ABSTRACT

SEBASTIAN, THOMAS. Methods for the Determination of Aerodynamic Parameters and Trajectory Reconstruction of the Orion Command Module from Scale Model Aeroballistic Flight Data. (Under the direction of Professor Robert Tolson).

Determination of aerodynamic coefficients and stability derivatives is necessary in defining a model of the Orion CEV dynamics. This involves reducing experimental data, which can include acceleration, angular rate, or orientation data. This sort of extraction of dynamics from experimental data is often performed on data gathered from experiments conducted on uninstrumented models at indoor ballistics ranges. The US Army Research Laboratory (ARL) has developed a high-g survivable stand-alone instrumentation package that can transmit in-flight measurements of acceleration, angular rate, and local magnetic field. This telemetry module (TM) was installed in a scale model of the Orion CEV, which was fired from a 175mm cannon at the ARL range. The instrumentation package was upgraded to include pressure transducers to measure forebody pressures. A minimum variance with a priori method was formulated to solve for both the “local” flight parameters of Mach number, angle of attack, and sideslip angle at each timestamp and the “global” parameters of scale factor and bias for each pressure transducer. Results using both simulated and experimental data indicates that these parameters may be estimated and used to compute stability coefficients. Low pressure differentials between symmetrically-opposed pressure transducers, however, increased uncertainty in the parameter estimates. Validation of this method of data generation and analysis supports a low-cost method of vehicle testing.

Methods for the Determination of Aerodynamic Parameters and Trajectory Reconstruction of
the Orion Command Module from Scale Model
Aeroballistic Flight Data

by
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Chapter 1

Introduction

1.1 The Vision for Space Exploration

In the aftermath of STS-107, the National Aeronautics and Space Administration (NASA) was tasked to pursue a new human exploration agenda. This new framework, the Vision for Space Exploration (VSE), was released to the public in February 2004. The fundamental goal of the VSE is to advance U.S. scientific, security, and economic interests through a robust and sustainable space exploration program. The following objectives were proposed to meet this goal [1]:

- Implement a sustained and affordable human and robotic program to explore the solar system and beyond
- Extend human presence across the solar system, starting with a human return to the Moon by the year 2020, in preparation for human exploration of Mars and other destinations
- Develop the innovative technologies, knowledge, and infrastructures both to explore

and to support decisions about the destinations for human exploration

- Promote international and commercial participation in exploration to further U.S. scientific, security, and economic interests

The Space Shuttle has been the workhorse of the U.S. manned space exploration program. The fleet has conducted operations since 1981, transporting crews to and from Earth orbit to conduct experiments, launch and repair satellites, and construct the International Space Station. While the shuttle fleet has done much to advance manned space exploration, the age of the fleet has called the safety of the vehicles into question.

1.2 Crew Exploration Vehicle

NASA has decided to retire the Space Shuttle fleet in 2010, after the assembly of the International Space Station is complete. To meet the objectives set forth in the VSE, a new vehicle, the Crew Exploration Vehicle (CEV), is being developed by NASA with support from industry partners.

The CEV, as defined by the VSE, must provide crew transportation for missions beyond low Earth orbit and begin human exploration missions by 2014. The CEV, unlike the Space Shuttle, will only transport humans, requiring a separate launch system for cargo and equipment.

Figure 1.1: CEV stack components

A capsule geometry was selected for the CEV. This design possesses a number of inherent advantages [2]:

- Safest, most reliable and affordable approach to meeting crew transportation requirements for exploration missions as compared to other methods
- Heat shield is protected (covered by the service module) until it is needed for reentry
- Is more aerodynamically stable for entry during both nominal auto guided entries and emergency abort entries
- Easier to integrate with a launch escape tower

The capsule shape has drawn comparisons to the Apollo missions of the 1960s and 1970s.

Figure 1.2: Size comparison between Orion CM and Apollo CM

It is 2.5 times the volume of Apollo and will carry four astronauts to the surface of the Moon and six to Mars, a feat Apollo and no other previously designed spacecraft could accomplish [3]. The CEV design will capitalize on advances in computers, avionics, and materials development.

1.3 Research Goals

While both the Apollo and CEV CMs are 70-degree cones with spherical heat-shields, differences in both size and weight distribution mean that it is not appropriate to linearly scale Apollo CM aerodynamic and stability parameters and apply them to the Orion CM. To this end, the Orion CM design has undergone a series of experimental tests to understand the dynamics during an abort-initiated descent and determine the key aerodynamic coefficients and parameters. Recovering the aerodynamic coefficients of the Orion CM

makes accurate modeling of the dynamics possible and allows for the refinement of guidance and control systems. Parameters of specific interest include the pitching moment coefficient, the pitch damping coefficient, trim angle of attack, coefficients of axial and normal force at zero angle of attack, and the derivatives of the pitching moment and normal force with respect to angle of attack. Development of a deterministic solution for these parameters, derived using data from experimental testing, will allow refinements of computational simulations that may be used to model vehicle stability and dynamics.

