ABSTRACT

SOLTMANN, LARS MICHAEL. Determination of Power Required through Accelerated Flight with Application to Unmanned Vehicles. (Under the direction of Dr. Charles Hall.)

Power required analysis is an important element of aircraft performance. The primary method for measuring this parameter is the classic PIW-VIW (power and velocity independent of weight) method which was developed in the first half of the 20th century. This method was originally developed for use with manned aircraft and therefore has some shortcomings when applied to unmanned aircraft, specifically those that are remotely piloted. The proposed method estimates the lift and drag models using the maximum likelihood technique which was applied to data acquired through acceleration and deceleration runs under the assumption that the pitch rate and lateral dynamics are negligible. The acceleration runs were performed by applying full throttle while the aircraft was flying just above the stall velocity and allowing it to naturally accelerate to maximum level flight velocity. The deceleration runs were performed in the opposite manner where the throttle was set to zero while the aircraft was flying at maximum velocity and allowed to naturally slow down until stall was reached. The collected data is reduced using the PIW-VIW technique whereby a single power curve is generated that is independent of aircraft weight and air density. Simulations indicate that deviations from the designed maneuvers, such as altitude changes and low pitch rates, are acceptable and do not change the resulting power curves. A hard limit on what constitutes low pitch rate was difficult to establish as system noise played a large role in determining the signal-to-noise ratio limit on the pitch rate. However, based on a model analysis it was concluded that the pitch rate could be safely ignored without consequences to the results. A low cost Raspberry Pi data collection system was designed for use on two test aircraft, the commercially available Nexstar and a re-configurable research aircraft called Astraeus. Data points using the classic PIW-VIW method were collected during the flight test program for comparison against the dynamic method. The results show good agreement between the deceleration runs and the PIW-VIW method for both aircraft. The acceleration runs generally had a poorer overall fit and a larger spread. This is due to excessive sensor noise caused by vibrations from the propulsion system. The acceleration runs were also more difficult to perform as the pilot was not always able to keep the thrust-induced pitching effects low during throttle up. A throttle ramp up was performed in an attempt to mitigate this effect and improve data quality; however, the analysis revealed no difference in the resulting power curves. Overall the dynamic power analysis was found to be a valid analysis tool for power required, especially when the deceleration run was used. Besides the decreased noise over the acceleration run, the deceleration run offers the advantage of neglecting the thrust model, especially when using aircraft with electric propulsion systems since they can be turned

off in flight resulting in zero residual thrust. This improves the accuracy of the resulting power required curves as well as saving time and effort since wind tunnel testing for propeller model generation can be avoided.

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Determination of Power Required through Accelerated Flight
with Application to Unmanned Vehicles

by
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DEDICATION

To my wife, Kimberly Soltmann. The sacrifices you have made in the interest of this research and my success as a graduate student mean more to me than you know. I am forever indebted to you for your selflessness and devotion.

BIOGRAPHY

Lars Soltmann was born in Raleigh, NC to Bernd and Renate Soltmann. He has a younger brother, Sven, who is also an aerospace engineer. Lars became interested in aerospace at a young age from building and flying model airplanes with his father. He graduated from Jesse O. Sanderson high school in 2005 and entered North Carolina State University in the fall of the same year. He obtained his Bachelors Degree in Aerospace Engineering in May 2009 and immediately began his graduate work on the Plank project. He spent two and a half years constructing Plank and performing the necessary ground tests to prepare it for flight. After graduating with his Masters Degree, Lars continued working in the Flight Research group on various projects, including this research. During his doctoral research he conducted contract work for VX Aerospace on the development and flight testing of a two-seat light sport aircraft with UAS variant. Lars is also a licensed licensed glider pilot and enjoys designing, building, and flying model aircraft.

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NOMENCLATURE

AR	Aspect ratio	R	Noise covariance matrix
C_D	Drag coefficient	S	Wing area
C_{D_i}	Lift induced drag coefficient	SE	Sensitivity matrix
C_{D_W}	Wind axis drag coefficient	T	Thrust
C_{D_0}	Zero lift drag coefficient	$\mathcal{T}$	Temperature
C_L	Lift coefficient	V	Velocity
$C_{L,minD}$	Lift coefficient at minimum drag	V_e	Equivalent velocity
C_{L_q}	Lift due to pitch rate	W	Weight
C_{L_0}	Lift at reference condition	X	Matrix of regressors
$C_{L\alpha}$	Lift curve slope	Z	Vector of measured outputs
$C_{L\delta e}$	Lift due to elevator angle	a	Acceleration
C_p	Pressure coefficient	b	Wing span
C_X	x-axis force coefficient	b'	Bias
C_Y	Side force coefficient	c_T	Thrust coefficient
C_{Y_W}	Wind axis side force coefficient	$\bar{c}$	Mean aerodynamic chord
C_Z	z-axis force coefficient	e	Oswald efficiency factor
C_l	Rolling moment coefficient	g	Gravity
C_m	Pitching moment coefficient	h	Altitude
C_n	Yawing moment coefficient	i, j, k	Indices
D	Drag	l_e	Length of tubing
$\mathcal{D}$	Dispersion matrix	m	Mass
D_{prop}	Diameter of propeller	n_0	Number of outputs
E	Energy	p	Roll rate
F	Force	q	Pitch rate
I	Moment of inertia	$\bar{q}$	Dynamic pressure
$\mathcal{I}$	Information matrix	r	Yaw rate
J	Cost	s	Standard error
K	Drag due to lift factor	t	Time
L	Lift	u, v, w	Body axis velocities
M	Moment	x	System states
$\mathcal{M}$	Mach number	y	System output
N	Total number of samples	z	Measured output
P	Power	Θ	Vector of unknown parameters
$\mathcal{P}$	Pressure	Ω	RPM

α	Angle of attack
β	Angle of side slip
γ	Heat capacity ratio
δ_e	Elevator angle
$\delta_{i,j}$	Kronecker delta
θ	Pitch angle
μ	Coefficient of viscosity of air
ν	Measurement noise
ξ	Scale factor error
ρ	Density
σ	Air density ratio
σ^2	Variance
ϕ	Roll angle
ψ	Yaw angle

Superscript

T	Transpose
-1	Inverse
$.$	Time derivative

$\hat{}$	Estimate
---------------------	----------

Subscript

A	Available
sl	Sea level
T	Thrust
g	Gravity
p	Propulsion system
req	Required
s	Standard condition
t	Test condition
x, y, z	Cartesian coordinate axes

Abbreviations

BHP	Brake horsepower
PIW	Power independent of weight
SNR	Signal to noise ratio
THP	Thrust horsepower
VIW	Velocity independent of weight

Chapter 1

Introduction

Aircraft performance is one of the most important elements in the design and analysis of an aircraft. While it is a broad topic, covering areas such as take-off, landing, climb, etc., it is a good indicator of how successful an aircraft will be to its customers and its designed mission. One aspect of performance that has always received considerable attention is power required. Power required is defined as the power or energy per unit time necessary to sustain level, unaccelerated flight at a given velocity and altitude. This is an extremely important performance parameter that, along with the power plant, determines a number of other parameters such as maximum level speed, climb performance, flight efficiency, etc.

Estimation of the power required curve during the design phase, or as a theoretical evaluation, is done through a drag analysis of the aircraft known as a drag build-up. To perform a drag build-up, the drag of individual vehicle components are estimated and summed to produce an estimate of the overall vehicle drag [20]. These build-up methods provide 'ball park' estimates but may not be entirely accurate. If a computational fluid dynamics (CFD) model of the aircraft exists, it can provide a much more accurate estimate of the overall vehicle drag and therefore, its power required. During the flight testing phase of a project it provides feedback to the designer on how well actual performance compares with designed and what the customer can actually expect in terms of overall performance.

Methods to measure this quantity have been developed and improved over time and were generally geared towards manned aircraft until unmanned aircraft became of interest within the last two decades. Since this time, the field of unmanned aerial systems (UAS) has seen a dramatic rise in interest, both militarily and privately. The majority of use in both the military and public/private sector is surveillance, with uses ranging from target tracking, search and rescue, agriculture crop monitoring, to geographical mapping. With such a dramatic increase in the number of UAS over the years, much research has gone into developing new technologies including test methods. Differences between manned and unmanned aircraft can be vast, espe-

cially when it comes to size and weight. The testing methods developed throughout the history of aviation were done so mainly for manned aircraft although they can generally be applied to UAS without too many problems. However, in certain instances modifications can, or need to be made, so that the methods can be easily applied to UAS. In the case of power required analysis, the classic methods can be applied without issues but some changes can be made to increase accuracy or to incorporate technologies that were not available at the time the original method was developed. For example, the majority of small UAS are powered by electric motors which historically found little use in aircraft propulsion. Internal combustion engines were the most common power plants used so the methods were developed around their performance. Testing methods were also designed to be performed by the pilot who had direct visual, sensory, and instrumentation feedback. While most UAS today have telemetry feedback of some kind, some small vehicles as well as research aircraft are remotely piloted with little to no feedback. This can complicate following certain test procedures that require somewhat accurate knowledge of the current aircraft states. Therefore, new testing methods have been and will continue to be developed catering to the ever growing popularity of UAS.

In order to develop or improve methods in any field of study, an understanding of the basics is required. Power required is derived from the equations of motion governing steady level flight. Figure 1.1 shows a free body diagram for an aircraft in level, unaccelerated flight with the assumption that the thrust acts along the body x -axis. Summing the forces along the

Figure 1.1: Free body diagram of an aircraft in steady, level, unaccelerated flight

x and y axis yields

$$\sum F_x = 0 = T \cos(\alpha) - D \quad (1.1)$$

$$\sum F_y = 0 = L + T \sin(\alpha) - W \quad (1.2)$$

where T is thrust, D is drag, W is weight, L is lift, and α is the angle of attack. Assuming small angles, the equations along the x and y axis simplify to thrust is equal to drag and lift is equal to weight respectively. Applying the definition of power then gives the well known equation for power required to maintain flight at a given condition

$$\begin{aligned} P &= F \cdot V \\ P_{req} &= DV = T_{req}V \end{aligned} \quad (1.3)$$

where P is power, F is force, and V is velocity. This is the main equation that power analysis is based on in design, testing, and this research. This equation can be expressed in a few different ways that yields important characteristics about the power required curve. Assuming that an aircraft is in steady, level, unaccelerated flight and its drag model follows the form

$$C_D = C_{D_0} + \frac{C_L^2}{\pi e AR} \quad (1.4)$$

where C_D is the drag coefficient, C_{D_0} is the zero lift drag coefficient, C_L is the lift coefficient, e is the Oswald efficiency factor, and AR is the wing aspect ratio. The definition of power from Eq. 1.3 can then be used to expand the power required equation to

$$P_{req} = \frac{1}{2}\rho V^3 S C_{D_0} + \frac{\frac{W^2}{\frac{1}{2}\rho V S}}{\pi e AR} \quad (1.5)$$

where ρ is the air density and S is the wing area. Taking the derivative with respect to velocity and setting it equal to zero yields

$$\begin{aligned} \frac{dP_{req}}{dV} &= \frac{3}{2}\rho V^2 S C_{D_0} - \frac{\frac{W^2}{\frac{1}{2}\rho V^2 S}}{\pi e AR} = 0 \\ &= \frac{3}{2}\rho V^2 S \left(C_{D_0} - \frac{1}{3} C_{D_i} \right) = 0 \\ C_{D_0} &= \frac{1}{3} C_{D_i} \end{aligned} \quad (1.6)$$

where C_{D_i} is the lift induced drag coefficient. Flight at this aerodynamic condition will achieve minimum power required. This also corresponds to the point at which endurance will be maximized based on the Breguet endurance formula [2]. The condition for maximum range can also be solved for using a similar approach. In this case, the equation for thrust required is used

along with the same drag model as before

$$T_{req} = D$$

$$T_{req} = \frac{1}{2}\rho V^2 S C_{D_0} + \frac{W^2}{\frac{1}{2}\rho V^2 S \pi e A R} \quad (1.7)$$

Taking the derivative with respect to velocity and setting it equal to zero yields

$$\frac{dT_{req}}{dV} = \rho V S C_{D_0} - \frac{4W^2}{\rho V^3 S \pi e A R} \quad (1.8)$$

Simplifying the equation by making the substitution

$$\frac{4W^2}{\rho^2 V^4 S^2} = C_L^2 \quad (1.9)$$

shows that the condition necessary for maximum range is

$$C_{D_0} = \frac{C_L^2}{\pi e A R} = C_{D_i} \quad (1.10)$$

These two conditions for maximum range and endurance are key performance metrics that depending on the customer/design requirements, will drive the overall system design. Airliners and cargo aircraft will generally be range driven designs so the aerodynamics will be tailored such that the cruise condition corresponds to the maximum range condition. It is also important for any pilot to be aware of the flight velocity corresponding to maximum range since in the event of an emergency, flying at this condition could be the difference between making it to a safe emergency landing spot and not. For other aircraft such as surveillance aircraft, the maximum endurance criterion plays a larger role in the overall design. These designs will generally have higher aspect ratio wings to increase efficiency and decrease drag. This flight condition is also important for a pilot to be aware of in the event that endurance needs to be maximized such as bad weather conditions in the landing area and the pilot having to wait for it to clear. It is also important to note that on a majority of aircraft, the velocity corresponding to minimum power required, maximum endurance, is not achievable as it falls below the stall velocity. In this case the minimum power required velocity will be the stall velocity. A power required analysis is generally presented along with a power available analysis so that a better idea of the overall performance can be conveyed. Doing so provides insight into another group of performance parameters used in climb analysis. One parameter that can be visually determined from the power required/available plot is maximum level flight speed. The point where the power required and available intersect is the maximum airspeed that can be achieved in steady level flight. An example plot of a power required and power available curve with the parameters discussed

earlier marked, is shown in Figure 1.2. The difference between the power available and required curves shows the excess power available which provides information on the climb performance of the aircraft but will not be discussed here as it is outside the scope of this research.

Figure 1.2: Power required and power available plot for the PLANK UAS

1.1 Motivation

It is evident from the basic analysis that power required is an important parameter that provides valuable insight into the expected performance of an aircraft. As previously mentioned, measuring this quantity is based on classical techniques that are well suited for manned aircraft. The application of these methods to certain unmanned aircraft, specifically remotely piloted vehicles with little or no telemetry feedback, requires a change in approach to suit the testing environment and limitations. The motivation for this research was to decrease test time and pilot work load by using dynamic flight maneuvers to extract the power required data across the entire airspeed envelope in a single pass when compared to the classic PIW-VIW method. The maneuvers used in this research were specifically geared toward remotely piloted vehicles so they had to be easy to perform and allow for deviations from the intended maneuver without significant consequences to the quality of the data. In prior research, the classic PIW-VIW method was the primary method used to measure power required and produced varying and inconsistent results due to violations of the assumptions. For a remote operator, the assump-

tions are difficult to maintain so this motivated the research to develop an improved approach to power required measurement.

Chapter 2

Literature Review

Power required, or more specifically drag, has been heavily researched since the early days of aviation. Many of the estimation techniques were developed in the early part of the 20th century in an effort to make aircraft faster, more efficient, and to better understand overall performance. A majority of these techniques have been refined over the years as aircraft designs and technology have improved but are still applicable today and some are actively recognized by the FAA for use in aircraft certification [12].

