ABSTRACT

KAKALEY, DANIEL EDWIN. An Offset Hub Active Vibration Control System for Mitigating Helicopter Vibrations during Power Loss: Simulation and Experimental Demonstration. (Under the direction of Dr. Gregory Buckner).

In recent years, hub active vibration control (HAVC) technologies have been developed to attenuate blade pass frequency vibration on helicopters. While these systems provide superior vibration control with reduced weight compared to passive options, in the event of electrical power loss they can exacerbate the vibration problem in a manner that is problematic for helicopter and tiltrotor aircraft. This paper presents an offset hub active vibration control system (OHAVCS) designed to attenuate vibration during a power loss by offsetting the centers of rotation of two imbalance masses. The equations of motion for this system are developed using Lagrangian methods; analysis, simulation and experimental validation of these equations indicate that offsetting the imbalance masses effectively mitigates 1/Rev vibrations during a rotor hub power loss while continuing to cancel N/Rev vibrations during normal operation. These offsets create stabilizing centripetal torques that rotate each imbalance mass to a unique equilibrium angle. Experimental data also indicate that these offsets do not hinder control of the imbalance masses during normal (active) operation, though they do increase system power requirements.

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An Offset Hub Active Vibration Control System for Mitigating Helicopter Vibrations during
Power Loss: Simulation and Experimental Demonstration

by
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DEDICATION

This study is dedicated to the three people who provided the inspiration for my scientific career: my great grandfather Dr. Claud E. Cleeton (physicist), my grandfather Ed Kakaley (mechanical engineer), and my father Russell Kakaley (mechanical engineer). My education today is a direct result of their support, inspiration, and encouragement.

BIOGRAPHY

Daniel Edwin Kakaley was born in Springfield, Massachusetts to Russell and Rebecca Kakaley. Shortly thereafter, his family relocated to Charlotte, North Carolina for the remainder of his childhood. After high school, he began working at Midrex Technologies in a machine shop and learned the importance of a college education. In 2010 he began his studies at North Carolina State University. During this time, he worked at NASA Glenn as an Aeronautics Academy Intern and Midrex Technologies analyzing the reduction of iron, under the supervision of Greg Hughes, who provided a tremendous example of hard work and inspiration. In the spring of 2013, he graduated with a B.S. in Mechanical Engineering and subsequently began working in Mechanical Research at Lord Corporation under the supervision of Dr. Mark Jolly and Russell Altieri. Concurrent with his work, he graduated with a B.S. in Physics in 2014. His work responsibilities focus on developing highly engineered vibration solutions. His work on helicopter vibration control has culminated in the Grover E. Bell Award at AHS International's 71st Annual Forum for his part on the Hub Mounted Vibration Suppressor (HMVS) team.

In a partnership between Lord Corporation and North Carolina State University, he conducted his Master's research under Dr. Gregory Buckner's guidance.

Shortly after beginning work at Lord Corporation, he met his fiancée, Elizabeth Medlock.

With her continual support and encouragement, he has continued his work at Lord Corporation and academic career at North Carolina State University.

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1. INTRODUCTION

Principle sources of helicopter vibration occur at the blade pass frequency (N/Rev), defined to be the number of blades (N) times the hub rotational frequency ($1/\text{Rev}$) [1-3]. Blade pass frequency vibrations reduce crew comfort and increase maintenance demands and operating costs [4]. Two primary approaches to vibration attenuation are currently utilized: active vibration control (AVC) systems and passive absorption technologies. The most common passive approach uses bifilar pendulum absorbers (commonly called bifilars) to attenuate vibrations at the helicopter hub [5-9]; this method has also been applied to reciprocating engine crankshafts [10-11]. Bifilars attenuate up to 90% of N/Rev vibrations while accounting for approximately 1% of the helicopter gross weight [12-13]. Figure 1 shows a typical construction schematic: a set of tungsten masses attached to bifilar arms by two journal bearings. The bifilar's masses and dimensions are designed so that its natural frequency matches specific N/Rev vibration frequencies; the system requires no complex electronics and provides a reliable mechanical method of attenuating vibration. However, bifilars require journal bearing lubrication before every flight, create increased drag on the helicopter, and do not cancel vibration effectively during high acceleration helicopter maneuvers (due to large non-linear pendulum oscillations).