Chapter 2

Experimental Methods for Obtaining Vehicle Dynamics

To observe the dynamics of the Orion CM, or any vehicle for that matter, flight tests are required. While wind tunnels are capable of testing a design over a range of Mach numbers and vehicle orientations, most require that the vehicle remain fixed to a sting, a mount that is fixed to the rear of the vehicle and attaches to the wind tunnel. While the data recovered may be used to develop a static aerodynamic database, dynamics cannot be observed in this fashion. Apparatus do exist to oscillate a model in angle and attach and pitch, yielding dynamics, but tests are limited by wall effects within the tunnel [4].

In contrast, vertical wind tunnels allow the observation of vehicle dynamics. Generating a column of vertical air, vertical wind tunnels suspend the test article in the flow-field and use cameras to obtain orientation data. These tests are limited in the range of testable Mach number and the test article must be manually injected into the flow by a skilled technician to achieve a desired initial orientation and angular rates [5].

Scaled flight tests using rockets have been used in the past to collect valuable stability

data. Instrumented test articles are launched aboard a rocket to a desired altitude, corresponding with a desired freestream pressure and density. The dynamically-scaled test article, accelerated to an initial velocity by the rocket, is released and instruments record the vehicle dynamics via accelerometers, angular rate gyros, magnetometers, and pressure transducers. The test article may be recovered by parachute, and the recorded data is extracted and analyzed [6]. These tests are expensive; multiple tests using various configurations may be cost-prohibitive. The instrumentation must be sized such that it may fit within the test article, limiting the types of sensors that may be employed.

2.1 Aeroballistic Range Testing

Models may be gun-launched to achieve the high test velocities required to characterize the projectile dynamics; an aeroballistic test. Modern aeroballistic tests on airborne vehicles and projectiles have been conducted since the 1940s. While perceived as being less accurate than other methods for obtaining aerodynamic data, aeroballistic tests have some significant advantages [7]:

- Relatively inexpensive method for conducting vehicle dynamics studies
- Requires no supporting sting and has minimal wall effects
- Minimal free stream disturbances
- Static and dynamic aerodynamic information is obtained by one test

Aeroballistic tests may be conducted in an enclosed aeroballistic range or an outdoor proving ground. Launch methods and data collection methods vary for tests conducted at either facility type, but share many similarities.

All aeroballistic ranges consist of the same basic elements: launcher, dump tank, sabot stripper, test section, and impact area.

Figure 2.1: Diagram of aeroballistic range

The launcher is typically a gun that has no internal projectile-stabilizing grooves (rifling), imparting little or no spin on the sabot/article package. The launcher relies on the combustion of powder or light gas to generate the acceleration needed to propel the test article to a desired exit velocity. Powder gas launchers are typically larger in diameter than light gas launchers allowing larger test articles. Light gas launchers can accelerate a test article to higher exit velocities. The dump tank, also known as a blast chamber, is the section of the range that the launcher fires into and it is designed to capture and contain gun gases and sabot fragments.

Any sabot fragments in the wake of the test article are prevented from entering the test section by the sabot stripper. The test section is the area of the range where measurements of the test article are collected. These measurements are typically visual data collected in the form of Schlieren images, or shadow-graphs, taken by photographic stations fixed along the test section at specific locations. This data can be used to determine the orientation of the vehicle, which in turn can be used to determine the angular rates due to aerodynamic forces and vehicle dynamics. Because the photographic stations are located at fixed positions, the times at which the test article enters the

imaging field of each station can be used to determine velocity. The deviation of the test article from the center of the image field can be used to determine normal forces, like lift. Schlieren images allow the experimental study of the dynamic test article flow field, which gives insight to shock and wake shapes, flow separation and reattachment, and flow transition. This information can be used to validate computational fluid dynamics (CFD) codes as flow field features are captured visually. The impact area is located at the end of the test section and is used to rapidly decelerate and recover the test article. The impact generally renders the test article unusable for future aeroballistic tests. This section may be used to perform impact studies, such as those conducted in the areas of meteor impact, debris impact, projectile penetration, and crater formation.

Test articles are machined to withstand shock loads that start at 10,000g and can be as high as 250,000g [7].

Figure 2.2: Examples of Aeroballistic Test Articles

High-mass test articles minimize deceleration, and in turn Mach number variation, but lighter models may be required for high Mach number tests.

2.2 Proving Grounds

Tests conducted at proving grounds occur in restricted, outdoor spaces, firing sabot-encased test articles out of a cannon at an initial orientation and Mach number.

Figure 2.3: ARL proving grounds

Proving grounds have a number of benefits. The cannon used to launch the sabot/article package can be much larger than is available on closed ranges, which translates into larger test articles and higher exit velocities. The volume outside of the launcher serves as the dump tank, sabot stripper, test section, and impact area. The absence of limiting walls and structures means that the test section can be as long or wide as the test warrants. This is extremely useful when studying test articles that generate a significant amount of lift with lateral force; such designs cannot be tested within a closed range without damaging the test section.