2.1 PIW-VIW Method

The classic PIW-VIW method is probably the most common and well known method for determining power required. The method was developed in the late 1930's and early 1940's as a way to obtain drag and power measurements for an aircraft utilizing basic instrumentation and flight maneuvers [3][7][21]. As was shown in the previous chapter, in steady level unaccelerated flight the assumption that thrust is equal to drag holds true and for manned aircraft it is generally easy to maintain these steady state conditions. Therefore, if engine, altitude, and airspeed data can be recorded, the thrust/power can be determined. The calculated power and velocity are then normalized with respect to weight and air density to generate a single curve that is independent of air density and airplane weight [12]. These two independent terms are known as VIW (velocity independent of weight and air density) and PIW (power independent of weight and air density). The derivation of this method relies on two key assumptions. The first assumption is that for a given angle of attack, the lift coefficient and drag coefficient are constant values. This means that the lift and drag for a given angle of attack at one condition, e.g. the test condition, is equal to the lift and drag at the same angle of attack at any other condition, like standard atmospheric conditions. This allows for a direct comparison of

the aircraft weights at two different conditions through the equation

$$\frac{W_s}{W_t} = \frac{L_s}{L_t} = \frac{0.5\rho_{sl}\sigma_s V_s^2 SC_L}{0.5\rho_{sl}\sigma_t V_t^2 SC_L} \quad (2.1)$$

where σ is the air density ratio and the subscripts t and s represent the test and standard conditions respectively. Canceling like terms, solving for V_s , and making the following substitution

$$V_e = V_t\sqrt{\sigma} \quad (2.2)$$

where V_e is the equivalent velocity. Equivalent velocity is defined as the velocity at sea level in standard atmospheric conditions for which the dynamic pressure is the same as the dynamic pressure at the true airspeed and altitude at which the aircraft is flying. The equation for VIW is given as

$$VIW = V_s = \frac{V_e}{\sqrt{W_t/W_s}} \quad (2.3)$$

The second key assumption of the PIW-VIW method plays into the PIW term and assumes that the propeller efficiency is constant. For aircraft with constant speed propellers, this is a relatively safe assumption [12]. Through a mathematically similar approach as for the VIW term, the ratio of standard to test condition thrust horsepower (THP) is given as

$$\frac{THP_s}{THP_t} = \frac{V_s (0.5\rho_{sl}\sigma_s V_s^2 S) / 550}{V_t (0.5\rho_{sl}\sigma_t V_t^2 S) / 550} \quad (2.4)$$

where the division by 550 is used to convert to horsepower. Using the constant propeller efficiency assumption, thrust horsepower can be replaced by brake horsepower (BHP) and substituting VIW for V_s yields

$$PIW = \frac{BHP_t \sqrt{\sigma_t}}{(W_t/W_s)^{1.5}} \quad (2.5)$$

Plotting PIW vs VIW provides the normalized power curve for the aircraft that can be used to determine the power under any weight and air density combination.

The flight test method used to acquire the necessary data for generating the PIW-VIW curve is relatively easy to perform. It does rely on smooth air as wind gradients and turbulence will upset the steady state conditions which can result in skewed data. The test is started at a low airspeed at the desired test altitude where ambient atmospheric conditions are recorded. The aircraft is then accelerated to maximum level flight speed. For accurate data, the aircraft must be at equilibrium, which may take anywhere from a few seconds to minutes to achieve depending on the aircraft. For most aircraft the phugoid mode is lightly damped and has a period on the order of minutes. It is critical that this mode be fully damped before data is

taken. Once steady state has been reached, airspeed, altitude, ambient temperature, RPM, manifold pressure, fuel quantity, fuel flow, and carburetor air temperature (if applicable) are recorded. Manifold pressure is then decreased by 1 inHg. Again, the airspeed must be allowed to stabilize before data can be recorded. This process is repeated until the back side of the power curve or stall is reached. The process is then repeated for different altitudes. The collected data is used in Eqs. 2.3 and 2.5 to determine VIW and PIW values for each test point. Since real measurements have noise associated with them, a normalization technique is used to transform the cubic power curve into a straight line for easy fitting. This is done by multiplying PIW by VIW and VIW by VIW^3 giving

$$PIW \times VIW = m_0 VIW^4 + b_0 \quad (2.6)$$

where m_0 and b_0 are the slope and intercept, respectively, determined from the resulting plot.

This technique was developed in the time before unmanned vehicles, but is still applicable. Depending on the type of vehicle this technique is applied to, slight modifications to the underlying equations and procedures are necessary. A detailed discussion of the changes can be found in Chapter 3. At this point it is evident that while this method is easy to perform and implement, it does have some drawbacks. The main drawback is that steady state conditions must be present for the PIW-VIW assumptions to be accurate. Since the phugoid mode requires a long wait time to be fully damped out, this causes the total test time to increase significantly. Also if any disturbances are encountered during this settling period, it may cause enough of an excitation that additional wait time is needed before accurate data can be taken. For manned or unmanned aircraft on a large test range where long runs of straight and level flight can be conducted, this does not present a problem. However, in a test range with limited space or where the aircraft has to remain within visual contact, this can present a problem. By the time the pilot needs to maneuver the aircraft to remain within the test range boundaries, most likely not enough time will have passed to have achieved steady state conditions, causing inaccurate data. Small aircraft are also more susceptible to atmospheric disturbances than larger aircraft making it much harder to maintain equilibrium conditions. The situation is further complicated by the fact that the aircraft states become harder for the pilot to discern as the distance between the pilot and aircraft increases.

2.2 W/δ Method

The PIW-VIW technique is well suited for flight tests at low subsonic velocities where the assumption of constant drag for constant true speed and density altitude is valid [23]. When jet powered aircraft became popular it became evident that compressibility effects could no longer

be neglected. Standardizing flight data taken during high speed runs using the PIW-VIW flight test procedure and reduction method resulted in skewed data. To account for compressibility effects a pressure altitude reduction method, known as the W/δ method, can be used instead of a density reduction method. In this method the aircraft is flown at constant values of weight over the pressure ratio δ .

Starting with the drag model given by Eq. 1.4, both sides are multiplied by $\bar{q}S$ to give drag

$$D = C_{D_0}\bar{q}S + \frac{C_L^2\bar{q}S}{\pi eAR} \quad (2.7)$$

where $\bar{q}$ is the free-stream dynamic pressure. Applying the lift equation ($L = C_L\bar{q}S$) under the assumption that $L = W$ results in

$$D = C_{D_0}\bar{q}S + \frac{W^2}{\pi eAR\bar{q}S} \quad (2.8)$$

The dynamic pressure can be written in terms of Mach number where

$$\bar{q} = \frac{1}{2}\gamma\mathcal{P}_t\mathcal{M}^2 \quad (2.9)$$

where γ is the heat capacity ratio, $\mathcal{P}$ is pressure, and $\mathcal{M}$ is the Mach number. Substituting this equation into Eq. 2.8 results in

$$D = \frac{C_{D_0(\mathcal{M})}\gamma\mathcal{P}_t\mathcal{M}^2S}{2} + \frac{2W^2}{\pi e_{(\mathcal{M})}AR\gamma\mathcal{P}_t\mathcal{M}^2S} \quad (2.10)$$

Both sides the equation are now multiplied by $\frac{\mathcal{P}_{sl}}{\mathcal{P}_t}$ whereby the pressure ratio is defined as $\delta = \frac{\mathcal{P}_t}{\mathcal{P}_{sl}}$

$$\frac{D}{\delta} = \frac{C_{D_0(\mathcal{M})}\gamma\mathcal{P}_{sl}\mathcal{M}^2S}{2} + \frac{2(W/\delta)^2}{\pi e_{(\mathcal{M})}AR\gamma\mathcal{P}_{sl}\mathcal{M}^2S} \quad (2.11)$$

This equation is only a function of $\mathcal{M}$ and W/δ meaning that for any given values of these parameters, the drag can be determined. It is important to note that this method does not account for the changing aircraft weight during flight and because of this a family of curves is generated instead of a single curve like in the PIW-VIW method. Lines of constant D/δ , or P_{req}/δ , are plotted against Mach number. The above equations are generally manipulated further to determine range and endurance curves for the aircraft as functions of Mach number and weight for a given W/δ [22]. These curves are useful for long range cruise conditions on high speed aircraft where the pilot can determine what Mach number is needed based on the current aircraft weight and altitude to maximize the range. The flight test procedure is very similar to the PIW-VIW method in that steady state conditions are set at the desired flight

conditions. In this method, however, additional prior planning is needed to predict the aircraft weight over time to determine what altitudes need to be flown at each test point in order to maintain a constant W/δ . Timing during these flights is critical as spending too much time at one condition may burn too much fuel and make the aircraft too light for the next test point, changing the W/δ . Once the steady state condition has been reached, engine, velocity, and atmospheric data are recorded for post flight data reduction.

The majority of operational UAS, especially those in the private sector, operate in the very low subsonic region so this test method is not directly applicable. However, it does represent a method for drag/power determination. A low subsonic version of Eq. 2.11 can be written by starting with Eq. 2.8 and using the standard definition of the dynamic pressure along with the definition of density

$$\rho = \frac{\mathcal{P}}{\mathcal{RT}} \quad (2.12)$$

where $\mathcal{R}$ is the universal gas constant and $\mathcal{T}$ is temperature. Using the same procedure as before, the low subsonic version of Eq. 2.11 can be rewritten as

$$\frac{D}{\delta} = \frac{C_{D_0} \mathcal{P}_{sl} V^2 S}{2\mathcal{RT}} + \frac{2\mathcal{RT} (W/\delta)^2}{\pi e A R \mathcal{P}_{sl} V^2 S} \quad (2.13)$$

In this case the equation is a function of W/δ , velocity, and temperature. Adding the temperature adds an extra element of complexity especially during large altitude changes or long tests. While this method could be applied to low subsonic UAS, it is not as useful or as easy as the PIW-VIW method.

2.3 Drag Polar Estimation

While the PIW-VIW technique has been the standard for many years, more modern techniques based on dynamic maneuvers have been developed as sensor and recording technologies have advanced. Since the power required is directly related to drag, the power required can be calculated if the drag polar can be determined. Estimating the drag polar from measured flight dynamics can be done in several ways. Many of these methods are based on model determination through optimization although some take a direct approach to solving for the drag through the equations of motion.

Many researchers have applied maximum likelihood techniques to model aircraft dynamics to determine the aircraft drag polar [5][10][18]. The general process is to apply the equations of motion to recorded flight data taken during a series of specifically designed maneuvers [19]. These maneuvers are designed to provide a statistically significant amount of information for each of the regressors so that a confident and accurate estimate can be made for each of the unknown

parameters. A more detailed discussion of the maximum likelihood estimation technique can be found in Chapter 3. A common maneuver used in drag polar estimation is the pushover/pull-up (roller coaster) maneuver, which is shown in Figure 2.1[5]. This maneuver is started by slowly

Figure 2.1: One cycle of the pushover/pull-up technique

pushing the nose of the aircraft down at constant rate of ≈ 0.1 g/sec until a certain load factor (between -1 and 0) has been achieved. The nose is then pulled up at the same rate until a certain positive load factor has been achieved (usually 2) [5][24]. The load factor range covered during the maneuver should cover most of the lift coefficient range of the aircraft. At this point, which equations of motion are applied to the data varies depending on the models the user is trying to apply. Another maneuver that can be performed to achieve the same results is the wind-up turn, shown in Figure 2.2. In this maneuver, it is much easier to maintain velocity

Figure 2.2: Wind-up turn maneuver

and altitude because it is performed at a constant altitude. The wind-up turn begins with the aircraft initiating a shallow turn and then slowly decreasing the turn radius to increase the load factor, usually by about 0.5-0.8 g/sec [5]. The turn is tightened until the aircraft reaches the maximum allowable load limit or the target load limit for the test. Model determination using the wind-up turn maneuver may require special care as the lateral dynamics may need

to be included, as opposed to the pushover/pull-up maneuver where the wings remain level the entire time and the lateral dynamics can be safely ignored. A similar argument can be made here about the application of these maneuvers to UAS as was done with the PIW-VIW method. These maneuvers were designed for manned aircraft where the pilot has direct real-time feedback of the load factor. More advanced UAS will be able to have the autopilot perform these maneuvers without problem, but for those UAS that have a ground based pilot, maintaining a constant g-force rate will be difficult even if telemetry is available. This is especially true for the pushover/pull-up maneuver since it is conducted along a straight line meaning the aircraft will travel a significant distance by the end of the maneuver, most likely putting it out of visual range or outside of the test boundaries.

A majority of researchers who use these maneuvers determine the drag polar through the axial and vertical force models

$$C_X = f(\alpha, \delta_e, V) \quad (2.14)$$

$$C_Z = f(\alpha, \delta_e, V) \quad (2.15)$$

$$C_D = -C_X \cos \alpha - C_Z \sin \alpha \quad (2.16)$$

where C_X and C_Z are the force coefficients along their respective axes and are functions of various combinations and orders of the given state variables [5] [10]. The output of the equations of motion based on the above axial and vertical force models are compared against easily measurable aircraft parameters to determine model conformity. The most common measurable aircraft parameters used for comparison are [10] [19]

$$z = [q \quad \dot{q} \quad \alpha \quad V \quad \theta \quad a_x \quad a_z]^T \quad (2.17)$$

where q is the pitch rate, θ is the pitch angle, and a is the acceleration along the given axis. The resulting drag polars have been compared to those obtained through wind tunnel testing and are in good agreement [5][10][13][24].

Another approach to drag polar estimation that is similar to maximum likelihood is estimation through least squares. Both techniques are optimal with respect to a given cost function, but their analysis process differs. In least squares it is assumed that the output is directly measurable along with the regressors. The only unknowns are the parameters for which the user applies a model and uses least squares regression to determine the unknown parameters based on the given model. The least squares model is given as

$$\hat{\Theta} = (X^T X)^{-1} X^T Z \quad (2.18)$$

where X is a matrix of regressors, Z is a vector of measured outputs, and $\hat{\Theta}$ is the vector of

estimated parameters [19]. In reference 24, the authors make the assumption that lift and drag can be directly measured using the equations

$$C_L = \frac{n_{za}W - T \sin \alpha}{\frac{1}{2}\gamma\mathcal{P}_t\mathcal{M}^2S} \quad (2.19)$$

$$C_D = \frac{T - n_{xa}W}{\frac{1}{2}\gamma\mathcal{P}_t\mathcal{M}^2S} \quad (2.20)$$

where n_{xa} and n_{za} are the longitudinal and vertical load factors respectively. The model to be estimated is

$$C_D = C_{D_0}(\mathcal{M}) + K_1(\mathcal{M}) + K_2(\mathcal{M})C_L^2 \quad (2.21)$$

where C_{D_0} , K_1 , and K_2 are the unknown parameters to be determined. Since this particular research investigated the drag polar throughout the transonic region, the unknown parameters were allowed to change with Mach number [24]. The same pushover/pull-up maneuver was used to generate a time history of the regressors with a suitable amount of information to accurately estimate the unknown parameters. The resulting drag polars are in good agreement with previous estimates for the aircraft. Although this research was conducted in the transonic and supersonic region, the above method could directly be applied to subsonic analysis by assuming that the unknown parameters remain constant with airspeed.