Figure 1: Exploded assembly view of UH-60 (Blackhawk) hub with bifilar pendulum absorber [14-15]

In recent years, AVC technologies have been developed as lighter and higher performance alternatives to passive systems. AVC systems have been estimated to reduce part removals by 5-10%, decrease structural fatigue, and enable longer missions [16]. Hub AVC systems employ imbalance masses rotating at the blade pass frequency to

cancel N/Rev vibration as measured by cabin accelerometers [17-20]. These systems provide superior vibration control with significant reductions in weight compared to bifilars (typically 0.5% of the aircraft gross weight). Figure 2a shows a prototype concentric hub AVC system (CHAVCS) developed by Sikorsky and LORD which mounts to an existing bifilar location, while Figure 2b highlights its internal components. The mechanical unit, which is mounted to the adapter plate, contains the imbalance masses, motors and bearings. The motor and control electronics are located above the mechanical unit, while the blade de-ice electronics are located inside the toroidal hub AVC assembly. Figure 2c details the imbalance mass assembly inside the mechanical unit. During normal operation, brushless motors position the imbalance masses to produce a net centrifugal force with a specific magnitude and phase that optimizes the attenuation of N/Rev vibrations. However, in the event of electrical power loss to these motors, the imbalance masses reach steady-state rotation at the hub rotational frequency and may create 1/Rev vibration at levels that can be problematic to helicopters and tiltrotor aircraft. For such applications, it is critical to consider design modifications to the CHAVCS that do not exacerbate 1/Rev vibrations during a power loss.

Figure 2. Prototype Sikorsky and LORD concentric hub AVC system: a) photograph showing external housing and components [21]; b) exploded view showing internal components; c) detailed view of imbalance masses

This paper presents a novel AVC approach to mitigating 1/Rev vibrations during a rotor hub power loss. By offsetting a pair of imbalance masses from the hub center of rotation, centripetal acceleration causes these masses to self-align such that the net imbalance force (and associated 1/Rev vibration) will be zero. This AVC system has the advantage of canceling blade pass frequency vibrations (N/Rev) during normal (active) operation, and mitigating hub frequency vibrations (1/Rev) during a power loss.

2. METHODS

The proposed modifications to a concentric hub AVC system (CHAVCS), wherein the bearing centers are offset equal and opposite distances l_i , are presented in Figure 3a. This “offset hub AVC system” (OHAVCS) can utilize similar imbalance masses, motors, and bearings found in the CHAVCS. During normal OHAVCS operation, the imbalance masses rotate at N/Rev and generate vibration cancelling forces. During a loss of electrical power to the rotor hub, however, motor torques are no longer transferred to the imbalance masses, and the offsets produce moments on the imbalances so that they rotate to opposite sides of the hub and statically balance at $1/\text{Rev}$, as shown in Figure 3b. Due to these offsets, controlling the imbalance mass positions during normal (active) operation requires larger motor torque and increased power consumption. By determining the optimal offset, both the weight of the OHAVCS motor power electronics and the additional power consumption can be minimized.

Figure 3. Offset hub AVC system (OHAVCS): a) Normal (active) operation at N/Rev; b) Operation during rotor hub power loss

An exploded view of the OHAVCS (Figure 4) reveals a similar construction as the CHAVCS, enabling fabrication and deployment with reduced manufacturing and maintenance costs.

Figure 4. Exploded view of offset hub AVC system (OHAVCS)

2.1 Dynamic Model of Offset Imbalance Mass

The dynamic response of an offset imbalance mass can be analyzed using the generalized diagram of Figure 5, which lumps the imbalance mass m_i at a point, neglecting the rotational inertia and kinetic energy of the distributed mass. The imbalance point mass m_i is supported by massless linkages l_1 and l_2 (oriented at angles θ_{i1} and θ_{i2} , respectively) and rotates in a vertical plane where gravitational acceleration g is assumed to act downward, corresponding physically to a turboprop or a tiltrotor in airplane mode.