Because it is impractical to instrument the entire test volume of an outdoor proving ground with photographic stations, data collection requires new solutions. Various systems exist to collect data during a test, but each has a corresponding set of limitations. Pulse-Doppler radar systems are used to determine the bearing, range, altitude,

and radial velocity of the test article [8]. These systems generate pulses of electromagnetic energy that are transmitted towards a moving target. The reflected signals are processed to measure the frequency shift between carrier cycles in each pulse and the original transmitted frequency, which leads to the determination of position and velocity. The resolution of these radar systems is usually not high enough to determine the body accelerations of the test article to a high degree of accuracy (observed differences on the order of $2 - 5^m/s^2$), and interference from sabot fragments and launcher ejecta introduces errors. In the past, on-board recorders collected data from body-mounted sensors, like accelerometers and angular rate gyros. This data would then be recovered post-impact and analyzed. Recovery of the test article may be difficult because some proving grounds fire into off-limit areas (due to unexploded ordinance).

2.3 On-board Instrumentation

The U.S. Army began investigating the use of low-cost micro-electromechanical sensors (MEMS) in high-g applications, like rockets and gun-launched projectiles. Focus was placed by the Army Research Laboratory (ARL) in assessing the viability of commercial-off-the-shelf (COTS) MEMS for use in high-g diagnostic and open range tests. Sensors that demonstrated survivability were then packaged into the Aeroballistic Diagnostic System (ADS). As the initial test articles used with this device were munitions, the sensor packaging was designed to have a fuse shape, hence the name diagnostic fuse, or DFuze [9].

Figure 2.4: DFuze housing and circuit boards

Within the packaging are sensor arrays that collect inertial measurement data, solar orientation (sun sensor), and magnetic field data. Other components included are signal conditioners, a battery, and a transmitter. Advances in COTS MEMS have led to more advanced versions of the DFuze, but these versions all had the same test limitations. The DFuze was designed for test articles with high axial spin rates, like munitions. It is inappropriate to use the same suite of sensors and packaging for a design expected to have a negligible spin rate, lifting trajectory, and blunt fore-body, like the Orion CM. Optical solar orientation sensors rely on bright conditions and high spin rates to resolve test article orientation unambiguously. Internal mounting requires a redesign of the housing and attention given to how to install it within the test article.

2.3.1 CEV Telemetry Module

NASA approached the ARL to determine if something akin to the DFuze sensor package could be designed to study the dynamics of the Orion CM. Performing proving ground tests on scale models of the CM in this fashion would reduce costs while generating useful data that may be used to determine vehicle flight parameters and coefficients. Because of the significant differences between the Orion CM and the munitions normally tested at the ARL the standard test procedure had to be reformulated. This included a

redesign of the test article electronics package. The redesigned electronics package was dubbed the CEV Telemetry Module (CEV-TM) [10].

Figure 2.5: Cutaway image of CEV-TM

The CEV-TM, like the DFuze, is a stand-alone instrumentation package that contains an Inertial Sensor Suite (ISS), consisting of accelerometers, angular rate gyros, and magnetometers, an encoder board, a transmitter with integrated patch antenna, and a battery-based power supply. The 16 channel, 12 bit, PCM (pulse coded modulated) NRZ-L encoder board has features that include a sub frame ID (SFID), 120ms of total PCM stream delay, and a single magnetometer auto set/reset approximately 1 second after the initial power-up. Although the SFID feature is not used for communication of analog data, it is required for correlating minor frames acquired from multiple telemetry ground stations. The PCM delay allows the transmission of in-bore sensor data after the package has cleared the muzzle. This prevents the loss of data during in-bore travel or plasma generated by the gun. The bit rate is approximately 3.3 mbits/s, with a 24-word

minor frame structure. Of the 16 channels, 15 are normally commutated and one channel is 6-times super commutated yielding approximate sample rates of 11.5 kHz and 68.8 kHz, respectively [10].

COTS MEMS, similar to those implemented on the DFuze, were used in designing the CEV-TM. Analog Devices accelerometers and angular rate gyros and Honeywell magnetometers were employed [11, 12, 13]. Simulations were run to predict the expected test article trajectory; this information was used to determine the required full scale of the sensors used on the CEV-TM. Sensor orientations supported a 3-axis right-hand-rule configuration. The flexibility of the CEV-TM system allows the modification of sensor ranges and type with relatively little effort. The outputs from these sensors are fed into a signal conditioning and power regulation circuit where signals are assigned appropriate voltage levels and otherwise modified for acceptance by subsequent circuitry. An on-board recorder stores data while the test article is within the gun and is unable to transmit reliably. This recorded data, along with the time-delayed in-flight data is encoded by the pulse-code modulation (PCM) circuitry and transmitted in the same way the ADM data is transmitted. An M/A-COM phase-locked transmitter, operating in the S-Band (2255.5MHz) at 250mW performs this function and sends the data to the receiving station [10].