The equations of motion can also be used to directly calculate drag without using optimization. Care must be taken when using these methods because they do not include or make any assumptions about measurement noise. Using a free body diagram, the sum of the forces along the wind axes give the equation for drag as [13]

$$D = T \cos(\alpha + \lambda_T) \cos \beta + T \sin \beta - D_{ram} \cos \beta + ma_x \quad (2.22)$$

where λ_T is the angle of the thrust line, D_{ram} is ram drag, and β is the angle of side slip. Two out of the four terms in equation 2.22 contain thrust which indicates a high correlation between the drag and the thrust measurement. To that end, the author in reference 13 did an extensive engine calibration in conjunction with the engine manufacturer to ensure an accurate thrust model [13]. The main goal of that research was to evaluate the drag characteristics of different configurations of the aircraft and engine inlet/exit geometry. Several maneuvers were performed during the flight test, each with a specific purpose. The maneuvers fell into two categories: dynamic and steady state. The dynamic maneuvers performed consisted of the same pushover/pull-up and windup turn maneuvers already discussed as well as acceleration and deceleration runs [13]. The dynamic maneuvers allow for several points on the drag polar to be determined in a single run, unlike the steady state maneuver, which is essentially

the same as the PIW-VIW method that provides only a single point on the drag polar. It is noted that the acceleration/deceleration maneuvers do not provide as many data points as the pushover/pull-up maneuver since they are not able to achieve the high lift coefficients at high velocities ($M \approx 1$). Therefore, the pushover/pull-up and windup turn maneuvers are favored when a comprehensive characterization of the Mach number/drag relationship is desired [13]. The author also emphasizes that the pushover/pull-up maneuvers must be performed slowly so as not to violate the quasi-steady state assumption. The equations used do not contain pitch effects so they must be kept low in order to be considered negligible. Although the case is made here that the pushover/pull-up and windup turn maneuvers are favored over the acceleration/deceleration runs, the opposite case could be made when applying these maneuvers to UAS. Acceleration/deceleration runs are much easier to perform by a ground based pilot and since the speeds are in the low subsonic region, the drag characterization with respect to Mach number is of no importance. Therefore, these runs can be used effectively in drag/power required analysis.

2.4 Energy Approach

Another approach to performance based flight testing that can be applied to drag estimation is energy analysis. Energy approaches are generally used in climb analysis of high performance aircraft but can be applied globally. The methods were originally formulated to deal with the change in true airspeed that occurs with large increases in altitude [12] [16]. In order to climb, an aircraft must have excess power, however, this excess power can also be used to accelerate instead of climb. The total energy of an aircraft in flight is given by the sum of the kinetic and potential energy as

$$E = \frac{1}{2}mV^2 + mgh \quad (2.23)$$

where E is energy, m is mass, g is the acceleration due to gravity, and h is altitude. Taking the time derivative of the equation yields

$$\dot{E} = mV\dot{V} + mg\dot{h} \quad (2.24)$$

Level or climbing acceleration is achieved when thrust is greater than drag so the first term in equation 2.24 can be rewritten and the equation becomes

$$\dot{E} = P_{excess} = mV(T - D) + mg\dot{h} \quad (2.25)$$

For a level acceleration, $\dot{h}$ is equal to zero and the equation can be simplified. A time history of the velocity can then be used to determine the rate of change of the velocity and determine the

excess power [12]. If the instantaneous thrust is known, the drag can be determined. It could also be used on a deceleration run where thrust is zero, further simplifying the equation. In the event that a change in altitude occurs, the second term of equation 2.25 must be included. While this approach works well in theory, it is often difficult to maintain a constant altitude and environmental effects as well as sensor noise make finding the time derivative of velocity difficult, thereby causing a poor estimate of drag. Pitch rates are also not included in this model leading to further error if significant pitching occurs. The rotational energy term could be added to 2.25 but in previous analysis conducted by the author the results have been poor.

Chapter 3

Theoretical Development

The motion of an aircraft is a complex interaction between forces and moments involving many variables. Mathematically describing such complex systems in detail is generally difficult so model based approaches are used to describe the motion using simplified mathematics. The power required values for an aircraft that are most often presented are the steady state values. An aircraft is rarely at steady state conditions so the power required at any given time is generally not equal to the steady state value, but a combination of the steady state value plus some delta due to maneuvering. Determining the instantaneous total power required is trivial but finding the component solely due to maneuvering is not. Model based design provides a relatively easy way to determine how the individual parts of the system interact to determine what the base power required must be at any time. To do this, a mathematical description of an aircraft is necessary.

3.1 Mathematical Model of an Aircraft

The equations of motion for an aircraft have been well studied and only a brief derivation will be given here, highlighting the assumptions made to obtain the final equations. A detailed derivation can be found in reference 19. The derivation assumes a standard right hand coordinate system with the origin at the center of gravity and the x, y, z axes pointed forward through the nose, out the right wing, and through the underside respectively. This coordinate system is fixed on the aircraft and is known as the body-axis coordinate system. The assumptions made to start the derivation are as follows [19]

1. The aircraft is a rigid body with fixed mass distribution and constant mass,
2. The air is at rest relative to the earth,
3. The earth is fixed in inertial space,

4. The earth's surface is assumed to be flat,
5. Gravity is uniform and does not change with altitude,
6. Thrust from the propulsion system acts along the body x -axis and through the center of gravity.

Using Newton's second law of motion, the general motion of an aircraft can be described by the equations

$$\mathbf{F}_{aero} + \mathbf{F}_g + \mathbf{F}_T = \frac{d}{dt}(m\mathbf{V}) = m\dot{\mathbf{V}} + \boldsymbol{\omega} \times m\mathbf{V} \quad (3.1)$$

$$\mathbf{M}_{aero} = \frac{d}{dt}(I\boldsymbol{\omega}) = I\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times I\boldsymbol{\omega} \quad (3.2)$$

where

$$\mathbf{F}_{aero} = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \bar{q}S \begin{bmatrix} C_X \\ C_Y \\ C_Z \end{bmatrix} \quad \mathbf{M}_{aero} = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \bar{q}S \begin{bmatrix} bC_l \\ \bar{c}C_m \\ bC_n \end{bmatrix} \quad (3.3)$$

$$\boldsymbol{\omega} = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad \mathbf{V} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad I = \begin{bmatrix} I_x & -I_{xy} & -I_{xz} \\ -I_{yx} & I_y & -I_{yz} \\ -I_{zx} & -I_{zy} & I_z \end{bmatrix} \quad (3.4)$$

$$\mathbf{F}_g = m \begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix}_{body} \quad \mathbf{F}_T = \begin{bmatrix} T \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{h}_p = \begin{bmatrix} I_p\Omega_p \\ 0 \\ 0 \end{bmatrix} \quad (3.5)$$

Within these definitions, $\bar{c}$ is the mean aerodynamic chord, b is the wing span, M is the moment, p is the roll rate, r is the yaw rate, Ω_p is the propulsion system RPM, I is the mass moment of inertia, C_Y is the side force coefficient, C_l, C_m, C_n are the rolling, pitching, and yawing moment coefficients, and u, v, w are the body axis velocities. By substituting equations 3.3 through 3.5 into equations 3.1 and 3.2 and completing the necessary algebra, the final force and moment equations are obtained.

$$\dot{u} = rv - qw + \frac{\bar{q}SC_X}{m} - g \sin(\theta) + \frac{T}{m} \quad (3.6)$$

$$\dot{v} = pw - ru + \frac{\bar{q}SC_Y}{m} + g \cos(\theta) \sin(\phi) \quad (3.7)$$

$$\dot{w} = qu - pv + \frac{\bar{q}SC_Z}{m} + g \cos(\theta) \cos(\phi) \quad (3.8)$$

$$\dot{p} - \frac{I_{xz}}{I_x} \dot{r} = \frac{\bar{q}SbC_l}{I_x} - \frac{(I_z - I_y)}{I_x} qr + \frac{I_{xz}}{I_x} qp \quad (3.9)$$

$$\dot{q} = \frac{\bar{q}SbC_m}{I_y} - \frac{(I_x - I_z)}{I_y} pr - \frac{I_{xz}}{I_y} (p^2 - r^2) + \frac{I_p}{I_y} \Omega_p r \quad (3.10)$$

$$\dot{r} - \frac{I_{xz}}{I_z} \dot{p} = \frac{\bar{q} S b C_n}{I_z} - \frac{(I_y - I_x)}{I_z} p q - \frac{I_{xz}}{I_z} q r - \frac{I_p}{I_z} \Omega_p q \quad (3.11)$$

The rotational kinematic equations are obtained by rotating the body axis rates into to the earth axes

$$\dot{\phi} = p + \tan \theta (q \sin \phi + r \cos \phi) \quad (3.12)$$

$$\dot{\theta} = q \cos \phi - r \sin \phi \quad (3.13)$$

$$\dot{\psi} = \frac{q \sin \phi + r \cos \phi}{\cos \theta} \quad (3.14)$$

where ϕ is the roll angle and ψ is the yaw angle. The above nine equations (3.6 - 3.14) define the non-linear motion of an aircraft through the atmosphere based on the assumptions made at the beginning of the derivation. These equations were formulated in the body axis coordinate system and were used in the development of the flight simulator. In the analysis of flight test data the force equations are not quite as useful because the body axis velocities u, v, w cannot be directly measured. In this case, rotating the body axis coordinate system to align with the incoming relative wind vector allows for u, v, w to be expressed in terms of V, α, β , which are directly measurable. The body axis velocities are related to the wind axis through the equations

$$u = V \cos \alpha \cos \beta \quad (3.15)$$

$$v = V \sin \beta \quad (3.16)$$

$$w = V \sin \alpha \cos \beta \quad (3.17)$$

$$V = \sqrt{u^2 + v^2 + w^2} \quad (3.18)$$

The body axis velocities are also related to the angle of attack and side slip angle through the equations

$$\alpha = \tan^{-1} \left(\frac{w}{u} \right) \quad (3.19)$$

$$\beta = \sin^{-1} \left(\frac{v}{V} \right) \quad (3.20)$$

Substituting equations 3.6 - 3.8 and 3.15 - 3.18 into equations 3.19 and 3.20 and performing the necessary algebra, the wind axis equations are given as

$$\begin{aligned} \dot{V} = & -\frac{\bar{q} S}{m} C_{D_w} + \frac{T}{m} \cos \alpha \cos \beta + g(\cos \phi \cos \theta \sin \alpha \cos \beta \\ & + \sin \phi \cos \theta \sin \beta - \sin \theta \cos \alpha \cos \beta) \end{aligned} \quad (3.21)$$

$$\begin{aligned}\dot{\alpha} = & -\frac{\bar{q}S}{mV \cos \beta} C_L + q - \tan \beta (p \cos \alpha + r \sin \alpha) - \frac{T \sin \alpha}{mV \cos \beta} \\ & + \frac{g}{V \cos \beta} (\cos \phi \cos \theta \cos \alpha + \sin \theta \sin \alpha)\end{aligned}\quad (3.22)$$

$$\begin{aligned}\dot{\beta} = & -\frac{\bar{q}S}{mV} C_{Y_W} + p \sin \alpha - r \cos \alpha + \frac{g}{V} \cos \beta \sin \phi \cos \theta \\ & + \frac{\sin \beta}{V} \left(g \cos \alpha \sin \theta - g \sin \cos \phi \cos \theta - \frac{T \cos \alpha}{m} \right)\end{aligned}\quad (3.23)$$

where C_{D_W} and C_{Y_W} are the wind axis drag and side force coefficients. If the side slip angles are sufficiently small, C_{D_W} and C_{Y_W} reduce to C_D and C_Y .

All of the above equations can be used as they are or further simplified by linearizing them about an operating point by Taylor series expansion. Since the power required analysis conducted in this research was not done in real-time and had no computational restrictions, the full nonlinear equations were used. In addition, the aerodynamic model equations (section 3.3.1) must be defined before the equations can be used. The aerodynamic models in general depend nonlinearly on present and past values of airspeed, angle of attack, rotational rates, etc., therefore, their dependencies can be difficult to characterize [19].

3.2 Maximum Likelihood Estimation

Given a set of aircraft parameters, it is possible to accurately model the response of the aircraft using the equations of motion. In practice, however, the aircraft parameters are generally unknown but can be estimated through theoretical methods or flight data analysis. Given a time history of measured quantities from the vehicle response, unknown parameters can be chosen and adjusted to match the model output to the measured response. A common method for solving these types of problems is the maximum likelihood estimation technique. This technique determines the value of unknown model parameters by maximizing the likelihood of a given measurement. This involves minimizing the weighted least squares difference between measured and model outputs [19]. Maximum likelihood methods for aircraft parameter identification generally take on two forms: output-error method and equation-error method [19]. The key difference between the methods are how they handle noise. The output-error method assumes no process noise but includes measurement noise while the equation-error method assumes that process noise is present but with error free measurements of the states. The research described here focuses on the output-error formulation with the assumption that the noise is white, Gaussian, stationary, and uncorrelated.

A brief derivation of the maximum likelihood estimation technique will be given here, highlighting the main points of the derivation. A more extensive derivation can be found in reference 19. The derivation begins with the definition of the governing equations of the system. Aircraft

motion can generally be described by a set of equations that take on the form

$$\dot{x}(t) = f(x, u_i, t, \hat{\Theta}) \quad (3.24)$$

$$y(t) = g(x, u_i, t, \hat{\Theta}) \quad (3.25)$$

$$z(t) = y(t) + \nu(t) \quad (3.26)$$

where $\hat{\Theta}$ is the vector of unknown parameters, x is a vector of state variables, y is a vector of the model outputs, z is a vector of measurements, u_i is a vector of inputs, t is time, and ν represents the measurement noise. Since this derivation focuses on the output-error method, the process noise is excluded. The measurement noise is assumed to be white with

$$E[\nu(i)] = 0 \quad (3.27)$$

$$E[\nu(i)\nu(j)^T] = R(i)\delta_{ij} \quad (3.28)$$

where i and j are indices, R is the noise covariance matrix, and δ_{ij} is the Kronecker delta. The likelihood function for a sequence of measurements $Z_N = [z(1) z(2) \dots z(N)]^T$ can be written as $\mathcal{L}[Z_N; \Theta]$. After successive applications of Bayes's rule over N samples and assuming that the noise is uncorrelated and normally distributed [19],

$$\mathcal{L}[Z_N; \theta] = \prod_{i=1}^N \mathcal{L}[z(i); \Theta] \quad (3.29)$$

It is generally more advantageous to minimize the negative logarithm of the likelihood function than it is to maximize it. Since the log function is monotonic, this is permissible and provides an estimate of the unknown parameter

$$\hat{\Theta} = \min_{\Theta} \sum_{i=1}^N -\ln \{\mathcal{L}[z(i); \Theta]\} \quad (3.30)$$

The system noise can be described as the difference between the measured and model output (Eq. 3.26) and approaches a Gaussian distribution as the sample rate increases. Making this assumption, the negative log of the likelihood function, after some algebraic manipulation, becomes

$$-\ln \{\mathcal{L}[z(i); \Theta]\} = \frac{1}{2} \sum_{i=1}^N \left[\nu^T(i) R^{-1} \nu(i) + \frac{N}{2} \ln|R| + \frac{N n_0}{2} \ln(2\pi) \right] \quad (3.31)$$

where n_0 is the number of output variables. The term on the far right is a constant and therefore has no effect on the optimization problem [19]. Differentiating Eq. 3.31 with respect to R and

setting it equal to zero allows for the minimum error condition to be solved for. The resulting cost function J used to evaluate the model is given as

$$J(\Theta) = \frac{1}{2} \sum_{i=1}^N \nu(i)^T \hat{R}^{-1} \nu(i) \quad (3.32)$$

where

$$\hat{R} = \frac{1}{N} \sum_{i=1}^N \nu(i) \nu(i)^T \quad (3.33)$$

$$\nu(i) = z(i) - y(i) \quad (3.34)$$

The cost function is minimized using a relaxation technique where $\hat{R}$ is computed and held constant while Eq. 3.32 is minimized with respect to Θ . The minimization of Θ can be done using any optimization technique chosen by the user. Once Θ has been minimized, $\hat{R}$ is recalculated and the process is repeated until the change in the cost function from the previous iteration is below some user defined threshold [19].