Figure 5. Generalized rotating imbalance mass schematic

Equations of motion for this imbalance mass can be derived using a Lagrangian approach [22], which requires potential and kinetic energy expressions based on imbalance mass position (x_i, y_i) and velocity squared (v_i^2) :

$$x_i = l_1 \cos \theta_{i1} + l_2 \cos(\theta_{i1} + \theta_{i2}) \quad (1)$$

$$y_i = l_1 \sin \theta_{i1} + l_2 \sin(\theta_{i1} + \theta_{i2}) \quad (2)$$

$$v_i^2 = (l_1 \dot{\theta}_{i1})^2 + (l_2 (\dot{\theta}_{i1} + \dot{\theta}_{i2}))^2 + 2l_1 l_2 \dot{\theta}_{i1} (\dot{\theta}_{i1} + \dot{\theta}_{i2}) \cos \theta_{i2} \quad (3)$$

Where $\dot{\theta}_{ij}$ is equivalent to $d\theta_{ij}/dt$. Kinetic energy T_i and potential energy U_i expressions for the i^{th} imbalance mass are:

$$T_i = \frac{1}{2} m_i v_i^2 = \frac{1}{2} m_i \left(l_{i1} \dot{\theta}_{i1} \right)^2 + \frac{1}{2} m_i \left(l_2 \left(\dot{\theta}_{i1} + \dot{\theta}_{i2} \right) \right)^2 + m_i l_1 l_2 \dot{\theta}_{i1} \left(\dot{\theta}_{i1} + \dot{\theta}_{i2} \right) \cos \theta_{i2} \quad (4)$$

$$U_i = m_i g y_i = m_i g l_1 \sin \theta_{i1} + m_i g l_2 \sin (\theta_{i1} + \theta_{i2}) \quad (5)$$

The link 1 offset angle θ_{i1} can be expressed in terms of the time-varying hub rotational velocity $\Omega(t)$ and its initial offset angle:

$$\theta_{i1}(t) = \theta_{i1}(t=0) + \int_{\tau=0}^{\tau=t} \Omega(\tau) d\tau \quad (6)$$

The kinetic energy and potential energy expressions are thus:

$$T_i = \frac{1}{2} m_i \left(l_1 \Omega \right)^2 + \frac{1}{2} m_i \left(l_2 \left(\Omega + \dot{\theta}_{i2} \right) \right)^2 + m_i l_1 l_2 \Omega \left(\Omega + \dot{\theta}_{i2} \right) \cos \theta_{i2} \quad (7)$$

$$U_i = m_i g l_1 \sin \theta_{i1} + m_i g l_2 \sin (\theta_{i1} + \theta_{i2}) \quad (8)$$

The Rayleigh dissipation function F_i can be defined to approximate linear damping in the system [23]:

$$F_i = \omega_n \zeta \dot{\theta}_{i2}^2 = \sqrt{\frac{l_1}{l_2}} \Omega \zeta \dot{\theta}_{i2}^2 \quad (9)$$

Where ζ is the dimensionless damping coefficient. The Lagrangian can then be expressed [22-23]:

$$L_i = T_i - U_i \quad (10)$$

$$\frac{d}{dt} \left(\frac{\partial L_i}{\partial \dot{\theta}_{i2}} \right) - \frac{\partial L_i}{\partial \theta_{i2}} = - \frac{\partial F_i}{\partial \dot{\theta}_{i2}} \quad (11)$$

Eqs. (7) through (11) can be combined to model the imbalance mass dynamics:

$$\ddot{\theta}_{i2} + 2 \sqrt{\frac{l_1}{l_2}} \Omega \zeta \dot{\theta}_{i2} + \frac{l_1}{l_2} \Omega^2 \sin \theta_{i2} = -\dot{\Omega} \left(1 + \frac{l_1}{l_2} \cos \theta_{i2} \right) - \frac{g}{l_2} \left(\cos (\theta_{i1} + \theta_{i2}) \right) \quad (12)$$

The mass's undamped natural frequency ω_n can be approximated by neglecting damping ($\zeta = 0$), gravity ($g = 0$), and hub acceleration ($\dot{\Omega} = 0$) in Eqn. (12):

$$\ddot{\theta}_{i2} + \frac{l_1}{l_2} \Omega^2 \sin \theta_{i2} = 0 \quad (13)$$

Applying the small angle approximation to Eqn. (13) results in a simple harmonic oscillator and provides an expression for the undamped natural frequency:

$$\ddot{\theta}_{i2} + \frac{l_1}{l_2} \Omega^2 \theta_{i2} = \ddot{\theta}_{i2} + \omega_n^2 \theta_{i2} = 0 \quad (14)$$

$$\omega_n = \sqrt{\frac{l_1}{l_2}} \Omega \quad (15)$$

It is worth noting the natural frequency in Eqn. (15) is identical to that of a monofilar pendulum absorber, which is used to attenuate rotor hub vibrations [6].