2.3.2 CEV-TM Calibration

The CEV-TM underwent a series of calibration tests before it was installed within the Orion CM test article to gauge sensor misalignments and output fidelity. On-board battery capacity is sufficient to support calibration procedures and aeroballistic testing operations. The on-board 3-axis magnetometer was subjected to a magnetic calibration

protocol via Helmholtz coils. Helmholtz coils are identical circular coils of wire placed along a common axis with a current running through them, generating a nearly uniform magnetic field between the two coils. A program was used to generate a predetermined array of magnetic fields of varying magnitude and orientation. Knowing the orientation of the instrumented test article in relation to the Helmholtz coils allowed sensor misalignments to be inferred from the calibration data. Performing tests using the assembled test article allowed the observation of magnetic interference generated by CEV-TM circuitry or any ferromagnetic components used to assemble the test article. Angular rate sensors were calibrated by placing the CEV-TM in a custom fixture centered on a centrifuge. This fixture positioned the module in various orientations, allowing outputs generated about each axis to be measured. A generic test profile was used to determine the input angular rates and the centrifuge was manually controlled to approximate these values. This calibration was also used to evaluate the effectiveness of the AO ring as a spin measurement device for certain magnitudes of roll, pitch, and yaw rates. Accelerometers were calibrated using a two-axis rate table; the CEV-TM was fixed to the table using the same fixture used in the angular rate gyro calibration. The tabletop rotated at a low angular velocity (approximately 20 to 30 rpm) while the table frame moved through a series of orientations, generating responses for the accelerometers due to gravitational acceleration.

Chapter 3

Post-Flight Analysis of CEV-TM Data

In addition to the data obtained by the CEV-TM sensor suite, radar data was collected using a Weibel Scientific azimuth and elevation pulse-Doppler tracking radar system operating in the X-band [10]. This radar data provided the position and velocity of the test article in the inertial reference frame using a gun downrange-altitude-drift coordinate system. The Mach number of the test article was obtained using radar and meteorological data obtained the day of the test, which included atmospheric pressure, temperature, and density at the test site.

Figure 3.1: Captured frames from high-speed tracking camera

Before a deterministic analysis could be applied, the CEV-TM data had to be pre-processed. This involved interpolating the data to the same timestamp and applying a running mean to remove noise from the sensor signals. Higher than expected angular rates resulted in the clipping of angular rate gyro data. The data in these saturated regions had to be extrapolated to yield a continuous solution. This was accomplished by identifying a saturated section of data, then fitting a parabolic curve over this region that had slopes at the endpoints equivalent to the slopes of the data at the endpoints. Because the frequency of the data was considerably higher than that of any of the physically-attributable oscillations, the sensor data was decimated to minimize processing time.

Figure 3.2: Filtered accelerometer output from CEV-TM-4

Figure 3.3: Filtered magnetometer output from CEV-TM-4

Figure 3.4: Filtered angular rate gyroscope output from CEV-TM-4

3.1 Transformations

Transforming the inertial velocities obtained by the radar to the body-fixed reference frame yields the velocity components that may be used to determine angle of attack, sideslip angle, and total angle of attack [14].

$$\begin{bmatrix} V_x & V_y & V_z \end{bmatrix} \rightarrow \begin{bmatrix} u & v & w \end{bmatrix}$$

$$\alpha = \arctan\left(\frac{w}{u}\right)$$

$$\beta = \arcsin\left(\frac{v}{\sqrt{u^2 + v^2 + w^2}}\right)$$

$$\alpha_T = \arccos(\cos \alpha \cos \beta)$$

A series of transformations must first be carried out. Many reference frames exist for describing projectile motion but they all follow the same basic definition, requiring an origin, a fundamental plane (which usually defines the z-axis) and a fundamental direction. Of the four used in the determination of the vehicle orientation three are inertial and the fourth is in the body reference frame. The range coordinate system is the system in which the radar data is presented; the origin is located at the muzzle, the fundamental plane is parallel to the local gravitational field vector, and the fundamental direction is perpendicular to this plane in the same direction as the gun. The x-axis (downrange) is defined by the projection of the gun barrel onto the surface of the Earth. The y-axis (altitude) is in the fundamental plane and points up, measuring altitude. The z-axis (drift) is the cross-product of the x and y axes and completes the coordinate system. The origin of this system is the gun barrel exit. The next inertial reference frame is the North-East Down (NED) coordinate system, and is oriented such that the x-axis is pointed North and the y-axis is pointed East. The cross-product of these two axes yields the z-axis, which points into the Earth. The third coordinate system is the magnetic field (B) coordinate system and is defined as having the z-axis in the direction of the local magnetic field unit vector, perpendicular to the fundamental plane [15]. The