3.3 Power Required Analysis

Now that the governing equations of the system and parameter estimation method have been defined, they can be combined to estimate the steady state power of the aircraft. The analysis that follows relies on two key assumptions

1. The pitching moment (pitch rate) at any given time is negligible.
2. The lateral dynamics at any given time are negligible.

These two simplifying assumptions were made based on the maneuvers used during data collection. A detailed description of the maneuvers is given in Chapter 4 but they are all conducted in the vertical plane only. Assuming negligible lateral dynamics significantly simplifies the analysis and the assumption is easy to implement in the flight test phase. The assumption of negligible pitch rate is also derived from the maneuvers as they are intended to be performed at a single altitude under quasi-steady state conditions. Justification for this assumption is given in Chapter 4. Using these assumptions, the aerodynamic models used in the equations of motion can now be defined.

3.3.1 Aerodynamic Models

For general analysis at low subsonic speeds with slow dynamics, linear aerodynamic models are sufficient. The aerodynamic models used in this research are given as follows

$$C_L = C_{L_0} + C_{L_\alpha} \alpha + C_{L_{\delta_e}} \delta_e \quad (3.35)$$

$$C_D = C_{D_0} + K (C_L - C_{L,minD})^2 \quad (3.36)$$

where C_{L_0} is the lift coefficient at the reference condition, C_{L_α} is the lift curve slope, $C_{L_{\delta_e}}$ is the lift due to elevator deflection, δ_e is the elevator angle, $C_{L,minD}$ is the lift coefficient at minimum drag, and K is the drag due to lift factor. The lift model generally includes the derivative C_{L_q} , the change in lift coefficient due to pitch rate, but due to the first assumption the term is left out. Some drag models also leave out the $C_{L,minD}$ term but, in general, the minimum drag of an aircraft does not occur at zero lift, but at some positive value so the term must be included [10]. The moment model (C_M) is also not included because of the second assumption. Since the pitch rate is considered negligible, the longitudinal moments of the aircraft will be essentially zero.

3.3.2 Data Reduction

The aircraft states concerned with motion in the vertical plane are generally given as [5] [10]

$$x = \begin{bmatrix} V \\ \alpha \\ q \\ \theta \end{bmatrix} \quad (3.37)$$

Since the pitch rate is assumed to be negligible, q and θ drop out making the state, output, and measurement vectors

$$x = \begin{bmatrix} V \\ \alpha \end{bmatrix} \quad y = \begin{bmatrix} V \\ \alpha \\ a_x \\ a_z \end{bmatrix}_{model} \quad z = \begin{bmatrix} V \\ \alpha \\ a_x \\ a_z \end{bmatrix}_{measured} \quad (3.38)$$

where the axial and vertical accelerations are given by the equations

$$a_x = \frac{C_X \bar{q} S + T}{m} \quad (3.39)$$

Table 3.1: Instrumentation error application

Variable	Bias Error	Scale Factor Error
a_x, a_z	X	—
α	X	X
V	X	X

$$a_z = \frac{C_Z \bar{q} S}{m} \quad (3.40)$$

where

$$C_X = C_L \sin \alpha - C_D \cos \alpha \quad (3.41)$$

$$C_Z = -C_L \cos \alpha - C_D \sin \alpha \quad (3.42)$$

It is important to note that Eq. 3.39 and Eq. 3.40 are written with respect to the center of gravity and do not include gravity. When comparing the equation outputs to the measured accelerometer data it is necessary to either remove gravity from the measurement or add it to the equation outputs since accelerometers cannot distinguish between gravity and pure accelerations.

The measurements taken by the accelerometers, as well as other sensors, are subject to random as well as systemic errors. Systemic errors due to calibration errors, low quality sensors, inaccurate trim values, etc. can be compensated for by adding bias and scale factor parameters called nuisance parameters [19]. The measurement equation model, Eq. 3.26, with systemic instrumentation error can be written as

$$z(t) = \xi y(t) + b' + \nu(t) \quad (3.43)$$

where ξ is the scale factor error and b' is the bias error. Not all sensors require both scale factor and bias error corrections. Table 3.1 lists the type of error added for each variable. The nuisance parameters are estimated along with the parameters of interest, but their values are of no interest. They serve only as correction factors to allow for a better model fit.

Estimation of the lift and drag models begins by using the initial parameter values and initial measurements to solve the lift and drag models (Eq. 3.35 and 3.36) and then solving Eq. 3.21, 3.22, and Eq. 3.39 through 3.42. The state update equation

$$x(t + \Delta t) = x(t) + \dot{x}(t)\Delta t \quad (3.44)$$

is used along with the sampling frequency to determine the aircraft's states and outputs. The estimated outputs are then compared with the measured values, the parameter estimates updated accordingly, and the process is repeated. Once a feasible solution has been found, a new covariance matrix, R , is determined and the process repeated until the cost function (Eq. 3.32) no longer decreases. As previously mentioned, any optimization method can be chosen to minimize the cost function. For this research, the built-in MATLAB optimization function *fmincon* was used, applying the default interior-point algorithm.

Once the iterative optimization process has successfully converged, the drag model was used to determine the true steady state power required curve for the aircraft using lift coefficient values corresponding to lift=weight at any given velocity. The power is then calculated using the equation

$$P_{req} = DV = C_D 0.5 \rho V^3 S \quad (3.45)$$

The power required values calculated are dependent upon the aircraft weight and atmospheric conditions present during the test. To remove these dependencies so that data from different tests and days can be compared, the same reduction procedure as outlined by the classic PIW-VIW method was used. In this case the actual power output is known, so *BHP* can be replaced by P_{req} and the equation becomes

$$PIW = \frac{P_{req} \sqrt{\sigma_t}}{(W_t/W_s)^{1.5}} \quad (3.46)$$

The dynamic method was compared against the classic method for validation since it is an accepted method for measuring power required. However, in this case some changes to the equations are needed. The classic method was designed around aircraft with reciprocating engines where the actual power output was unknown and could only be estimated. In this case, the actual power output from the motor/propeller combination is known. At any point in time the propulsion system power output is equal to

$$P_A = TV \quad (3.47)$$

The thrust from the propeller can be calculated using the equation

$$T = c_T \rho \Omega_p^2 D_{prop}^4 \quad (3.48)$$

where c_T is the thrust coefficient and D_{prop} is the propeller diameter. Through wind tunnel testing, a propeller model relating thrust coefficient and advance ratio(j) was established which allows the thrust to be calculated at any airspeed and RPM combination. A detailed explanation of the wind tunnel test used to generate the propeller model can be found in Appendix

A. Substituting the known power output for the estimated power, the PIW equation can be rewritten as

$$PIW = \frac{BHP_T \sqrt{\sigma_t}}{(W_t/W_s)^{1.5}} = \frac{TV \sqrt{\sigma_t}}{(W_t/W_s)^{1.5}} \quad (3.49)$$

The VIW equation remains the same where

$$VIW = \frac{V}{\sqrt{W_t/W_s}} \quad (3.50)$$

Chapter 4

Flight Simulator

4.1 Simulator Design

The flight simulator was designed to speed up development of the analysis method and to quickly test potential maneuver designs for effectiveness. The aerodynamic models from the previous chapter combined with the equations of motion provide the basis for the flight simulator. MATLAB Simulink was used to implement the simulator, which is shown in Figure 4.1. The aerodynamic models used in the flight simulator are based on those of the Astraeus UAS. Details about the aircraft can be found in Chapter 5. The models were derived from a combination of theoretical analysis as well as previous flight test data. The simulator uses a fixed-step solver utilizing Euler's method at 0.02 sec increments.

The Nonlinear Aircraft Model block in Figure 4.1 contains the nonlinear equations of motion and is the core block of the simulator. The equations of motion are contained in a function block that is looped back into itself to update the aircraft states. The states are updated according to Eq. 3.44. The initial conditions for the simulation are set in an external file by the user and include initial velocity and altitude. Since the simulator was setup to test maneuvers that start from a steady state condition with motion happening primarily in the vertical plane, the initial conditions solver automatically solves for the angle of attack and motor RPM to maintain steady level flight at the given condition.

The propulsion system block contains the propeller model for the aircraft based on wind tunnel testing. Several functions within the block control the RPM to provide an increasing throttle ramp, decreasing throttle ramp, manual input, or pass through from an outside controller input.

The altitude/elev control block contains various functions to control the elevator based on the desired maneuvers. The user can select between a PID controller for altitude, pitch, and pitch rate control, or preprogrammed inputs such as elevator pulses, doublets, and manual

Figure 4.1: Overview of the flight simulator block diagram in Simulink

positioning. Simulated pilot input is achieved through manipulation of the PID gains within the controllers as well as through preprogrammed inputs. A detailed discussion on simulated pilot input is given later in this chapter.

The velocity control block is an optional block that allows the user to set a predetermined velocity profile or single target value. A PID controller varies the RPM in order to control the vehicle's velocity. A model of the brushless motor used is included in the controller to more accurately model the motor's response to any given input. The model was generated from physical testing during a previous project.

The last control block is the lateral control block. This block also contains PID controllers to actuate the ailerons and rudder based on the given target roll and yaw values. For this research the target values were always set to zero bank and zero yaw. In both the altitude/elev block and lateral control block, the control surface commands were sent through a servo model that was based on a model generated from physical testing on a similarly sized servo.

4.2 Maneuver Selection

Previous research on drag polar estimation used several different maneuvers to collect the necessary data. The most common maneuvers are the pushover/pull-up, slow down, and windup turn [5]. These maneuvers are specifically designed to achieve a wide range of lift coefficient values so that a complete drag polar can be generated. Since power required is directly propor-

tional to drag, these maneuvers can also be used to determine power required. However, most of these maneuvers require relatively tight control and work well for manned and autonomous aircraft but not remotely piloted aircraft. The roller coaster maneuver requires the aircraft to slowly pitch up at a rate of 0.1 g/sec until 2 g and then nose over at -0.1 g/sec until 0 g is achieved. This is relatively easy for a skilled pilot sitting in the cockpit to perform and even easier for an autonomous aircraft. For a remotely piloted vehicle with no telemetry feedback, this maneuver is extremely difficult to perform as the pilot has no way of knowing the g-force rate. Similarly, the windup turn maneuver also requires a linear g-force increase in a constant altitude turn. Again, without feedback this is extremely difficult for a remote pilot to judge. The slow down maneuver starts with the aircraft in trimmed steady level flight and then sets the throttle to low or idle and allows the aircraft to naturally slow down while providing control input to maintain altitude and keep the wings level. This maneuver is much easier for a remote pilot to perform so it was selected as a candidate for testing. The inverse of this maneuver, starting at a low velocity and applying full power to accelerate at a constant altitude can also be used to determine the power required. If the thrust at every instant in time is known, then the thrust available can be determined and the aircraft's acceleration will reflect the difference in available and required power of the system. This maneuver was selected as the second test candidate.

4.3 Simulation of Pilot Input

The simulator was not designed to operate in real time and, therefore, can not utilize actual pilot input during the maneuvers. However, several methods and models exist to simulate human response to tracking in non-real time systems [1] [8]. These models are fairly complex and more complicated than what is required in this case. Initial simulation runs of the acceleration and deceleration maneuvers have shown that the total run times are relatively short, less than 20 sec, and the response of the aircraft is of more importance than the pilot input. If the aircraft performs the maneuver without any deviation in the vertical plane, the power required can easily be determined by summing the forces along the x -axis. However, in reality no pilot, or even controller, will be able to perform the deceleration or acceleration maneuver perfectly. The goal of the simulations is to determine to what degree deviations are allowable before the method no longer provides reasonable results. Therefore, accurately modeling a human pilot is not essential.

Previous research performed in the Flight Research group as well as observations during flight testing indicate that the human response to altitude control can be modeled by a PID controller with poorly tuned gain values or by following a predefined altitude profile modeled by the summation of sine waves with decay. Distance from the aircraft also plays a large role

in the ability to maintain altitude. As the distance between the aircraft and pilot increases, it becomes increasingly more difficult for the pilot to correctly judge the aircraft's states such as climb rate, altitude, and bank angle. Therefore, some maneuvers may end up at a higher or lower altitude than they started, and/or with a gentle turn unintentionally executed somewhere during the maneuver. The simulator will test the method's robustness using these deviations.

The two types of human control that were implemented in this simulator were those mentioned above, poor gain values on the altitude PID controller and altitude following. A well tuned controller provides excellent control with very little deviation in altitude during the entire maneuver. Low gain values cause large but slow deviations from the target altitude which are undesired but keep the pitch rate low. High gain values cause small but quick deviations which hold altitude well but may violate the negligible pitch rate assumption. The gains can also be scheduled to provide a combination of these responses and more accurately simulate a pilot who is trying to maintain a constant altitude. Gain values were chosen on a trial and error basis since an optimal response was not required.

Altitude following uses a well tuned altitude controller to follow a preprogrammed altitude profile that mimics a human pilot. Altitude profiles from previous PIW-VIW and neutral point determination flights have shown that the target altitude profile can be expressed as the summation of sine waves. The exact formulation is somewhat arbitrary and only needs to conform to a few loosely defined guidelines. The target profile should not have a dh/dt value whose magnitude is too large and the total altitude gain or loss, as well as the peak value, should also not be too large. The definition of too large is left to the user's particular situation and will most likely be defined by the aircraft used in testing.

4.4 Noise Identification

The raw flight simulator output contains no process or measurement noise. To increase the realism of the data and to test the robustness of the method and the effect of noise, noise was added to the output according to measured noise variance values. System noise measurements were made with the data system installed in the aircraft and the system set to record all sensors for two minutes while the aircraft remained stationary on the ground. Noise values were also estimated from data collected during an initial flight test of the data system to check functionality. It was found that all sensors except the accelerometer had noise values that were approximately equal to those found during the bench test. Noise values were estimated from steady gliding flight with the motor turned off, so as not to introduce electrical or vibrational noise. The noise variance from the flight test data was estimated using a smoothing spline model whose smoothing weight was determined by minimizing the generalized cross-validation score [4]. Table 4.1 contains the noise variance values used in the simulations. Noise is assumed to be

Table 4.1: Bench test measured sensor noise variances

Sensor Parameter		Variance
$a_{x,y,z}$	(ft/s ²)	0.19
p, q, r	(rad/s)	0.005
α, β	(rad)	0.008
V	(ft/s)	0.96

white Gaussian.

4.5 Simulation Results

The simulation results are broken down into three cases: one to validate the simulator and power required reduction method, one to test scenarios likely to occur during the flight test phase, and one to investigate the negligible pitch rate assumption. All cases provide results for both the acceleration and deceleration runs as these will be the maneuvers used during the flight test phase. The first case covers the results from simulated pilot input that exhibits excellent altitude control. This represents a near perfect data run that was used to validate the simulator and data reduction methodology. The next case shows results for data runs where the pilot control was marginal and in one case finished at the same altitude (or very close to it) and in another case, finished at a higher or lower altitude. The last case investigated the assumption of negligible pitch rate and when it can and cannot be assumed to be negligible.