Gravity's effects on the system response θ_{i2} can be approximated analytically. When Eqn. (12) is simplified by neglecting hub rotational acceleration ($\dot{\Omega} = 0$) and assuming that $\theta_{i1}(t = 0) = 0$, the following results:

$$\ddot{\theta}_{i2} + 2\sqrt{\frac{l_1}{l_2}} \Omega \zeta \dot{\theta}_{i2} + \frac{l_1}{l_2} \Omega^2 \sin \theta_{i2} = -\frac{g}{l_2} (\cos(\theta_{i1} + \theta_{i2})) \quad (16)$$

Applying an angle sum identity [24] to Eqn. (16), and substituting in Eqn. (15) results in the following:

$$\ddot{\theta}_{i2} + 2\sqrt{\frac{l_1}{l_2}} \Omega \zeta \dot{\theta}_{i2} + \omega_n^2 \sin \theta_{i2} = -\frac{g}{l_2} (\sin(\Omega t) \cos \theta_{i2} + \cos(\Omega t) \sin \theta_{i2}) \quad (17)$$

Since damping will eventually cause θ_{i2} to undergo decayed oscillation towards zero, the small angle approximation can be applied to Eqn. (17) yielding:

$$\ddot{\theta}_{i2} + 2\sqrt{\frac{l_1}{l_2}} \Omega \zeta \dot{\theta}_{i2} + \left(\omega_n^2 + \frac{g}{l_2} \cos(\Omega t) \right) \theta_{i2} \cong -\frac{g}{l_2} \sin(\Omega t) \quad (18)$$

This differential equation cannot be evaluated analytically due to the time varying coefficient of θ_{i2} . However, a constant coefficient can be approximated using the following expression which is inserted into Eqn. (18):

$$\bar{\omega}_n = \frac{1}{\sqrt{2\pi}} \left(\int_0^{2\pi} \left[\omega_n^2 + \frac{g}{l_2} \cos(\Omega \tau) \right]^2 d\tau \right)^{1/4} \quad (19)$$

$$\ddot{\theta}_{i2} + 2\sqrt{\frac{l_1}{l_2}}\Omega\zeta\dot{\theta}_{i2} + \bar{\omega}_n^2\theta_{i2} \cong -\frac{g}{l_2}\sin(\Omega t) \quad (20)$$

This approximated coefficient $\bar{\omega}_n$ helps account for the effective increase in natural frequency associated with gravity. While damping causes the homogeneous solution to Eqn. (20) to decay to zero, the amplitude of the particular solution $\theta_{i2,p}$ does not:

$$|\theta_{i2,p}(t)| = \frac{g/l_2}{\left[\left(\bar{\omega}_n^2 - \Omega^2 \right)^2 + 4\frac{l_1}{l_2}\zeta^2\Omega^4 \right]^{1/2}} \quad (21)$$

The amplitude of this particular solution accurately quantifies gravity's effect on the imbalance mass, which is typically quite small since the hub rotational frequency Ω is much greater than the imbalance's natural frequency ω_n .

The centrifugal force CF_i exerted on the rotor hub by the imbalance mass m_i can be expressed:

$$CF_i = m_i l_{i2} \left(\Omega + \dot{\theta}_{i2} \right)^2 \left(\cos(\theta_{i1} + \theta_{i2}) \hat{X} + \sin(\theta_{i1} + \theta_{i2}) \hat{Y} \right) \quad (22)$$

Where $\hat{X}$ and $\hat{Y}$ are unit vectors associated with the x and y axes in Figure 5. The net centrifugal force CF exerted by two imbalance masses in the OHAVCS configuration is:

$$\begin{aligned} CF = & m_1 l_{12} \left(\Omega + \dot{\theta}_{12} \right)^2 \left(\cos(\theta_{11} + \theta_{12}) \hat{X} + \sin(\theta_{11} + \theta_{12}) \hat{Y} \right) \\ & + m_2 l_{22} \left(\Omega + \dot{\theta}_{22} \right)^2 \left(\cos(\theta_{11} + \theta_{22} + \pi) \hat{X} + \sin(\theta_{11} + \theta_{22} + \pi) \hat{Y} \right) \end{aligned} \quad (23)$$

Where $\theta_{21} = \theta_{11} + \pi$ accounts for the equal and opposite offsets.