fundamental direction is the North axis projected onto this fundamental plane. In this coordinate system, the magnetic field unit vector is defined as $\vec{B} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$. This intermediate coordinate system allows the magnetometers to be used directly in attitude determination. The final coordinate system is the body-fixed coordinate system in the body reference frame. This system is fixed such that the origin is the projection of the center of mass onto the axial line of symmetry. The fundamental direction (x-axis) is along the vehicle axial line of symmetry and points toward the heat shield. The y and z-axes are in the fundamental plane which is orthogonal to the fundamental direction. The z-axis intersects the mass offset. Because this test article did not have a mass offset, the z-axis was defined by the orientation of the CEV-TM package in the sabot. The y-axis is consistent with right-hand-rule coordinate systems.

Figure 3.5: Rotation between range and NED coordinate systems

A 2-1 rotation sequence (first rotate about the range y-axis, then the intermediate x-axis) rotates the range coordinate system to the NED coordinate system.

$$DCM_{Range \rightarrow NED} = \begin{bmatrix} \cos \gamma & 0 & -\sin \gamma \\ \sin \gamma & 0 & \cos \gamma \\ 0 & -1 & 0 \end{bmatrix} \quad (3.1)$$

where γ is the azimuth of the gun. Rotation from the NED to the B-system involves a 3-2 rotation sequence.

$$DCM_{NED \rightarrow B} = \begin{bmatrix} \cos \delta \sin i & \sin \delta \sin i & -\cos i \\ -\sin \delta & \cos \delta & 0 \\ \cos \delta \cos i & \sin \delta \cos i & \sin i \end{bmatrix} \quad (3.2)$$

where δ and i are magnetic declination and inclination, respectively.

Figure 3.6: Rotation between NED and magnetic field coordinate systems

Rotating from the B-system to the body coordinate system involves a 3-2-1 rotation [15]. This direction cosine matrix (DCM) was used later on in the derivation of the kinematic relations.

$$DCM_{B \rightarrow body} = \begin{bmatrix} \cos \psi \cos \theta & \sin \psi \cos \theta & -\sin \theta \\ \cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi & \sin \psi \sin \theta \sin \phi + \cos \psi \cos \phi & \cos \theta \sin \phi \\ \cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi & \sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi & \cos \theta \cos \phi \end{bmatrix} \quad (3.3)$$

where ψ , θ , and ϕ are the Euler angles corresponding to rotations about the z-axis, intermediate y-axis, and intermediate x-axis (3-2-1).

Figure 3.7: Rotation between magnetic field and body coordinate systems

Orientation can be determined through the relationship between the angular rates and the Euler angles and by the definition of the magnetic field coordinate system. From examination of the DCM and the definition of the magnetic field vector in the B-system, relations between normalized sensor measurements and Euler angles can be written.

$$\theta = \arcsin(-B_x)$$

$$\phi = \arctan\left(\frac{B_y}{B_z}\right)$$

Note that ψ cannot be unambiguously determined from the magnetometers alone. A second orientation vector, such as one obtained by sun sensors, is needed to clearly define the orientation of the body. The DCM between the magnetic field coordinate system and

the body-fixed coordinate system allows the kinematic relations to be written as follows:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\ 0 & \cos \phi & -\sin \phi \\ 0 & \frac{\sin \phi}{\cos \theta} & \frac{\cos \phi}{\cos \theta} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

where p , q , and r are the angular rate measurements of the body. From here, the expression that relates the time derivative of ψ to the other two Euler angles and the angular rates can be extracted [16].

$$\dot{\psi} = \frac{q \sin \phi + r \cos \phi}{\cos \theta}$$

Integrating this expression yields the final Euler angle.

$$\psi = \psi_0 + \int \dot{\psi} dt = \psi_0 + \int \frac{q \sin \phi + r \cos \phi}{\cos \theta} dt$$

This set of Euler angles now completes a chain of rotations from the range system to the body, permitting the velocities measured by the radar to be rotated to the body coordinate system.

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = DCM_{B \rightarrow body} DCM_{NED \rightarrow B} DCM_{Range \rightarrow NED} \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} \quad (3.4)$$

These body-centered velocities can now be used to calculate angle of attack, sideslip angle, and total angle of attack.

Knowledge of the initial orientation of the vehicle yields a complete solution. Because

of the high-g shock loads and the subsequent settling time of the angular rate gyros, this initial orientation is unknown. What is known is that the total torque derived from the moments of inertia and the angular rates and the total angle of attack should be in phase. An iterative method was used to determine the ψ_0 needed to shift the values of total angle of attack into phase with total torque. Comparing the normalized (to maximum value) angles of attack to the normalized torque leads to the minimum error solution.

Figure 3.8: Comparison of normalized phase-correcting parameters

Once a minimum error solution was reached, the corresponding set of Euler angles and flight angles were retained.