4.5.1 Case 1 - Validation and Near Ideal Runs

The results from the near perfect data run for both deceleration and acceleration runs are in excellent agreement with the theoretical results. The acceleration run did not have nearly as good altitude control as the deceleration run due to the sudden increase in the vertical component of the thrust at low airspeeds. Figures 4.2 through 4.4 show the raw simulator output for the deceleration run with the maximum likelihood model outputs using noise-corrupted simulator data. The power required curve generated by the model is shown in Figure 4.4 along with the theoretical model. The difference in the curves is on the order of 1 ft-lbs/s, making the curves nearly indistinguishable. Acceleration run data is shown in the same manner in Figures 4.5 through 4.7. In the cases that follow, only the altitude and power plots will be shown as these are the most important plots.

(a)

(b)

(c)

(d)

Figure 4.2: Flight simulator output for ideal deceleration run

(a)

(b)

(c)

(d)

Figure 4.3: Maximum likelihood estimation output for ideal deceleration run

Figure 4.4: Power required for ideal deceleration run

4.5.2 Case 2 - Moderate Altitude Deviations

The cases shown here represent altitude deviation profiles that are likely to occur during flight testing. As previously mentioned, the distance between the pilot and aircraft will cause difficulties in judging altitude changes and whether or not the aircraft is at, or has been brought back to, the same altitude that it started from. It is likely that a sinusoidal oscillation around the target altitude will occur and that the starting and finishing altitudes will differ. The first set of results (Figure 4.8) shows an acceleration run where decaying oscillations around the target altitude occur. The power plot shows a very good match between the theoretical and estimated power curves. Figure 4.9 shows the same case but for a deceleration run. Here the amplitude of the altitude oscillations increased as the aircraft slowed down. Again, the power plot is in agreement with theory, but with a small deviation of around 2 ft-lbs/s in the middle of the velocity range. Several additional cases were run for both acceleration and deceleration runs in which the power required plots were all in agreement with theory.

The second set of sub-cases was for acceleration and deceleration runs where the altitude at the end of the maneuver was different from that at the start. For the acceleration run the final altitude was higher than the starting value and for the deceleration run it was lower. Figures 4.10 and 4.11 show the altitude profiles and power required estimates. Again, the plots show a good agreement with theory. Several different altitude profiles were run with similar results including those where the acceleration run ended lower and the deceleration run ended higher. It can be concluded that altitude changes and finishing altitude do not affect the resulting power required analysis, assuming that a significant change in air density has not occurred.

(a)

(b)

(c)

(d)

Figure 4.5: Flight simulator output for near ideal acceleration run

(a)

(b)

(c)

(d)

Figure 4.6: Maximum likelihood estimation output for near ideal acceleration run

Figure 4.7: Power required for near ideal acceleration run

Figure 4.8: Altitude profile and estimated power required for the return to altitude case for an acceleration run

Figure 4.9: Altitude profile and estimated power required for the return to altitude case for a deceleration run

Figure 4.10: Altitude profile and estimated power required for the final altitude offset case for an acceleration run

Figure 4.11: Altitude profile and estimated power required for the final altitude offset case for a deceleration run

4.5.3 Case 3 - Pitch Rate Assumption

In Chapter 3, two key assumptions were made in the analysis development, one of which was that the pitch rate dependent terms can be considered negligible. The main reason for this assumption was that the maneuvers designed to be used with the analysis method are performed at a constant altitude where changes in the pitch angle and angle of attack occur very slowly over the duration of the maneuver causing a very low pitch rate. In the case of the ideal deceleration, the simulator shows a pitch angle change of about 8 deg over 20 sec, which equates to a pitch rate of 0.4 deg/s. This value is too low to measure based on the variance of the sensor noise. However, during the acceleration run the peak pitch rate was determined to be 6.8 deg/s, which is measurable even though this value can be considered negligible in this case. At some point however, the pitch rate can no longer be considered negligible, and must be included in the model. The analysis that follows determines the pitch rate limit.

The pitch rate affects the lift model, and therefore the drag model, directly through the derivative C_{Lq} . The significance and accuracy of this parameter, or any parameter for that matter, is determined by the information matrix $\mathcal{I}$. The information matrix is defined as

$$\mathcal{I} = -E \left(\frac{\partial^2 \ln \mathcal{L}}{\partial \Theta \partial \Theta^T} \right) \quad (4.1)$$

but can be rewritten as

$$\mathcal{I} = \mathcal{D}^{-1} = \sum_{i=1}^N SE(i)^T R^{-1} SE(i) \quad (4.2)$$

using the assumptions made in the theoretical development where the dispersion matrix $\mathcal{D}$ is equal to the covariance matrix when the noise is white Gaussian and where SE is the sensitivity matrix [18]. As the information content in the signal decreases, i.e. the signal-to-noise ratio of the pitch rate decreases, the information matrix becomes ill-conditioned causing the covariance estimate of the parameter to go to infinity through the inversion process. The magnitudes of the individual covariance values are used for testing parameter significance through hypothesis testing. In the case of pitch rate, the hypothesis for testing is given as

$$\begin{aligned} H_0 : C_{Lq} &= 0 \\ H_1 : C_{Lq} &\neq 0 \end{aligned} \quad (4.3)$$

The t -statistic

$$t_0 = \frac{\hat{C}_{Lq}}{s(\hat{C}_{Lq})} \quad (4.4)$$

where

$$s(\hat{C}_{Lq}) \equiv \sqrt{\text{Var}(\hat{C}_{Lq})} \quad (4.5)$$

is used as the metric to determine whether or not the null hypothesis can be rejected [9]. If $|t_0| > t(\alpha^*/2, N - n_p)$ where n_p is the number of parameters, then C_{Lq} is statistically different from zero. The value for α^* used in the tests is 0.05. Generally, in flight testing, the sample size is significantly greater than the number of model parameters such that the value of $t(\alpha^*/2, N - n_p)$ tends towards 1.96. For all of the simulator and flight test results this holds true, so 1.96 was used as the cutoff value.

Since the information content of the pitch rate signal will determine whether or not C_{Lq} is significant, it was the primary variable used for testing. Information content is directly related to signal magnitude. However, it alone is not sufficient to determine significance. Since the real world is not noise free, random noise must be accounted for as it has the possibility of 'drowning out' a signal depending on its magnitude. The signal-to-noise ratio (SNR) is a much better variable for use in testing as it accounts for both. The signal-to-noise ratio is defined as

$$SNR = \frac{P_{signal}}{P_{noise}} \quad (4.6)$$

However, in this analysis we can use an alternate form of the definition that is more useful. In the case of these maneuvers, the pitch rate is a zero mean signal and by definition the additive noise is as well, so the equation can be written in terms of variance instead of power

$$SNR = \frac{\sigma_{signal}^2}{\sigma_{noise}^2} \quad (4.7)$$

where σ^2 is the variance. For the simulated data, the SNR value is easily calculated because the two components that make up the final signal (true signal + additive noise) are known. For the flight test data, the variances are estimated using a different technique that is explained in detail in Chapter 7. In addition to the SNR effect on the significance of C_{Lq} , maneuver effects were also considered since reproducibility by the pilot can be difficult. Three different maneuvers were tested to determine what effect, if any, maneuver type had on the outcome. The three maneuvers used were those shown in Figures 4.9 through 4.11. For each test, the maneuver and magnitude of the noise on each output variable remained constant while the magnitude of the noise on the pitch rate signal was increased or decreased. Noise variance was based on the values in Table 4.1. The outputs were run through the maximum likelihood technique and the estimated value for C_{Lq} along with its variance were used in Eq. 4.4 to determine significance. During testing it was noted that the magnitude of the noise on all of the other parameters played a role in the significance of C_{Lq} . Therefore, each maneuver was also run with all sensor noise values increased by 50% for the second case and decreased by 50% for the third case. The resulting SNR values for each maneuver and noise variance factor are shown in Figure 4.12. Within the plot, the different line types, solid, dashed, or dotted, represent the three different

Figure 4.12: Flight simulator derived SNR cutoff for various system noise levels

maneuvers performed. The plot shows two important trends: one, that the maneuver type has little effect on the significance of C_{Lq} , and two, that the overall system noise has a large impact

on the significance of C_{Lq} . The first trend verifies that parameter significance is independent of maneuver type, i.e. the pilot does not have to worry about how the maneuver is performed giving robustness to the method. The second trend presents a problem because real world noise is not always perfectly constant white Gaussian. Small changes in the noise variance has a relatively large effect on parameter significance cutoff. It will be shown later in Chapter 7 that the resulting flight test data verifies these trends. Based on these simulation results and assuming that the noise variance values are equal to those given in Table 4.1 then, as long as the pitch rate remains under ≈ 35 deg/s, C_{Lq} can be considered negligible. However, small changes in the actual noise variance will significantly alter this value indicating that there most likely does not exist a hard pitch rate cutoff for when C_{Lq} is and is not significant.

Chapter 5

Experimental Hardware Development

5.1 Aircraft

Two aircraft were used in the validation of the dynamic power required analysis method, the Astraeus UAS and the Hobbico Nexstar. The Astraeus UAS was designed and constructed by the 2010/2011 Aerospace Engineering senior design class as a reconfigurable multi-purpose UAS. Design requirements dictated that a high speed and low speed configuration sharing a common fuselage and empennage be designed with specific stall speed requirements for each configuration. To this end, the aircraft was an excellent candidate for use as a test bed for method validation, because the UAS has two different wings. The power required for the entire aircraft changes depending on which wing is currently being flown. Thus, two different power required curves can be generated without moving the data system from one vehicle to another. The high speed version is shown in Figure 5.1. The payload section of the aircraft was originally designed for a fairly large avionics box allowing for plenty of room for a small data collection system with separate power supply. Only a single modification to the payload section was necessary to receive the new avionics tray which consisted of four mounting holes in the forward and aft section. The pitot-static tube was also replaced with a custom five port probe for angle of attack and side slip measurements. The construction and calibration of this probe is detailed in section 5.3.

The Nexstar is a commercially available RC aircraft made by Hobbico. It is a standard high wing trainer that exhibits good flying characteristics and is shown in Figure 5.2. Since this aircraft was not designed with flight testing in mind, a few modifications were necessary. A pylon was added to the outer section of the right wing to mount the five port probe along with modifications to the wing root to accommodate the pressure tubing. The motor and speed

Figure 5.1: Astraeus UAS, high speed configuration

Figure 5.2: Hobbico Nexstar

Table 5.1: Dimensions for the Astraeus and Nexstar UAS

Parameter	Astraeus(LowSpeed)	Astraeus(HighSpeed)	Nexstar
b , ft	7.75	5.5	5.73
$\bar{c}$, ft	0.753	0.793	0.870
S , ft ²	5.81	4.35	4.99
CG_x , ft*	0.390	0.342	0.277
W , lbs	6.57	5.83	8.47

*Measured along the wing root airfoil.

controller were also upgraded to compensate for the increased weight due to the addition of the flight computer and supporting hardware and extra flight batteries. Table 5.1 gives the relevant dimensions of both Astraeus and the Nexstar.

5.2 Data Acquisition System

The design of the data collection system went through several iterations based on the data requirements of the method, available sensors, currently available systems, and previous experience with other systems.

5.2.1 Data Requirements

To utilize the analysis method described in Chapter 3, the data system must record certain flight parameters. The parameters, shown in the list below, are those needed to solve Equations 3.21, 3.22, 3.35, 3.39, and 3.40.

$$T \quad V \quad a_x \quad a_z \quad \Delta t \quad p \quad q \quad r \quad \alpha \quad \beta \quad \delta_e \quad \theta \quad \phi$$

These parameters are needed for the power required analysis utilizing accelerating or decelerating runs. However, since the classic PIW-VIW method was also performed, an additional flight parameter was needed, namely $\dot{h}$. A direct measurement of this value can be obtained through GPS, but it can also be determined from an altitude profile. Therefore, h and $\dot{h}$ were also added to the data requirements list.

Another important factor in the power required analysis, or any flight test analysis for that matter, is the data sample rate. Several factors go into sample rate selection, but the two key factors here are aliasing and reconstruction. Aliasing is an effect that causes two signals to be indistinguishable from each other and occurs when sampling a signal at less than twice its

frequency. In an environment where multiple frequencies of interest exist, the data acquisition system must be set to sample at a rate of at least twice the highest frequency of interest [26]. For aircraft dynamics, the mode that generally exhibits the highest frequency is the short period mode. While this research is not directly interested in measuring the short period mode, several parameters such as angle of attack and pitch angle, are affected by this mode, and these quantities are used in the underlying equations. For the Astraeus UAS, the short period mode is 1.1 Hz. Data on the dynamics of the Nexstar does not exist but since it is comparable in size, shape, and weight, it is assumed that its short period mode is of similar frequency. Therefore, the minimum sampling frequency to avoid aliasing is 2.2 Hz. However, while this ensures that the signal can be recovered, a good reconstruction of the signal will require more points. It is generally suggested to sample at a rate of 10 times the highest frequency to obtain enough points for a good reconstruction [11]. Using these requirements and guidelines, the data system sample rate was set at 25 Hz.

5.2.2 Hardware Design

In the early stages of system definition and layout, the ArduPilot was initially selected because it had been previously used in senior design testing as well as a few other graduate projects, so it was a known system. However, based on the data requirements, the ArduPilot software as well as hardware needed to be modified. Although the software for the system is open source, robust, and well tested, it is extremely complex and difficult to follow, especially to those who are not proficient at programming. The ArduPilot is also based on an ATmega microcontroller which, in its current state, is already being pushed to its performance limits. Given the aforementioned list of issues/modifications, it was determined that the ArduPilot would not be the best choice.

The next candidate for the data collection system was the NAVIO autopilot shield for Raspberry Pi by EMLID Ltd. The NAVIO is a snap on shield for the Raspberry Pi that contains the same type of sensors as the ArduPilot, but powered a significantly more powerful processor. The Raspberry Pi is a credit card sized computer developed by the Raspberry Pi Foundation for the purposes of advancing education in the field of computers, computer science, programming, etc. Its size, low cost, relatively powerful processor, low power consumption, and expandability made it the candidate of choice to serve as the data collection system. In addition, it has a large community following with wide support. EMLID also developed a patch for the Raspberry Pi operating system that allows it to run in near realtime, so time sensitive tasks can be completed accurately. In the event that running ArduPilot becomes necessary/desirable, they have (along with the APM community) developed a version of the ArduPilot code that can run on the Raspberry Pi. Expandability is also much easier on the Raspberry Pi as the GPIO connector is easily accessible.

Table 5.2: List of sensors on the flight data system

NAVIO Sensors	Data Collected
MPU9250	$a_x, a_y, a_z, p, q, r, \theta, \phi, \psi$
u-blox M8 GPS	NED velocity, altitude, position, course
MS5611	barometric altitude and temperature
Additional Sensors Needed	
Differential pressure transducers (3)	Angle of attack, side slip angle, airspeed
Absolute pressure transducer	altitude
Microcontroller	RPM

The NAVIO shield contains most of the sensors needed to meet the data requirements but not all. Table 5.2 contains a list of the sensors contained on the NAVIO as well as the additional sensors needed. Although the NAVIO already contains an absolute pressure transducer, it does not have a hose barb and lacks the ability to mount one on it, so a second transducer with a hose barb was added. Unless specific provisions are made to ensure that the pressure inside of the fuselage is equal to static pressure, a separate static measurement must be made.