2.2 Simulations

Simulations were conducted to explore the dynamic concepts introduced previously. Initial condition responses of imbalance mass position θ_{i2} were simulated using Eqn. (12) and the ODE45 solver in MATLAB 2012a. The first case assumed that the imbalance masses rotated in a horizontal plane, such that no component of gravity was in the plane of rotation ($g = 0$), and that no change in hub rotational velocity occurred ($\dot{\Omega} = 0$). The second case assumed that the imbalance masses rotated in a vertical plane ($g = 9.81 \text{ m/s}^2$), and that no change in hub rotational

velocity occurred ($\dot{\Omega} = 0$). For both Cases 1 and 2, a single imbalance mass was simulated with four specific initial conditions for θ_{i2} : 45, 178, -178, and -45 degrees. Large transients in hub rotational velocity are known to occur during helicopter autorotation [25-27]. Consequently, the third case considered hub rotational acceleration only ($g = 0$, $\dot{\Omega} \neq 0$) and included a simulation of two imbalance masses. The force output of the two imbalances masses during hub rotational acceleration was calculated using Eqn. (23) and the initial conditions $\theta_{i2} = 90$ degrees and $\theta_{22} = -90$ degrees which correspond to the maximum OHAVCS centrifugal force. A summary of parameters associated with these simulations is compiled in Table 1.

Table 1: Summary of parameters associated with simulation Cases 1-3

Parameters	Case 1	Case 2	Case 3
Initial Hub Rotational Frequency $\Omega(t = 0)$	5.5 Hz	5.5 Hz	5.5 Hz
Hub Rotational Acceleration $\dot{\Omega}$	0 cycles/s ²	0 cycles/s ²	1.1 cycles/s ²
Offset Distance l_1	0.5 in	0.5 in	0.5 in
Linkage l_2	12 in	12 in	12 in
Gravity g	0 m/s ²	9.81 m/s ²	0 m/s ²
Damping Coefficient ζ	0.075	0.075	0.100
Initial Conditions for θ_{i2}	45, 178, -178, -45 deg	45, 178, -178, -45 deg	90, -90 deg
Number of Masses	1	1	2

2.3 Experimental Testing

To experimentally validate the analytical concepts and simulation results, tests were conducted on a “modified CHAVCS” fixture (Figure 6). This fixture consists of a concentric hub AVC system (CHAVCS), offset by means of an adapter plate, with a spindle motor drive that rotates the CHAVCS at a spindle velocity Ω equivalent to the hub rotational frequency. A spindle bearing, which supports the CHAVCS, joins the fixed frame and the rotating frame. At the hub mounting interface, the adapter plate (Figure 7a) is inserted to offset the CHAVCS precisely 0.25 inches; the resulting “modified CHAVCS” system is shown in Figure 7b.

Figure 6. Photograph of modified CHAVCS test fixture

Figure 7. Design and implementation of modified CHAVCS adapter plate: a) drawing showing the offset hole pattern; b) implementation on test fixture with exaggerated offset

Inside the modified CHAVCS shown in Figure 8a, the imbalance mass m_i rotates counterclockwise at speed $\dot{\theta}_{i2}$ and is located at angular position θ_{i2} . The imbalance mass positions (θ_{12} and θ_{22}) in Figure 8 are measured using integrated Hall effect sensors which are accurate to within ± 6 degrees. Note that the modified CHAVCS features only one offset, as shown in Figure 7b, instead of the two equal and opposite offsets of the proposed OHAVCS. As shown in the schematic comparison of Figure 8, offset distance l_1 is equivalent to the adapter plate offset in the experimental fixture. Distance l_2 is equivalent in both diagrams and represents the distance between the

imbalance mass m_i and the center of its respective ball bearing. Both systems feature dynamics governed by Eqn. (12), and can be used to test its validity.

Figure 8. Schematic illustrating differences between the modified CHAVCS test fixture and the proposed OHAVCS: a) modified CHAVCS with single offset used in lab testing; b) OHAVCS design with equal and opposite offsets

Experimental data was collected to assess the performance and practicality of an OHAVCS during an electrical power loss. The modified CHAVCS was oriented horizontally ($g = 0$) and there was no hub acceleration during testing ($\dot{\Omega} = 0$). Angular position of the imbalance mass θ_{i2} and electrical power consumption of the modified CHAVCS motors were acquired at 500 Hz using CANbus and a custom LabVIEW virtual instrument (National Instruments, Austin, TX). First, the CHAVCS (with no adapter offset) was powered to nominal operating conditions: a spindle rotational frequency of $\Omega = 4.3$ Hz and an imbalance mass angular velocity of $\dot{\theta}_{i2} = 12.9$ Hz. Imbalance mass position θ_{i2} was monitored during system startup. These nominal operating conditions were maintained for five minutes to eliminate all mechanical, thermal and electrical system transients. An identical test

protocol was utilized for the modified CHAVCS. To simulate the loss of electrical power on the rotor hub, all phases of the brushless DC motor were disconnected from the modified CHAVCS while at nominal operating conditions. Disconnecting the motor phases allowed no current to flow through the motors, and ensured no energy was dissipated electrically. The imbalance mass position was monitored until its velocity $\dot{\theta}_{i2}$ reached zero, and its final position was documented. This process was conducted multiple times for statistical consistency.