Figure 3.9: CEV flight angles generated by minimum error solution

3.2 Determination of Aerodynamic Coefficients

The aerodynamic coefficients are determined from the normal and axial forces and torques observed in the vehicle.

$$F_N = m\sqrt{a_{CM,y}^2 + a_{CM,z}^2}$$

$$F_A = ma_{CM,x}$$

The magnitude of the velocity is used to determine the dynamic pressure, and the geometry of the vehicle is used to determine the reference area, which are both used to nondimensionalize the coefficients.

$$V_{\infty} = \sqrt{V_x^2 + V_y^2 + V_z^2}$$

$$q_{\infty} = \frac{1}{2} \rho_{\infty} V_{\infty}^2$$

$$S = \pi r^2$$

Nondimensionalizing the forces and torques leads to the following coefficients:

$$C_N = \frac{F_N}{q_{\infty} S} \quad , C_A = \frac{F_A}{q_{\infty} S}$$

$$C_l = \frac{L}{q_{\infty} S d} \quad , C_m = \frac{M}{q_{\infty} S d} \quad , C_n = \frac{N}{q_{\infty} S d}$$

Using the angle of attack as a rotation angle, the coefficients of lift and drag are determined [14].

$$C_L = C_N \cos \alpha_T - C_A \sin \alpha_T$$

$$C_D = C_A \cos \alpha_T + C_N \sin \alpha_T$$

The derivatives of the coefficients of lift and the moment coefficients are taken with respect to the angle of attack, yielding stability terms. Note that for static stability, $C_{m_{\alpha}}$ must be less than zero. The trim angle is the angle of attack at zero lift.

3.3 Performance of Methodology

Figure 3.10: CEV normal and lifting force coefficients

Figure 3.11: CEV pitching and yawing moment coefficients

Note that there are fairly distinct regions illustrated, particularly for the normal force coefficient, that indicate a shift from supersonic to transonic and subsonic dynamics, occurring around 0.5 seconds. The negative C_{m_α} and positive C_{n_β} imply static longitudinal and lateral stability. Coefficients were compared to available values generated by the EXTRACTR 6-DOF maximum likelihood estimator [17].

Table 3.1: Comparison of coefficients between analytical method and EXTRACTR

	Analytic	EXTRACTR	Percent Difference
C_{N_α}	0.23	0.20	15%
C_{m_α}	-0.10	-0.10	0%
C_{A_0}	1.21	1.47	18%
α_{trim}	4.05	3.70	9%

EXTRACTR is the tool currently being used by Aberdeen Proving Grounds for the CEV data reduction and analysis, and it works by selecting model parameters, or coefficients, that best fit an aerodynamic model to the experimental data. The results of the method outlined in this paper compare favorably with the results generated by EXTRACTR.

Chapter 4

Aeroballistic Testing with Pressure Transducers

After the initial series of tests, it was decided that the project may benefit in transitioning from the use of a 120mm smooth-bore cannon to a 7-inch (approximately 180mm) High Altitude Research Program (HARP) gun. This change in test apparatus enables the use of a larger test article, with more internal volume available for instruments and a greater ballistic coefficient, thus extending the duration of usable free-flight data.

The increased internal volume of the test article enables the upgrade of the telemetry module, supporting 32 channels of data rather than 16. As with the CEV-TM, this new sensor suite has 6 channels dedicated to Analog Devices accelerometers and angular rate gyros, with two of each sensor type monitoring each of the three body axes. The ranges of the sensors were determined using simulations of an expected aeroballistic test article trajectory. One 3-axis set of accelerometers and angular rate gyros had ranges large enough to ensure that sensors were not saturated. Two sets of 3-axis Honeywell magnetometers were used instead of one as before, allowing redundant measurements to be

made on each primary axis throughout the flight. For the purposes of these aeroballistic tests, one of the magnetometers was subjected to a single auto set/reset approximately 1 second after initial power up while the second unit was subjected to a series of pre-programmed set/resets regularly throughout the flight to investigate changes in biases and scale factors that may have occurred during the flight. Four accelerometers were placed at a radial offset from the symmetric axis and the outputs were summed in an attempt to approximate the spin rate of the body. One channel was used to monitor the on-board battery.

This increase in the number of recordable channels and the overall increase in test article size supported the installation of MEMs-based pressure transducers within the telemetry module, the now renamed CEV pressure telemetry module or CEV-PTM [18].

Figure 4.1: Diagram of CEV-PTM

Vehicle orientation and Mach number may be determined from gathered pressure data, allowing an independent determination of these parameters. Nine pressure transducers were mounted in a cruciform configuration parallel to the symmetric axis. Machined pressure taps channel air to the pressure transducers. One of the pressure transducers was not exposed to external flow conditions and served as a dummy to measure acceleration effects that may bias the transducer outputs. Temperatures of two of the

transducers were measured and recorded on two channels to check that the pressure transducers were operating within temperature fluctuation tolerance levels.