Instead of mounting the additional sensors on separate breakout board(s) away from the Raspberry Pi, an additional shield with the extra sensors was designed to be mounted between the Raspberry Pi and NAVIO. This saves space and weight and decreases the complexity of the design. Before the sensors shield could be designed, the exact make/model of the extra sensors needed to be identified. The angle of attack and side slip sensors were chosen based on the the five port probed design requirements and is detailed in section 5.3. The sensor chosen for both angle of attack and side slip measurements was the Measurement Specialities MS4515DO with a sensing range of ± 2 inH₂O and an accuracy of 0.25% full span.

The airspeed sensor was chosen based on previous flight test data collected on Astraeus. During flight testing, the maximum speed observed was 103 ft/s on either configuration, which equates to a dynamic pressure of 2.4 inH₂O under standard conditions. Ideally, a sensor whose range is the same as that observed during testing would be chosen, but no digital differential sensors with a range around 2.4 inH₂O could be sourced. A digital type sensor, specifically one using the I2C protocol, was chosen over an analog sensor because of the ease of use, simplified circuit layout, and reduced sensitivity to interference. The sensor chosen to measure airspeed was the Honeywell SSCDRRN005ND2A5, which has a sensing range of ± 5 inH₂O and a 0.25% full scale span error.

RPM, in conjunction with airspeed, is needed to determine the current thrust output of the

propulsion system. Wind tunnel testing of the propellers for both planes (Aeronaut 12x8 for Astraesus and APC10x5 for the Nexstar) was conducted to generate a thrust model for each propeller. The thrust coefficient for various combinations of advance ratios at nearly constant RPM values was mapped out allowing for calculation of the instantaneous thrust value given the RPM and airspeed. Appendix A contains a description of the testing procedure and results since the tests were performed both before the research began and by a third party during the research. Both aircraft use a Castle Creations Phoenix Edge Lite 75 electronic speed controller (ESC) which has a configurable auxiliary output line that can be set to fire a logic change on every electrical commutation. A micro-controller was used to read the time between logic changes and then send that information to the Raspberry Pi over the serial port. During initial flight tests, it was noted that the RPM signal was rather noisy (± 100 RPM) which, when combined with the airspeed for thrust measurements, caused thrust values of ± 0.5 lbs from the mean. Using an oscilloscope, it was found that the RPM pulses generated by the ESC varied anywhere from ≈ 5 to ≈ 100 microseconds depending on the RPM. To smooth out the RPM signal, a decade counter was added to average 20 pulses and find the time for $1.43(20/14)$ revolutions instead of $0.071(1/14)$ revolutions. This decreased the RPM noise to ± 15 RPM. Figure 5.3 shows the oscilloscope output of the RPM signal before and after the decade counter.

Figure 5.3: ESC RPM output pulses, with and without decade counter

Once all of the additional sensors had been identified, the NavDAQ shield was designed and fabricated. A real time clock was also added to the shield even though it was not part of the data requirements. This was done to allow data to be time stamped with the current time since

the Raspberry Pi contains no real time clock. Figure 5.4 shows the layout of the NavDAQ shield. The final assembled flight computer is shown in Figure 5.5 and is mounted in the aircraft using

Figure 5.4: Custom PCB for additional sensors, known as NavDAQ

three vibration isolators so that the ethernet and one set of USB ports remained accessible for either a wired or wireless connection to download the collected data. The computer was powered by a 2-cell lithium polymer battery (Lipo) connected to a voltage regulator set to 5.1 V. This was done to isolate the computer system from the propulsion system for protection against significant voltage changes, inductive spikes, and to keep the system running once the main flight batteries have been depleted. The computer system consumes 3.7 W with the data collection script running which yields a maximum run time of 2.7 hours. Total system weight is 0.9 lbs including battery and regulator.

5.2.3 Software Design

The APM software was not designed to be an engineering analysis tool and, therefore, has many shortcomings in this respect. Modifying the code is possible since it is open source, however, the complex nature of the code makes it very difficult for non-proficient programmers to navigate. The code also contains many extraneous features so a new data acquisition program was written in C++ to be simple and expandable. EMLID provides open source C++ drivers for all of the sensors so only a few additional libraries needed to be written for the added sensors. Figure 5.6 shows a basic flow chart of the data acquisition program. Upon execution of the program, the user configuration file is read for information on sample frequency, which sensors are active, thread priorities, and some debugging options. The output file and sensors are then initialized

Figure 5.5: Assembled flight data system

Figure 5.6: Data acquisition program flow chart

and the necessary sensor threads created. Not all sensors are capable of being sampled at 'high' rates. For example, the GPS update rate can be set anywhere from < 1 Hz up to 10 Hz. This is much less than the 25 Hz update rate set by the data requirements. On the other hand, the attitude heading reference system (AHRS) algorithm requires a very high update rate in order to accurately determine orientation. This is due to the integration of angular rates that occurs in the algorithm. Integration over short time steps is generally more accurate as noise has less of an influence. The AHRS algorithm provided by EMLID runs at just over 800 Hz with no other CPU load. Therefore, to meet the needs of the sensors and not cause timing problems in the main loop, certain sensors were given separate threads to allow them to run at their required speeds. Other sensors such as the pressure transducers and RPM micro-controller, can be sampled on demand since their update rates far exceed the necessary sample rate. Once the threads have been successfully setup, the program enters the main loop. In this loop, a time stamp is taken at the top of the loop as a reference point and then all sensor data is read and written to a file. After writing to the file, a second time stamp is taken and the difference between the first and second one calculated. Based on the user set sampling frequency, the system will sleep for the remaining time necessary to achieve the desired loop frequency. To avoid searching through lengthy data records, data is only written to the output file when the 'GEAR' switch on the transmitter is set high. If the switch is low, the program skips over the write to output file function.

The program was originally written to also process the incoming sensor data by converting it to the correct units and applying any necessary calibrations. Bench testing indicated that the data processing functions required an excessive amount of system resources to a point where the sample rate was barely above the required 25 Hz. To reduce system load, all data processing was removed and only raw data was recorded. This brought the maximum sample rate up to 115 Hz. The data processing was completed post flight in MATLAB.

5.3 Five Port Probe

The analysis method requires knowledge of the angle of attack and side slip angle. Several methods exist to determine these quantities and can broadly be classified as either direct or indirect methods. The two main direct methods are mechanical vanes, such as an alpha beta vane, and pressure measurements, such as the five port probe. Indirect methods include estimation from inertial data using mathematical models. Each of the methods have their pros and cons. Mechanical vanes work by aligning themselves with the local flow through aerodynamic forces. This system is popular on manned aircraft but presents some problems as the vehicle size decreases. On large aircraft, the dynamics of the vehicle are much slower than the dynamics of the vane. The dynamic modes of aircraft generally increase in frequency as the size and

mass decrease. Depending on the vane’s size, moment of inertia, and friction, these frequency bands can overlap and cause problems during data analysis. Previous research conducted in the NCSU Flight Research Group has shown that mechanical vanes for aircraft in the size class used in this work have natural frequencies that fall within the achievable frequency range of the aircraft, causing significant lag in the readings [27]. One method to correct for this is the use of an analog lead circuit before sampling. However, it increases the complexity and weight of the system. Another disadvantage of mechanical vanes is the resulting increase of the overall drag of the vehicle. For large aircraft the drag increase is negligible, however, for small aircraft the increase in drag from the vanes and masts can contribute significantly to the overall drag of the vehicle. These vanes also contain precise moving parts which require maintenance and can easily be damaged on a rough landing.

The pressure measurement method uses multiple pressure taps located in a circular pattern around the tip of a boom. The pressure difference experienced by the taps due to the air flow angles can be mapped back to the local flow conditions through a calibration. Since this method can be utilized on a pitot-static tube, it causes significantly less drag, if any, additional drag compared to the mechanical vanes. The downside is that the length of the tubing between the pressure tap and transducer can cause significant lag between event and measurement. Both of the direct measurement methods require careful calibration to yield accurate results as their placement is generally close to the aircraft where the local flow conditions may not be equal to free stream conditions.

The indirect measurement methods are the least intrusive since the sensors needed to estimate the air flow angles exist on most full scale aircraft and almost all UAS. The equations of motion are used to derive the equations for the time rate of change of angle of attack and angle of side slip, $\dot{\alpha}$ and $\dot{\beta}$. These equations use the aircraft velocity and inertial data to determine the derivatives which are time stepped using a set of initial conditions (generally the start of the maneuver) to reconstruct the air flow angles. This method is easy to implement as it relies entirely on software. The resulting accuracy is highly dependent on data quality and initial conditions. Utilizing the frequency domain allows for bias to be removed but drift may still be present if the sensors have high noise. This method also works best over short time spans since it is based on user set initial conditions [17].

For this project, the five port probe was chosen over the mechanical vane or mathematical estimation for several reasons. Since the aircraft are relatively small, the addition of the mechanical vane system would result in a fairly significant increase in overall drag along with the vane dynamics issue. Astraeus utilizes skids for landing instead of conventional landing gear, so the vanes could easily be damaged during landing either through direct contact with the ground or impact shock. The mathematical estimation method was also not chosen because previous research conducted by the author found the method to yield poor results when using

lower quality sensors. Since sensors of similar quality are being used in this work, the results would most likely be poor as well.

The five port probe was constructed in-house using five pieces of 0.625 in stainless steel tubing and one piece of 0.219 in aluminum tubing. The five pieces of stainless tubing were arranged in a 'cross' pattern using a jig and secured with adhesive. The aluminum tubing was then slid over the assembly and secured with adhesive. The front portion of the probe is shown in Figure 5.7 with labeled ports.

Figure 5.7: Five port probe with labeled port numbers

5.3.1 Calibration

Calibration of the probe was conducted in the NCSU subsonic wind tunnel using the alpha/beta rig to set the angle of attack and side slip angle. These probes are generally calibrated using pressure coefficients derived from the direct measurement of the pressure at each port. The pressure coefficients are defined as [28]

$$C_{p,\alpha} = \frac{p_2 - p_4}{p_1 - \bar{p}} \quad (5.1)$$

$$C_{p,\beta} = \frac{p_3 - p_5}{p_1 - \bar{p}} \quad (5.2)$$

$$\bar{p} = \frac{p_2 + p_3 + p_4 + p_5}{4} \quad (5.3)$$

where C_p is the pressure coefficient for the given set of ports. Typically the free stream dynamic pressure is used as the reference pressure for nondimensionalization. However, using the above definitions was found to reduce scatter in the calibration data versus dividing by the dynamic pressure alone [14]. The five port probe used in this research utilizes only three differential pressure transducers and can, therefore, only measure the pressure difference between ports 2 and 4 and ports 3 and 5, not their actual pressure. This was done to reduce weight and complexity in the data system. Therefore, the traditional definition of the pressure coefficient was used

$$C_{p,\alpha} = \frac{p_2 - p_4}{\bar{q}} \quad (5.4)$$

$$C_{p,\beta} = \frac{p_3 - p_5}{\bar{q}} \quad (5.5)$$

Since this is not ideal, an extended calibration was conducted at multiple velocity, angle of attack, and side slip angle combinations. The calibration velocities were set at 15, 35, 55, 75, 95, and 115 ft/s. The maximum and minimum airspeed values were derived from previous flight test data on Astraeus where the stall speed and the maximum level flight speed were found to be 19 and 105 ft/s respectively. The angle of attack and side slip range were set at -4 to 14 deg and -6 to 6 deg respectively. These were also selected based on previous flight test data. A plot of the calibration range is shown in Figure 5.8.

Figure 5.8: Five port probe test points used during calibration

The recorded calibration data indicates poor results at 15 ft/s, as shown by the significant scatter in Figure 5.9. Additional runs were conducted at 20 and 25 ft/s and it was found that

Figure 5.9: Angle of attack pressure coefficient versus angle of attack and side slip angle for all calibration velocities

the scatter was significantly reduced at 20 ft/s and essentially eliminated by 25 ft/s. From the calibration data it is evident that the probe essentially has a lower limit for which it can accurately measure angle of attack and side slip. Since velocities below 20 ft/s caused large changes in the pressure coefficients, 20 ft/s was set as the velocity limit for which angle of attack and side slip could be measured. Several types of models can be applied to the data depending on the user's requirements. Since the flight data will be post processed, computation time and resources are relatively unimportant, therefore, a third order model based on reference 6 was used to fit the calibration data. A model analysis using JMP Pro, a statistical modeling and analysis program, was conducted in order to determine fit and parameter significance. For both the angle of attack and side slip models, not all terms were found to be significant so a backwards elimination technique was applied to reduce the model to only significant terms [19]. The final angle of attack and side slip models are given as

$$\begin{aligned} \alpha = & A_0 + A_1 C_{p,\alpha} + A_2 C_{p,\beta} + A_3 C_{p,\beta}^2 + A_4 C_{p,\alpha} C_{p,\beta}^2 \\ & + A_5 C_{p,\alpha}^3 + A_6 C_{p,\alpha}^3 C_{p,\beta}^3 + A_7 V + A_8 V^2 \end{aligned} \quad (5.6)$$

$$\begin{aligned}
\beta = & B_0 + B_1 C_{p,\alpha} + B_2 C_{p,\beta} + B_3 C_{p,\alpha}^2 + B_4 C_{p,\beta}^2 \\
& + B_5 C_{p,\alpha}^2 C_{p,\beta}^2 + B_6 C_{p,\alpha}^2 C_{p,\beta} + B_7 C_{p,\alpha}^3 + B_8 C_{p,\beta}^3 \\
& + B_9 C_{p,\alpha}^3 C_{p,\beta}^2 + B_{10} C_{p,\alpha}^2 C_{p,\beta}^3 + B_{11} V + B_{12} V^2
\end{aligned} \tag{5.7}$$

where A and B are the calibration coefficients. An analysis of the residuals indicates a measurement accuracy on the angle of attack and side slip of 0.33 and 0.29 deg respectively under a 95% confidence interval.

5.3.2 Pressure Lag

Pressure lag is a major problem with multi-port probes that is not encountered by any of the other air flow angle measurement methods. Depending on the length and diameter of the tubing used to transmit the pressure from probe to transducer, a significant time delay may be introduced that causes data analysis to yield inaccurate results. This is especially true with capillary tubing [25]. Time lag has been shown to increase with decreasing diameter and decreasing pressure. Since the tubing used in the five port probe in this research is 0.0625 in in diameter with relatively low pressure differences across the transducer, corrections need to be made. The time lag was calculated using the equation [25]

$$t_{lag} = \frac{128\mu l_e}{\pi d^4} \left[\frac{V_1}{p_1} \ln \frac{(p_0 - p_1)(p + p_1)}{(p - p_1)(p_0 + p_1)} + \frac{3V_d}{p_0 - p_1} \ln \frac{p_0 + p_1}{p + p_1} + \frac{V_d}{p_0 - p_1} \ln \frac{p_0 - p_1}{p - p_1} \right] \tag{5.8}$$

where μ is the coefficient of viscosity of air, l_e is the total length of tubing, and V_d is the volume displacement. The pressure system onboard the aircraft does not change in volume due to applied pressures, so V_d is zero causing the second and third terms inside the brackets to drop out. Due to the asymptotic nature of equation 5.8, the time lag for the pressure at the sensor face to reach 99% of that at the port inlet was determined. Astraeus contains a total of 1.8 ft of tubing while the Nexstar contains 3.9 ft causing time lags of 0.02 ± 0.005 sec and 0.09 ± 0.04 sec respectively depending on the airspeed, angle of attack, and side slip angle. Gauss interpolation was used to determine pressure values at the data system sample intervals.