3. RESULTS

3.1 Simulated Responses

A typical Case 1 simulated time response of θ_{i2} is presented in Figure 9. This response decays to the same equilibrium angle ($\theta_{i2} = 0$) regardless of its initial condition. While different decay rates were obtained by varying system parameters l_1 , l_2 , Ω and ζ , θ_{i2} decayed to the same offset angle ($\theta_{i2} = 0$) regardless of system parameters.

Figure 9. Simulated Initial Condition Response for $\theta_{i2}(t)$: Case 1 ($g = 0$)

The time response of θ_{i2} with gravity included (Case 2) is presented in Figure 10. The imbalance mass angle θ_{i2} still settles to a predetermined angle, but there is a slight perturbation resulting from gravity. A comparison of θ_{i2} 's frequency spectrum with and without gravity is shown in Figure 11, which was calculated using Fast Fourier Transforms (FFTs) for an initial condition of 45 degrees. The large resonant peak observed at 1.1 Hz is the imbalance mass natural frequency ω_n , which precisely matches the result obtained analytically using Eqn. (15). These simulation results indicate that the effects of gravity occur at the hub rotational frequency Ω , 5.5 Hz; its amplitude can be calculated using Eqn. (21). The frequency spectrum including gravity shows the existence of beats (at 4.4 Hz and 6.6 Hz) resulting from modulation of the imbalance mass natural frequency ω_n and the hub rotational frequency Ω . In Figure 11, the large resonant response at the imbalance mass natural frequency ω_n and

the relatively small amplitude of gravity's effects indicates that it does not dominate the hub AVC system response (provided that the ratio of g/l_2 is kept at an appropriate level), furthermore signifying that gravity will not degrade OHAVCS performance.

Figure 10. Simulated Initial Condition Response for $\theta_{i2}(t)$: Case 2 ($g = 9.81 \text{ m/s}^2$)

Figure 11. Frequency spectrums with and without the effects of gravity (Cases 1 and 2). Circles denote harmonics associated with gravity

Simulated time responses of imbalance mass position during hub rotational acceleration (Case 3) are presented in Figure 12. Each mass's time response is plotted with respect to θ_{11} such that θ_{12} and $\theta_{22} + 180$ degrees are the two plotted variables. Initially, the imbalance mass positions decay toward their respective offset directions. Once the increase in hub rotational velocity occurs, the imbalance masses rotate away from their equilibrium angle temporarily before returning. This motion was not significant compared to the decay from the initial condition response.

Figure 12. OHAVCS during hub rotational velocity change (Case 3): a) Imbalance mass positions; b) Hub rotational velocity

The total centrifugal force CF exerted by the imbalance masses as quantified by Eqn. (23) is plotted in Figure 13. Initially, the centrifugal force decreases to approximately zero from its initial value as the angles of both masses begin decaying toward the direction of their respective offsets, 180 degrees apart. The magnitude of the force CF depends on the angle between the imbalance masses m_1 and m_2 , and each imbalance mass's velocity $\dot{\theta}_{i2}$. Eventually, the angle between the masses reaches a steady state condition of 180 degrees and the force CF is zero. During the transient in hub rotational velocity, the force output into the rotor hub does not change and the angle between both imbalance masses remains at a constant 180 degrees indicating that the static balancing of the OHAVCS is not affected by hub rotational acceleration and furthermore an autorotation.

Figure 13. OHAVCS during hub rotational velocity change (Case 3): a) Normalized centrifugal force output of imbalance masses; b) Angle between imbalance masses

3.2 Experimental Results

A comparison of experimentally-measured power consumption for the CHAVCS and modified CHAVCS is presented in Figure 14, while comparisons of voltage and power responses during startup of the CHAVCS and modified CHAVCS are plotted in Figure 15.