4.1 Pressure Transducers

Aeroballistic tests conducted with fore-body pressure monitoring for the purpose of determining test article flight parameters have never been documented before. The high shock loads encountered during firing and the effects such loads may have on a pressure transducer requires thorough research into available pressure transducers and their limitations and physical experimentation on pressure transducers as a prelude to flight testing.

Most pressure transducers operate by measuring the deflection of a diaphragm. Calibration curves are used to determine how much pressure is required to induce an observed diaphragm deflection. This can be accomplished electronically in the form of a semiconductor by measuring the change of a electronic property (resistance or capacitance, for example) due to the deflection of a piezoelectric material [19]. This allows for the miniaturization of pressure transducers to a scale more suitable to the volumetric constraints of aeroballistic test articles.

Estimates for operating conditions and design constraints must be determined to select an appropriate pressure transducer. Expected pressure ranges were obtained using isentropic and normal shock relations. These relations assume adiabatic flow, but are acceptable for the range of expected Mach numbers. From these relationships, a Mach range of 0.5 to 4 yielded stagnation pressures of 1.2 to 21 atmospheres, respectively. Aeroballistic tests using the CEV-TM indicated shock loads on the order of 10,000g; as the pressure transducers were mounted parallel to the symmetric axis, acceleration

effects on the diaphragms must be considered. The transducer package size must be small enough to fit within the volumetric constraints of the test article. The size of the pressure transducer affects its offset from the CM forebody, determining the depth of the pressure tap. This is important as aeroballistic tests collect data in real-time; the potential exists for lag times to appear in collected pressure measurements.

The Kulite XCEL-100 series of sealed gage pressure transducers, manufactured by Kulite Semiconductor Products, Inc., best fit the operating criteria of the Orion CM aeroballistic tests [20].

Figure 4.2: Kulite XCEL-100 pressure transducer

Sealed gage refers to the calibration of the transducer relative to sea level pressure. These transducers are capable of providing temperature measurements of the internal silicon diaphragm. This measurement is meant to digitally compensate the sensor output for errors related to thermal fluctuation.

4.2 Pressure Transducer Calibration

The test article has been designed to support up to ten pressure transducers, one being a “blind” transducer (receives no airflow) that was used to quantify pressure errors due to acceleration. In general, the manufacturer-supplied sensitivity calibration curves were used for each transducer. Pre-flight calibration procedures have been documented to ensure that the transducers are functioning before and after they have been potted (permanently fixed within the test article) and that the ports are clear. The test article was placed within a pressure vessel and a static calibration over stepped pressure values performed to a maximum allowable pressure (supply air limitation) of 827,370 Pascals (120psi). Sensor data was telemetered out of the pressure vessel to a receiving station and recorded. This calibration allowed the evaluation of the seals between transducer ports; the dummy transducer should measure no pressure change if the seals are sufficient. The custom calibration curves and configuration of each transducer warrants this system-level static calibration, as each transducer relies on a compensation circuit made up of three resistors, with each transducer having unique resistor values.

The first calibration test indicated that the seal surrounding the dummy transducer was not sufficient, as a lagged pressure response was observed.

Figure 4.3: CEV-PTM-01 static pressure calibration response

This calibration provides information regarding expected noise levels of the sensors. The noise appears to have a normal distribution and a standard deviation on the order of 0.25% of the full-scale limit, which for these sensors is 3450 kiloPascals (500psi).

Figure 4.4: Noise levels and histogram of #3 pressure tap residuals

Chapter 5

Constructing an Aerodynamic Database

To determine flight parameters of the Orion CM test article from aeroballistic pressure measurements, an aerodynamic database must be constructed. An aerodynamic database contains information that relates pressure measurements at various locations on a body to parameters of interest, like Mach number and vehicle orientation. The data used to construct an aerodynamic database typically comes from static testing, such as those conducted by wind tunnels. This data may be used to validate CFD codes. Such wind tunnel tests were conducted on the Orion CM geometry.

5.1 Wind Tunnel Testing

As part of a nationwide effort to construct an aerodynamic database for the Orion CM, wind tunnel tests were conducted at two NASA facilities, the NASA Ames and Langley Unitary Plan Wind Tunnels (AUPWT and LUPWT, respectively) [21].

Figure 5.1: NASA Ames Unitary Plan Wind Tunnel

Both facilities are closed-circuit, variable-pressure, continuous operation wind tunnels with multiple legs. For the AUPWT, subsonic Mach number control in the 11x11 leg is achieved with a combination of compressor drive speed control and variable-camber guide vanes at the compressor inlet. Supersonic Mach number control additionally involves setting the flexible wall nozzle to achieve the proper area ratio. This leg of the tunnel can be operated between Mach number of 0.2 to 1.45. The 9x7 leg can vary Mach number between 1.5 and 2.5 by means of a sliding block inlet which forms the floor of an S-shaped duct. Sliding the block forward or back controls area ratio, changing the flow conditions in the test section [21].