Chapter 6

Flight Test Program

The flight test program was conducted at Perkins field located inside the North Carolina State University Beef Cattle Research Facility in Butner, NC. Figure 6.1 shows a layout of the field and test area. The runway is paved asphalt and is 454 x 50 ft.

Figure 6.1: Flight test facility at Perkins Field

The flight tests conducted on the aircraft fell into three categories: steady state, deceleration, and acceleration. Atmospheric data was recorded before and after each flight to ensure constant conditions and to account for changing conditions during the test. Conditions were recorded on site using a weather station and were assumed to be the same at altitude since the test

altitudes did not exceed 200 ft and were generally around 100 ft which is a negligible difference in density of $6.9\text{E-}6$ slug/ft³ based on a standard atmosphere. All tests were performed in the morning on calm days to reduce the impact of wind.

The steady state maneuver was performed specifically to collect data for use in the classic PIW-VIW method. Three separate flights were required to collect enough data to generate a single power curve for the aircraft. The three flights were divided up by velocity range. The first flight was at high velocities, the second at moderate velocities, and the last at low velocities. This was done because the battery capacity was not large enough to conduct all flights on a single charge. Within the high velocity group of flights, full throttle runs were conducted first since the battery was near maximum capacity and, therefore, had the greatest power output. Several large holding patterns with shallow turns, as shown by the blue line in Figure 6.1, were performed to allow the aircraft to reach the trim condition at maximum throttle before data was taken. Data was recorded in 5 sec intervals to obtain an average of the conditions and this was repeated four times per velocity. The throttle was then retarded by $\approx 5\%$ on the transmitter and the aircraft allowed to reach the new trim condition again before the procedure was repeated. Having the shallow turns within the steady state runs was not ideal but since the aircraft was remotely piloted, it had to remain within visual contact and within the designated test area. This highlights the problem when using the classic PIW-VIW method on remotely piloted UAS.

The deceleration runs were started in a similar manner to the first PIW-VIW run where the aircraft was allowed to stabilize at maximum level flight speed before the start of the maneuver. The aircraft was then piloted to one side of the test range, data collection engaged, throttle set to zero, and then allowed to naturally slow down while crossing the test range. The speed controller on board the aircraft was set to engage the motor brake immediately upon throttle cut so that the propeller would not windmill and, in the case of Astraeus, fold back. During this maneuver the pilot provided elevator and aileron input to keep the wings level, keep pitching as low as possible, and to maintain constant altitude. Upon the onset of stall, throttle was applied and the aircraft was recovered.

Acceleration runs were performed in the opposite manner. The aircraft was stabilized at a slow, but comfortable airspeed and piloted toward one end of the test range. Full throttle was applied and the aircraft allowed to naturally accelerate to maximum level flight speed while the pilot again used the controls to maintain course and altitude. Later runs were conducted with a ramped throttle input as it decreased the pitching due to thrust during the step input to full throttle and for the Nexstar, decreased the small torque roll caused by the motor.

A total of 11 test sessions were conducted over the course of the flight test program. The first two test sessions were used to check system hardware and software functionality, as well as feasibility of conducting level acceleration and deceleration runs. The next three test sessions

began the data collection flights and included incremental upgrades to the hardware and software. A majority of the data collected during these flights was unusable due to minor software issues. Between the fifth and sixth test session, the system was upgraded from the Raspberry Pi B with NAVIO to the Raspberry Pi B+ with NAVIO+. The reason for the system upgrade was to make use of the improved sensors on the NAVIO+ as well as the more favorable mounting pattern on the Raspberry Pi B+. The last six test sessions provided the majority of the usable data. Three of these test sessions were dedicated to collecting data for use in the classic PIW-VIW method for comparison. The remainder of the test sessions were for acceleration and deceleration runs.

The primary aircraft used during the test program was the low speed version of Astraeus because of its payload accessibility, ease of hand launch, and well documented flight characteristics. A total of 3.9 flight hours were put on the low speed version of Astraeus over the course of the test program. The Nexstar had the next most flight hours with a total of 1.5. The high speed version of Astraeus had the least flight time at 0.58 hours. An incident occurred during the hand launch of the last flight test resulting in damage to the nose of the aircraft as well as the five-port probe. Overall the flight test program contained 36 data flights with 6.0 hours of flight time between the three aircraft.

Chapter 7

Flight Test Results

The flight test results presented in this chapter are broken down into two main sections, classic PIW-VIW results and dynamic analysis results. The dynamic analysis is further broken down into maximum likelihood estimation (MLE) results and parameter estimates, and acceleration and deceleration run results. Before any analysis was completed on the acquired data, the raw data from the flight data system was preprocessed to correct for orientation, offsets, and to apply calibrations. The data system orientation changes by 180 deg between the Nexstar and Astraeus due to differences in mounting options and accessibility. Therefore, the raw data must first be corrected for the change in orientation. Some sensors, particularly the gyroscopes and pressure transducers, have offsets that can cause errors in the data analysis. These offsets were recorded before and after flight so that they could be removed from the raw data. The last function performed by the preprocessor was to apply calibration curves to certain parameters. The data system does not directly record control surface position so a calibration curve was applied to PWM data to determine control surface position. Also, since thrust was not directly measured the propeller model was applied to the velocity and RPM data to determine thrust. Once preprocessing was complete, the analysis was started.

7.1 PIW-VIW

The first set of data analyzed was that collected using the classic PIW-VIW method. This provides a known and trusted basis for comparison against the dynamic power required analysis. It was noted during the flight test phase that one of the key assumptions of the method, steady level flight where lift is equal to weight and drag equal to thrust, proved somewhat difficult to achieve. Since the aircraft were remotely piloted with no telemetry feedback, slight changes in velocity, altitude, and bank angle were difficult for the pilot to discern, especially as the distance between the pilot and aircraft increased. The size of the flight test area also made it difficult

to have long periods of wings level flight before the aircraft needed to be maneuvered in order to remain within the test boundary. Many of the data sets included steady climbs or descents of a few feet per second. This caused an artificial increase or decrease in the calculated power required. These can be easily accounted for using the equation

$$\begin{aligned} 0 &= (T - D) V + m g \dot{h} \\ D V &= T V + m g \dot{h} \end{aligned} \tag{7.1}$$

Other data sets had too large or inconsistent variations in velocity and or altitude and had to be eliminated. Of all the data collected, approximately one out of every three data points had to be eliminated due to assumption violations. Therefore, it is necessary to collect additional data when using this method, which requires longer test times. The final results from the classic method for each of the aircraft tested are shown in Figure 7.1.

7.2 Maximum Likelihood Results and Parameter Estimates

The second set of data analyzed included the rest of the data collected through the dynamic maneuvers (acceleration/deceleration). The data runs were processed individually with the initial starting values for the optimization algorithm set to those estimated for the specific aircraft. Estimations for Astraeus came from the critical design review written by the NCSU senior students who designed the aircraft. The values for the Nexstar came from estimations using the USAF Datcom. Since the optimization algorithm relies on the relaxation technique, the stopping criteria was set as the change in cost from the previous iteration. The break value for the difference in cost was set at 0.01. This value was chosen as it provided a compromise between speed and accuracy. Values greater than an order of magnitude above this value caused quick run times but inaccurate parameter estimates. Values an order of magnitude below caused excessive run times (greater than 30sec) with no change in the final parameter estimate values over the 0.01 value. The MLE outputs were coplotted with the measured outputs along with their residuals to compare and examine model fit. Examples of these plots are shown in Figures 7.2 through 7.5 for a deceleration run (a) and an acceleration run (b) on the Nexstar. Overall, the model fit was fairly good for both acceleration and deceleration runs with only a few differences between them. One major difference between the runs is visible in Figures 7.4 and 7.5. The magnitude of the accelerometer noise is much higher in the acceleration run than the deceleration run. This trend was recorded on both aircraft, however, the increase in magnitude of the accelerometer noise was much higher for the Nexstar. Despite the increase in noise, the generated power required curve, shown in Figures 7.6, still provides a good fit when compared to the deceleration run and the PIW-VIW data. The cause of the noise was traced to vibration

(a)

(b)

(c)

Figure 7.1: Classic PIW-VIW flight test results: (a) Astraesus LS (b) Astraesus HS (c) Nexstar

Figure 7.2: MLE velocity comparison with residuals for the Nexstar: (a) deceleration, (b) acceleration

Figure 7.3: MLE angle of attack comparison with residuals for the Nexstar: (a) deceleration, (b) acceleration

Figure 7.4: MLE axial acceleration comparison with residuals for the Nexstar: (a) deceleration, (b) acceleration

Figure 7.5: MLE vertical acceleration comparison with residuals for the Nexstar: (a) deceleration, (b) acceleration

Figure 7.6: MLE derived power required plot for the Nexstar with PIW-VIW comparison: (a) deceleration, (b) acceleration

from the motor/propeller combination. Tests were conducted to determine whether the noise was electrical or vibrational by physically disconnecting the data system from the aircraft but still keeping it electrically connected. Sensor noise was shown to be equal to power off values even during throttle sweeps, indicating vibrational noise was the culprit and not electrical noise. The maximum RPM seen by the Nexstar during an average acceleration run was around 16,100, while for Astraesus it was only around 9,800. This is due to the 6-cell battery setup used in the Nexstar vs. the 3-cell setup used in Astraesus. This also corroborates the difference in noise magnitude seen in both aircraft. Figure 7.7 shows an acceleration run for both aircraft and clearly demonstrates the difference in noise magnitude. Even though both propellers were balanced before installation, small imbalances and/or dynamic imbalances will most likely still exist and be much worse at higher RPM values. Also since Astraesus is in a pusher configuration, the motor is much further from the accelerometers which allows the airframe to absorb some of the vibration. One other observation of note is the distribution of residuals between the model and measured data. If the model being applied to the system is adequate and the measured data is only corrupted by white Gaussian noise, then the residuals should have an even random distribution around zero across time [19]. If the model is inadequate or the measurement noise is colored instead of white, 'patterns' will appear in the residuals. Some of the residual plots shown in Figures 7.2 through 7.5 exhibit some degree of non-uniformity. While a majority of model residual issues can generally be traced to either model deficiency or colored noise, some can be due to physical phenomena. A good example of this is Figure 7.5a. Here the magnitude of the residuals decrease over time. This is most likely due to increased vibration noise at higher

Figure 7.7: Power on noise comparison during an acceleration run for Astraesus and Nexstar

airspeeds caused by small protuberances on the aircraft surface. Comparing the residuals to the airspeed plot shows a high correlation between residual magnitude and airspeed. The remainder of the residual plots were analyzed for model deficiency and checked for colored noise.

7.2.1 Model Analysis

As previously mentioned, if a model is deficient the residuals will assume a non-random pattern. Figure 7.3 shows an example of this where the magnitude of residuals are relatively constant in time but there appears to be an underlying pattern. Since the rate of change of the angle of attack is directly related to pitch rate, this could indicate a deficiency in the model, specifically C_{Lq} , which would indicate that the negligible pitch rate assumption has been violated. This can be checked by using the same procedure that was applied in Chapter 4 to determine the significance of C_{Lq} using the flight simulator. All flight test data was re-run with C_{Lq} included in the lift model. The same t -statistic that was used in Chapter 4, was calculated for C_{Lq} along with the SNR of the pitch signal and co-plotted with the flight simulator results in Figure 7.8. During the simulation runs, noise was added to the output to mimic real flight data which turned the SNR into an exact quantity. For the measured flight test data it is not known what portion of the measurement is the signal versus the noise. A technique developed by Garcia was used to separate the signal from the noise by using a smoothing spline model whose smoothing weight was determined by minimizing the generalized cross-validation score [4].

The results are relatively inconclusive as there is not a clear SNR cutoff for determining

Figure 7.8: SNR values for measured flight data as well as simulated data

significance and a given SNR value could cause C_{Lq} to be significant or not depending on the run. This indicates that there are other underlying factors because the simulator runs show evidence of a strong correlation between the noise on the other measurements and the significance cutoff of C_{Lq} . Between the deceleration and acceleration runs, the noise on the accelerometers greatly exceeded the 1.5 scale factor used on the simulated data. One thing to note on the plot is that even when C_{Lq} is significant, its t -statistic is not very large compared to the cutoff indicating that although it is statistically significant, it is weakly so. Looking at the parameter estimates and their confidence intervals in Figure 7.9 makes this much clearer. The parameter numbers follow the order given in Table 7.1, which is presented in the next section. Of interest in these plots is parameter number seven which is C_{Lq} . These plots show that even when C_{Lq} is significant, its value is highly variable between runs and its confidence interval is generally around half of its value if not more. Estimating this parameter with any accuracy cannot be done using these runs which is one of the reasons for the assumption of negligible pitch rate. Another reason for leaving the pitch rate term out of the lift model is that when the runs were analyzed with and without C_{Lq} , the model outputs and remaining parameter values did not change significantly. Figure 7.10 shows the difference between the model with C_{Lq} and without for both an acceleration run and deceleration run. The results show very little difference between the outputs. The angle of attack is a direct function of the lift model and pitch rate, so any real differences between including or excluding C_{Lq} would manifest themselves there. However, the angle of attack output varies by less than ± 0.003 rad when C_{Lq} is included. Some

Figure 7.9: Estimated lift and drag model parameter values and corresponding confidence intervals

larger differences are noted in the accelerometer plots but the mean difference is still around zero and the magnitude of the noise is relatively high on those sensors which will lead to larger residuals.

From these plots it can be concluded that C_{Lq} is not a significant term and can be safely left out of the model. Determining a practical cutoff for pitch rate is much more difficult as shown by Figure 7.8. A low pitch rate does not guarantee insignificance but for reasonably low pitch rates, significance is essentially unimportant. The definition of reasonably low is somewhat arbitrary but for this research it was found that pitch rates in the range of $\pm \approx 50$ deg/s produced more power required curves with a favorable fit.