Figure 14. Power usage of hub AVC system with and without offset

Figure 15. Power during start up: a) Filtered bus voltage; b) Overall power usage

A significant steady state power increase (from 323 W without the offset to 637 W with the offset) is evident in the responses of Figure 14. This power increase originates from two sources: the centrifugal displacement of bearing oil and the oscillatory torque exerted on the imbalance mass. The offset in the modified CHAVCS causes the oil to be centrifugally forced to the outside of the bearing, causing its components to rotate through a more viscous medium and requiring more motor power for a given rotational speed. Similarly, the oscillatory torque associated with the offset imbalance masses requires more motor power to track a given reference command.

While both systems exhibit similar drops in voltage due to startup power draw (Figure 15), the modified system's power exhibits oscillatory behavior for several seconds afterward. This torque's effect on motor power is more apparent at low speeds, which occur during startup. During startup, an increased power consumption of approximately 17.0 W at 2.5 Hz (the imbalance mass velocity $\dot{\theta}_{i_2}$ is 2.5 Hz) is observed. If the associated motor torque, 0.54 N-m on each of the two imbalance masses, is extrapolated to nominal steady state operating conditions ($\dot{\theta}_{i_2} = 12.9$ Hz), this increased power represents approximately 88 W, or 27.9 percent of the increase

in steady state power. The remainder of this power increase, 226 W, arises from centrifugal oil displacement in the ball bearing. In future OHAVCS designs, the amount of bearing oil can be reduced (to decrease power consumption) without adversely affecting bearing lubrication.

The measured imbalance mass positions θ_{12} and θ_{22} are shown in Figure 16 and 17 for two separate tests where the motor phases were disconnected from the modified CHAVCS. The time at which the motor power was disconnected is defined as $t=0$. The experimental data from both tests shows that the imbalance mass position decays to the same predetermined angle within 5 seconds. The angle between the two imbalance masses in the modified CHAVCS decays to 0.05 rad (2.9 deg) in Figure 16 and to 0.10 rad (5.7 deg) in Figure 17, within the Hall effect sensor's error margin. The final observed angles in the first and second tests were 197.0 rad and 190.7 rad. The 2π modulus of these angles is approximately 2.2 rad, which indicates that the same angle of rest was observed in both tests.

Figure 16. First simulated rotor hub power loss results: a) Imbalance mass position $\theta_{12}(t)$ and $\theta_{22}(t)$; b) Angle between imbalance masses $|\theta_{12}(t) - \theta_{22}(t)|$

Figure 17. Second simulated rotor hub power loss results: a) Imbalance mass position $\theta_{12}(t)$ and $\theta_{22}(t)$; b) Angle between imbalance masses $|\theta_{12}(t) - \theta_{22}(t)|$

With the tendency of the OHAVCS imbalance masses to rotate to a fixed angle during a rotor hub power loss, there is also a tendency to resist rotations during startup. Figure 18a compares the motion of the imbalance masses and the reference command during a modified CHAVCS startup test and shows overlapping measured and reference angles. The time at which the actuator was commanded to startup is defined as $t=0$. The initial angle of the imbalance masses before startup is 2.2 rad, identical to the final angle measured after the simulated rotor hub power loss test. The linear increase in angle indicates that the motor control loops are successfully controlling the mass angles despite the offset. Figure 18b plots the tracking error during this event, which shows a small oscillatory error from the centripetal acceleration on the mass, confirming the control loops' effectiveness. The tradeoff for effective control loops is that there is an oscillatory power draw which was measured during start up, however, the ability to control the position of the imbalances masses is fundamental to nominal operation of the OHAVCS and supersedes any power implications.

Figure 18. Modified CHAVCS startup: a) Imbalance mass angles $\theta_{12}(t)$ and $\theta_{22}(t)$ and their respective reference commands; b) Imbalance mass angle tracking error

3.1 Model Parameter Estimation

A process of parameter estimation was used to calculate the linear damping coefficient ζ in Eqn. (12) to match experimental data taken during two tests when the modified CHAVCS power was disconnected from the motors. Consequently, the damping coefficient ζ provides an estimate for future simulations of an OHAVCS during a power loss. The model initial conditions (θ_{12} , θ_{22} , $\dot{\theta}_{12}$, and $\dot{\theta}_{22}$) were determined from the experimental data when power was disconnected, and the system parameters l_1 , l_2 , and Ω were known constants from solid models or operating conditions. The simulation modeled the motion of the imbalances masses for 20 seconds after the power was disconnected.

