lines at each axial position represent the limits of packing fraction and flow rate oscillations. The blue colored horizontal lines represent the standard deviations at each data point. The black vertical dotted line is the average value throughout the tube. From the

standard deviations at each axial location, it can be observed that the fluctuation intensities increase with increasing axial location. They remain small near the orifice but rapidly increases as one moves away from the orifice to higher axial locations. The percentage variation of packing fraction is 10% while the flow rates vary by almost 6 times the average value. It can be seen that the minimum flow rates at higher axial locations are 0, which means the particles become stagnant at some instances during the flow. On the other hand, at locations near the orifices, there exists a continuous flow. This also supports the assertion that the intensity of density waves increases as they propagate from orifice to the top of the tube.

Despite of the increasing fluctuation, the time averaged flow properties still remain uniform across the length of the tube. But it may be that these fluctuation amplify and leading to varying bulk properties in longer tubes. Since it would be difficult to understand the internal flow profiles for these geometries, it would be difficult to experimentally observe these waves. Moreover, the frequencies of these oscillations are high, which would be difficult to observe with the naked eye. In addition, the dense nature of the flows makes laser techniques ineffective. The reason these waves were clearly seen in the simulations is because of the distinctive colors of particles with different velocities. An attempt was made to record the flow through a glass tube with a high speed camera to identify the traces of density waves. At such a slow motion, the intermittent nature of the flow was highlighted clearly. It was seen that the particles do become stagnant for a small amount of time and continue to move along the tube. This momentary stagnation could directly be related to the stagnation of particles at the bottom part of a density wave. More studies are currently being done to quantitatively understand this phenomenon.

Figure 5.25 Axial profiles of time-averaged packing fraction and particle flow rates (number of particles crossing a cross-section) along with standard deviations and oscillation limits

In figures 5.26 and 5.27, axial profiles were plotted versus orifice sizes to observe how the fluctuation intensities vary with flow rates. The tube diameter and lengths in both cases were

7.747mm and 0.3m respectively. Particles of diameter 300microns were used in these plots.

The different mass flow rates were 2.6g/s, 3.6g/s, 5.6g/s and 7.8g/s.

Figure 5.26 Axial profiles of time-averaged particle flow rates along with standard deviations and oscillation limits for flow rates of (a) 2.6g/s (b) 3.6 g/s (from left to right)

Figure 5.27 Axial profiles of time-averaged particle flow rates along with standard deviations and oscillation limits for flow rates of (a) 5.6g/s (b) 7.8 g/s (from left to right)

In all the cases, similar observations were made. The red horizontal lines are the amplitudes of fluctuations (greater and less than the mean value) and the blue horizontal lines the standard

deviation of the fluctuations. The fluctuations caused by the density waves increased with increasing axial locations. Additionally, the intensities of the amplitudes of these fluctuations increased with increasing flow rates. This can collectively be observed from all these plots. One common and peculiar observation in all these plots is that the time averaged values remain very much similar to the overall average value. So, it can be understood that the density waves may lead to fluctuations in the systems but the average behavior still remains the same.

Though the density wave phenomenon is just a flow feature of dense granular media it can also result in effecting the thermal properties when used as heat transfer fluids. Despite of the fact that the time averaged values of the flow behavior remains uniform throughout the tube length, the fluctuations may lead to different bulk properties at different axial locations. For example, the bulk density can be obtained from the packing fractions using the following equation:

$$\rho_{bulk} = \varphi_p \rho_p + \varphi_a \rho_a \quad (26)$$

In addition, the heat transfer behavior is also dependent on the velocity profiles of the flow. Though both the parameters are independently oscillating inside the tube and independently have uniform time-averaged profiles in axial direction, their fluctuation may be out of phase with each other and thus the heat transfer behavior may be different at different axial locations. As a result, more study needs to be done on this issue and careful observations needs to be made for characterizing the thermal behavior of these flow systems.

Chapter 6

Conclusions

Throughout the work, the different characteristics of gravity-driven dry granular flows in cylindrical tubes were studied using experimental and computational analyses. With an aim of using dense particles as a high-temperature HTF, the main focus was kept on the flow characterization of dense regimes with an approximate packing fraction of 60%. It was identified that in the dense regimes, the length of tube has no influence on the overall flow. In addition, it was also understood that the particle size has minimal influence on the dense flow regimes, given that their contact properties remain same. It was identified that flow rates should be within certain limits to avoid a non-choked configuration at the orifice that could lead to a dilute flow inside the tube. More work needs to be done to quantify this un-choking phenomenon in a global sense.

To obtain the velocity, packing fraction, pressure and shear force profiles, computational studies were conducted by modelling these particles using DEM simulation using a Lagrangian approach. These results could be considered novel as the dense flow regimes ($10^{-3} < I < 1$) have not extensively been studied either experimentally or computationally. The mean axial velocity profile, which has a near constant velocity core at the center and a shear-band near

wall followed a Gaussian curve fit. It was concluded that the Gaussian profiles may be characteristic feature of dense flow regimes in cylindrical tubes but not of quasi-static flow profiles. Unlike quasi-static flow regimes, the shear band thicknesses increased with increasing flowrates. However, they remained constant with the tube diameter. Wall normal and shear stresses had similar behavior to static granular beds in tubes where their values saturate after a certain depth. The effective friction, defined by Mohr-Coulomb law was found to increase with increasing inertial number. This behavior, which is a characteristic distinction from quasi-static regimes confirmed that the present configuration lies in the dense regime. Finally, it was also identified that the packing fractions are maximum at the flow center and minimum at the wall with identical behavior for all flow rates in dense regimes.

The computational results also showed that the Beverloo coefficients C and k do not vary with particle or tube diameter when the contact properties are kept constant. It was also identified that the Beverloo coefficients C is a direct function of the frictional properties, with particle-wall friction being dominant over particle-particle friction. Since the main objective of the research was to evaluate the use of particles as heat transfer fluids (HTFs), the particle-wall contact behavior was also studied. It was found that with smaller particulate sizes, the particle-wall contact points and the overall particle-wall contact area increases which could result in better heat transfer properties. In addition, it was also identified that the particle-wall contact decreases with increasing flow rates.

Finally, studies were done with long tubes to understand the characteristics of velocity fluctuations inside the tube at higher axial locations. It was identified that density wave structures propagate in the upstream direction leading to oscillating flow characteristics. The

intensity of these wave structures was found to increase with increasing axial position. It was also understood that the fluctuation intensity increases at higher flow rates. More study needs to be done before quantifying this phenomenon.

All these observations helped solidify our understanding the rheology of dense-regime granular flows in circular tubes. More work and parametric studies need to be conducted before formulating constitutive laws that could enable the development of hydro-dynamic analogies and effective flow properties for these flows.

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