7.2.2 Parameter Estimates

The maximum likelihood estimates for the lift and drag model parameters(excluding C_{Lq}) are shown in Table 7.1. These represent the average values based on all of the acceleration and deceleration runs. These were determined under the assumption of white Gaussian noise. While the assumption generally applies, it has been shown that the error bounds determined under this assumption are generally too optimistic [18]. The discrepancies between the scatter of repeated measurements and the Cramer-Rao lower bounds are due to colored noise which violates the assumption about the independence of the noise samples. An in depth analysis by Maine and Iliff on the spectral density of residuals from flight test data showed significantly higher power in the residuals at lower frequencies ($\approx \leq 1Hz$) [15]. This relatively high power band falls within the frequency band of the aircraft dynamics and depending on the color of the noise, will cause

Figure 7.10: Differences in model output when including C_{Lq} for acceleration and deceleration run on Nexstar: (a) velocity, (b) angle of attack, (c) axial acceleration, (d) vertical acceleration

Table 7.1: Parameter estimates under white Gaussian noise assumption

Parameter	Astraeus LS	Astraeus HS	Nexstar
C_{D0}	0.029 ± 0.007	0.031 ± 0.003	0.027 ± 0.003
K	0.034 ± 0.01	0.087 ± 0.02	0.095 ± 0.04
C_{L0}	0.47 ± 0.03	0.13 ± 0.03	0.47 ± 0.03
$C_{L\alpha}$	7.8 ± 0.4	5.5 ± 0.2	5.9 ± 0.4
$CL_{min,D}$	0.31 ± 0.06	0.13 ± 0.05	0.37 ± 0.05
$C_{L\delta e}$	0.005 ± 0.002	0.004 ± 0.001	0.007 ± 0.002

Table 7.2: Parameter estimates under colored noise assumption

Parameter	Astraeus LS	Astraeus HS	Nexstar
C_{D0}	0.029 ± 0.008	0.031 ± 0.004	0.027 ± 0.003
K	0.034 ± 0.02	0.087 ± 0.02	0.095 ± 0.04
C_{L0}	0.47 ± 0.04	0.13 ± 0.03	0.47 ± 0.04
$C_{L\alpha}$	7.8 ± 0.5	5.5 ± 0.3	5.9 ± 0.4
$CL_{min,D}$	0.31 ± 0.07	0.13 ± 0.06	0.37 ± 0.05
$C_{L\delta e}$	0.005 ± 0.002	0.004 ± 0.002	0.007 ± 0.003

the residuals to exhibit 'patterns.' A check was made on the residuals by re-analyzing them under the assumption of colored noise with C_{Lq} removed since it was found to have little effect on the model outputs. The colored noise analysis is essentially the same as that for white Gaussian but uses a modified expression of the covariance matrix[18]

$$Cov(\hat{\Theta}) = D \left[\sum_{i=1}^N SE(i)^T R^{-1} \sum_{j=1}^N \mathcal{R}_{vv}(i-j) R^{-1} SE(j) \right] D \quad (7.2)$$

where

$$\mathcal{R}_{vv}(k) = \mathcal{R}_{vv}(-k) = \frac{1}{N-k} \sum_{i=1}^{N-k} \nu(i) \nu(i+k)^T \quad (7.3)$$

is the autocorrelation matrix for the output residuals. If the noise is truly white Gaussian, then $R_{vv} = R$ and Eq. 7.2 simplifies to the standard covariance equation. Table 7.2 contains the parameter error bounds when colored noise is assumed. The error bounds are generally larger under this assumption indicating that colored noise is present in the system. However, the differences in the magnitudes indicate that the white Gaussian assumption is relatively safe and that colored noise is most likely not the cause of the non-uniformities found in the residual plots. In this case, any non-uniformities are most likely due to excessive sensor noise which appears to exhibit a fairly even power distribution across the measurable frequency range and is most likely diluting the effects of colored noise.

7.3 Power Required

Power required plots for each aircraft were generated from the above parameter estimates using the procedure outlined in Chapter 3. The curves for each run are co-plotted with the classic PIW-VIW data in Figure 7.11 for comparison. Acceleration data for the high speed version of Astraeus does not exist due to an incident that occurred on take-off which caused damage to

(a)

(b)

(c)

Figure 7.11: Dynamically generated power required results: (a) Astraeus LS (b) Astraeus HS (c) Nexstar

the nose of the aircraft and five port probe. These plots contain several key observations that highlight some problems and advantages and are discussed in detail in the following sections.

7.3.1 Acceleration Runs

One obvious observation from the plots is that the acceleration runs have a much higher spread and poorer overall fit than the deceleration runs. This is most likely due to one or both of these factors: sensor noise and thrust measurement.

Power Sensitivity due to Noise

As previously mentioned, during the analysis it was found that the acceleration runs had substantially more noise associated with the accelerometer measurements than the deceleration runs. Analysis of the noise between deceleration and acceleration runs showed an increase in noise variance by about 10 times. Since the effects of imbalances become worse at higher RPM values it puts the Nexstar at a disadvantage when using the acceleration run due to its 6-cell setup. A simple analysis of the maximum likelihood estimates also shows how the noise affects the results. One of the properties of the estimate is that it asymptotically approaches the normal distribution where

$$\hat{\Theta} = \Theta + \nu \quad \nu \sim (0, \sigma^2) \quad (7.4)$$

As σ^2 increases, the estimate is increasingly diluted by the noise which increases the probability of the estimate falling further from the mean. This was also confirmed by simulated flight data where the same acceleration run was analyzed 50 times, 25 times with the average accelerometer noise level found during the deceleration run and 25 times with the average noise level found during the acceleration run (Nexstar). The power required plot generated from the simulator data in Figure 7.12 shows a similar trend to the flight test data indicating a relatively tightly packed group for the low noise runs and a noticeable spread for the high noise runs. Since the propulsion system vibration mainly affects the accelerometers, only their noise was changed.

Power Sensitivity due to Thrust

The acceleration runs require the thrust to be known in order to estimate the aircraft power curve. Since the value of the thrust directly affects Eqs. 3.21 and 3.22, the accuracy of its measurement will change the accuracy of the power estimate. The sensitivity of the power required due to thrust can be calculated by solving equation Eq. 3.21 for C_D , then determining the power required using equation Eq. 3.45, and lastly taking the derivative of P_{req} with respect to T yielding

$$\frac{dP_{req}}{dT} = V \cos \alpha \cos \beta \quad (7.5)$$

Figure 7.12: Flight simulator derived power required curves showing effect of system noise

The strong dependence on the velocity can be directly seen by the increase in spread of the acceleration derived power curves as the speed increases. From this it can be concluded that accurate thrust measurements are important, and that the power required curve estimates are less accurate at high velocities. This also holds true for the deceleration runs even though the thrust is zero. Since power required increases with the cube of the velocity, any errors related to velocity will be much larger at higher velocities.

Throttle Response

In the theoretical development and maximum likelihood analysis the assumption was made that the thrust acts in line with the center of gravity, thereby causing the pitching moment due to thrust to be zero. In reality the thrust line will most likely be offset, but will generally be close to the center of gravity so pitch changes due to thrust will be small. It was noted during the flight test program that pitch changes did occur for both aircraft during the acceleration and deceleration runs. The pitch changes that occurred upon motor shutoff were generally very small or insignificant. For the acceleration runs, however, moderate to large pitch changes were sometimes encountered during throttle up. The Nexstar always had a much larger pitch change when compared to Astraesus as it has a greater thrust output and larger thrust line offset. Two methods were used in an attempt to deal with these unwanted pitch changes.

The first method moved up the start time of the data set if a large pitch change occurred and the analysis provided an unrealistic power required curve. Analyzing the data after the

pitch change occurred has shown to provide mixed results. Doing this removes a relatively large portion of low speed data since the aircraft's acceleration is at a maximum during this time. In a few cases this has provided good results while in others it has removed enough data to where the parameter estimates along with the power required curve are no longer accurate. Unless absolutely necessary, as much data as possible should be used in the analysis.

The second option for dealing with pitch changes is to modify the throttle input during acceleration. The acceleration runs were done using what is essentially a step input on the throttle. This often produced pitch profiles like the one seen in black in Figure 7.13. Pitch rates

Figure 7.13: Nexstar pitch response to a stepped vs ramped throttle input

of up to 60 deg/s were noted during some runs with relatively large altitude changes occurring. By using a ramp input it is possible to significantly decrease the amount of pitching that occurs during acceleration while at low speeds. Figure 7.14 shows the two throttle inputs that were used on the Nexstar. The aircraft response to the ramp input can also be seen in Figure 7.13 (red). Using this technique significantly decreased the amount of pitching that occurred at the start of the acceleration runs. Pitch rates were generally ≤ 15 deg/s through the entire run. The ramp input was only applied to the Nexstar as this was not a problem on Astraesus. Power required curves were generated using both throttle inputs and are shown in Figure 7.15. The plot is inconclusive as to whether or not the ramp input provides a better analysis. This is most likely due to excessive system noise caused by the propulsion system. Since both ramp and step

Figure 7.14: Throttle input: step vs ramp

Figure 7.15: Power required comparison between stepped and ramped throttle input

inputs had equal noise variances once they reached full throttle and since the noise was already rather high, any advantage to the ramp input was most likely lost in the noise. The pilot did note, however, that acceleration runs using throttle ramps were much smoother and easier to perform than those using the step input. This inconclusive result also confirms the assumption of negligible pitch rate. Since the plot shows no quantifiable difference between the two throttle inputs even though the pitch rates on some runs were quite high, it can again be concluded that the pitch rate must not have a significant effect on the measured power required curve.

7.3.2 Deceleration Runs

The deceleration runs provide the more accurate and consistent results when compared with the acceleration runs. They are also in good agreement with the data points obtained through the classic PIW-VIW method. Since these runs were made with the motor off and propeller stationary/folded back, there is very little vibrational noise which is a major advantage of the deceleration run. The effect of the noise was shown in the acceleration section but it becomes very clear when comparing these runs that sensor noise plays a large role in the quality of the results. Another advantage of the deceleration run that is not so obvious from the power required plots is that the thrust terms in the maximum likelihood analysis are zero. This is one of, if not, the most significant benefit of the deceleration runs over the acceleration runs and classic PIW-VIW method. Determining the thrust in flight can be a major challenge. The classic PIW-VIW method relies on engine manufacture data and the assumption of constant propeller efficiency of 0.80 (when using a constant speed propeller) to estimate output power [12]. This may not be accurate depending on engine age and operating conditions. In this research, a propeller thrust model was generated from wind tunnel data which proved to be time consuming and required a significant amount of specialized hardware, which may not always be easy to obtain. By eliminating the need to know the thrust, time and effort are saved and estimate accuracy is improved. However, the thrust is only truly zero when the engine or motor is shut off. For vehicles with an electric propulsion system, this is trivial. For reciprocating or jet powered aircraft, it may not be practical or safe to shut down the propulsion system. In that case, a residual thrust model may need to be included in the analysis but that is outside of the scope of this research.

Using the deceleration runs only, the curves can be averaged to obtain a final power required curve for each aircraft. This was done using only the deceleration runs because they provided a much more consistent and accurate result. The final curves with confidence intervals and PIW-VIW data points are shown in Figure 7.16. The curves show a very good match with the classic method results with the confidence interval limits generally matching the spread of the PIW-VIW points.

(a)

(b)

(c)

Figure 7.16: Final power required curves: (a) Astraesus LS (b) Astraesus HS (c) Nexstar

Chapter 8

Conclusions

The main focus of this research was on the development of a method for power required analysis for unmanned vehicles, specifically those that are remotely piloted, using dynamic maneuvers. This method has been shown to be a valid analysis tool providing data of equal quality when using the deceleration method as compared to the classic PIW-VIW method. A maximum likelihood analysis was performed on the flight test data with the assumptions that the pitch rate and lateral dynamics remain insignificant throughout the entire maneuver. The maneuvers performed to collect the data for analysis were acceleration and deceleration runs as these were the easiest to perform for a remote operator with no telemetry feedback. Simulation has shown that altitude changes and low pitch rates do not have a significant effect on the quality of the analysis. The flight test results confirm this but are unable to provide a concrete limit on the pitch rate. A statistical analysis on the pitch rate dependent term found that even when it was significant, its accuracy and error bounds were highly variable and that the remaining outputs were essentially unchanged when the term was excluded.

A majority of the benefits of the dynamic power analysis have been discussed in the previous section but a few additional points can be made. The dynamic power analysis is a much faster and easier option when compared with the classic PIW-VIW method. During the flight test program, three flights on each aircraft were dedicated to collecting PIW-VIW data points. Each flight lasted between 13 and 20 minutes depending on the speed range flown and four data points were collected per velocity, some of which were eliminated due to assumption violations. Total flight time to collect all PIW-VIW data was around 70 min for each aircraft. The dynamic power analysis flights in comparison were shorter. The average flight completed 10 data runs and lasted just under 10 mins. For the average flight, 10 power curves were generated from 10 min of flight time instead of one power curve from 70 min of flight time. This presents a large savings in test time which results in lower testing costs.

This method also works well when the test range is relatively small. The classic PIW-VIW

method is built on the assumption of steady state flight which is difficult to achieve in a small test range since the distance between boundaries may not be enough to allow the aircraft to reach steady state conditions. The dynamic power runs performed in this research covered approximately 1000 ft during the maneuver which fit well within the boundaries of the test range. Since the maximum likelihood technique uses the non-linear equations of motion of the aircraft, deviations from the desired maneuver are acceptable provided the lateral and pitch assumptions are not violated. The PIW-VIW method requires much tighter control which is difficult to achieve for a ground based pilot.

8.1 Future Work

While the method is functional, it brings with it questions about different applications and improvements. One immediate area that was made clear by the acceleration runs is data quality. Sensor noise played a very large role in the accuracy of the final power required curves. The two obvious ways to mitigate sensor noise would be to design a better vibration mount for the flight data system and propulsion system, and to use higher quality sensors. The data and damping mounts were comprised of off the shelf low cost RC hardware. These provide 'good enough' damping and results for hobbyist but can be borderline adequate for research purposes. Nonetheless, even with this low cost hardware, the deceleration runs can provide fairly accurate results. Most aircraft also have some sort of vibration mounts on the engine/motor, which Astraeus and the Nexstar did not.

Another possibility that has the potential to provide better results would be to have an autopilot perform the dynamic runs instead of a human pilot. It is difficult to say whether or not this would provide a big improvement to the quality of the data since the equations of motion account for dynamics. It would, at the very least, support the assumptions made by the analysis on pitch rate and lateral dynamics. The PIW-VIW method would also greatly benefit from an autopilot because velocity and altitude control would be much better.

While this research was designed around unmanned aircraft, there is no reason why this method could not be directly applied to manned aircraft. The big issue would be thrust modeling, specifically for the deceleration runs, as they may require a residual thrust model to maintain accuracy. The acceleration run could be used without problem, provided the thrust model for the aircraft is known.

Overall the dynamic power analysis was shown to be a valid analysis tool for power required with the deceleration method favored due to the elimination of the thrust term and significantly decreased vibrational noise.

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APPENDIX

Appendix A

Propeller Testing

The propeller models used in this research were obtained by the NCSU senior design class that designed and constructed Astraeus (Aernaut CAM 12x8) and by Dr. Stearns Heinzen at Naval Research Labs (APC 10x5). Both tests were conducted using the same testing procedure and similar hardware. The propeller and brushless motor were mounted on a load cell that measures thrust and torque. A shroud covers the load cell and motor so that the flow does not influence the load cell measurements. RPM was measured using the auxiliary output line of the speed controller. Power for the motor was supplied by a high current power supply during the APC 10x5 test and by multiple flight batteries connected in parallel during the Aeronaut 12x8 test. The load cell, as well as the speed controller input and auxiliary line, were fed into a computer where control, monitoring, and data collection was done with LabView. Each test began with the ambient atmospheric pressure recorded. The wind tunnel was started and the dynamic pressure set to the first data point. Throttle was set to 100% and the RPM and tunnel dynamic pressure allowed to stabilize. Data was then recorded which consisted of averaging 100 samples from each measurement to obtain an average value for thrust, torque, RPM, etc. before nondimensionalization. This procedure was repeated for increasing dynamic pressure values until the propeller no longer generated thrust. Dynamic pressure was then decreased and the same data collection procedure repeated for decreasing dynamic pressure values. During the test conducted by the senior design class, only full throttle runs were completed and it was assumed that Reynolds effects due to RPM were negligible. The test conducted by Dr. Heinzen included various RPM levels to capture Reynolds effects. The resulting propeller data used to generate the models used in this research are shown in Figure A.1 and A.2.

Figure A.1: Aeronaut CAM 12x8 wind tunnel test results: (a) thrust coefficient (b) power coefficient (c) efficiency

Figure A.2: APC 10x5 wind tunnel test results: (a) thrust coefficient (b) power coefficient (c) efficiency