3.1.1 Damping Coefficient Optimization

The differential Eqn. (12) was solved keeping all parameters constant except the damping coefficient ζ . For each value of ζ , the discrete L2 Norm [28] of the error between the experimental and simulated values was calculated for the last 10 seconds of simulation. This error was plotted against the damping coefficient ζ to observe the

minima of the curve in Figure 19 (for the first test) and in Figure 20 (for the second test). The real system damping term is non-linear, but a linear approximation was valid to approximate the number of revolutions and the elapsed time before the imbalance masses reached a predetermined angle. For each imbalance mass and each test a specific damping coefficient of ζ was optimized to minimize the error: the values ranged between $\zeta = 0.079 - 0.088$. A damping coefficient ζ in this range can be used for future simulations of an OHAVCS.

Figure 19. L2 Norm of simulation error as a function of the damping coefficient ζ during the first rotor hub power loss: a) error for m_1 ; b) error for m_2

Figure 20. L2 Norm of simulation error as a function of the damping coefficient ζ during the second rotor hub power loss: a) error for m_1 ; b) error for m_2

3.1.2 Model Results with Optimal Damping Coefficients

The initial condition response of Eqn. (12) with optimal damping coefficients for each imbalance mass and each test was simulated. A comparison of simulated and experimental data is plotted in Figure 21 and 22 for a time interval of 20 seconds after power is disconnected from the motors. The time at which the motor power was disconnected is defined as $t=0$, and the time at which the simulation is within π of the mass's final angle is marked by a dashed vertical line. All the imbalances settle out to a consistent predetermined angle after approximately 30 revolutions: via simulation in 8 seconds and experimental result in under 5 seconds. The empirical linear damping model does show error since in reality the system is non-linear, but this approximation is sufficient for an accurate system model which predicts the number of revolutions and the time before the imbalance masses achieve a statically balanced condition. Additional practical design features such as electronic motor braking can be applied to the OHAVCS causing the imbalances to be slowed to near zero velocity in seconds, further mitigating the 1/Rev vibration.

Figure 21. Comparison of simulated and experimental data for first rotor hub power loss:
a) Imbalance mass position θ_{12} (t); b) Imbalance mass position θ_{22} (t)

Figure 22: Comparison of simulated and experimental data for second rotor hub power loss:
a) Imbalance mass position θ_{12} (t); b) Imbalance mass position θ_{22} (t)

Comparing closer to the final angle of the imbalances (between 180 and 200 rad) results in Figure 23 and 24. The difference between experimental data and simulation is noticeable at this level of detail. This error results from a higher level of damping near the equilibrium angle of the mass (where the oil is accumulating). The imbalance masses are rotating through a more viscous medium near this angle subsequently causing a higher damping

coefficient. This could be modeled by a damping coefficient ζ which is a function of imbalance mass angle θ_{i2} . Despite the error at this resolution, the simulation predicts the number of revolutions before the mass comes to rest, and the approximate time until the mass comes to rest.

Figure 23: Comparison between 180 and 200 rad of simulated and experimental data for first rotor hub power loss: a) Imbalance mass position θ_{12} (t); b) Imbalance mass position θ_{22} (t)

Figure 24: Comparison between 180 and 200 rad of simulated and experimental data for second rotor hub power loss: a) Imbalance mass position θ_{12} (t); b) Imbalance mass position θ_{22} (t)

4. DISCUSSION

Simulation and experimental results indicate that offsetting the imbalance masses in a CHAVCS is a viable approach for mitigating 1/Rev helicopter vibrations during a rotor hub power loss while continuing to cancel N/Rev vibrations during normal operation. Analysis of the system dynamics confirms that this behavior is independent of gravity's effects and hub rotational acceleration. The imbalance mass's decay to a predetermined angle is confirmed first by analysis of Eqn. (12), and then via simulation and experimental testing using a modified CHAVCS. The offset creates a stabilizing centripetal torque that rotates each imbalance mass to a unique equilibrium angle of 2.2 rad (126.1 deg). This equilibrium angle was confirmed using two different motor and imbalance sets during multiple tests. The estimated linear damping coefficient of the imbalance mass system, ζ , varied between 0.079 – 0.088. Experimental data also indicates that the offset can be designed without hindering the control of the imbalance masses to cancel N/Rev vibration. However, with an introduced offset of 0.25 inches, the steady state system power required nearly doubled.

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