Scale models of the Orion CM were used in the wind tunnel tests. Each model was designed to receive a standard 6-component force and moment balance at a 45-degree angle relative to its axis of symmetry. The models were sized with respect to blockage considerations in each of the wind tunnel test sections to minimize wall effects. Pressure taps were machined over the heat shield and back shell with a configuration of three radial and two circumferential rows of taps. Tap locations for each of the test models match,

with respect to overall model dimensions. Static pressures at each tap are measured using multiple (facility-dependent) calibrated pressure tap scanning devices, connected to each tap using pressure tubing, which is housed within the sting apparatus to minimize aerodynamic interference. Boundary layer trip dots were applied to all models. This consisted of an approximated circle (trip dot tape can only be applied in a straight line) of trip dots which enclosed the stagnation point over its expected range of variation with angle of attack.

Figure 5.2: Wind tunnel CM model with sting assembly and trip dots

The sting arm supports the model and maintains its orientation relative to the flow-field. It is aligned with the wake flow over the Mach number and angle of attack range to be tested to minimize aerodynamic interference effects. The wake of the model was determined using CFD to determine the optimal location for the sting. Tests strove to keep the model as close to the test section center-line as possible while changing angle of attack. Test loads were applied to the model-sting apparatus before testing to determine change in sting deflection given an applied load. The strut angle was not offset for the deflection correction; instead, the correction was applied in post-processing.

Each model was tested over a Mach number range of 0.3 to 2.5 and a Reynolds number range of 1.5×10^6 to 5.3×10^6 . Reynolds number values play a significant role in the transition from laminar to turbulent flow and in the subsequent changes in force and moment values. The sting was mounted to a knuckle-sleeve assembly which rotated the model through a range of angles of attack. Angle of attack is considered equivalent to total angle of attack, unless accompanied by another angle. Limitations on the sting range-of-motion prevented sideslip angle variation [21]. As this is an axisymmetric body, symmetry arguments may be used in determining resultant forces, moments, and pressure distributions given a sideslip angle. Small deflections due to loads acting on the sting arm induced small sideslip angles on the order of one-tenth of a degree and are considered negligible.

5.2 Computational Fluid Dynamics Simulations

Computational fluid dynamics (CFD) relies on discretized mathematical models and iterative processes to approximate the expected flowfield surrounding a body, given a set of initial parameters. Such models can be used to interpolate over the parameters used and the results obtained from physical experimentation, like wind tunnel tests. These models must first be validated by these tests before results can be used in further analysis.

Various CFD codes exist and appropriate uses are bounded by the assumptions inherent in the algorithms and structures of the codes and the sort of body being analyzed. All codes are various attempts to iteratively solve the Navier-Stokes equations.

OVERFLOW-D is a general purpose CFD code that is based on the well-known OVERFLOW code, but has been significantly enhanced to accommodate moving body applications, facilitate accuracy control via solution adaptation, and efficiently utilize

scalable computing resources. It employs a discretization model that partitions the flow-field into near-body and off-body regions. The near-body region includes the surface geometry of all bodies being considered and the volume of space extending a short distance above the respective surfaces. The off-body region encompasses the near-body domain and extends to the far-field boundaries of the problem, with grid resolution determined based on proximity to near-body components [22]. The OVERFLOW-D solver was used to approximate the flowfield surrounding the Orion CM geometry over an extended range of Mach and Reynolds numbers and angles of attack, allowing comparison to wind tunnel data and extrapolation beyond the capabilities of available wind tunnel facilities.

Pressure data obtained for the wind tunnel tests and CFD simulations was reported as pressure coefficients corresponding to a particular tap location or grid-point, respectively. Tap locations and grid-points are reported in Cartesian coordinates. To facilitate direct comparison, and because the data of interest are obtained from the three radial rows of taps extending from the apex of the heatshield to the shoulder, clock and cone angles were used to describe tap coordinates.

Figure 5.3: CFD and wind tunnel test range and grid/tap locations on Orion CM fore-body

Clock angle is defined as the clockwise angle between the plane bisecting the lower hemisphere of the heatshield at the pressure tap about the axis of symmetry. Cone angle is the angle between the axis of symmetry and the pressure tap, with the center of the resulting arc defined as the origin of the sphere of which the heatshield is an arc-surface.

Figure 5.4: CFD and wind tunnel pressure coefficient variation at cone angle of 19.45° -degrees

Clearly, there is a discrepancy between the two sets of data. This difference increases as the distance to the shoulder decreases and as the angle of attack and Mach number increase. This is possibly due differences in where laminar-to-turbulent transition points occur. Still, the range of CFD-simulated Mach numbers and angles of attack captures more of the window of expected test window than the wind tunnel tests and must be part of any solution. The wind tunnel data more accurately determines the laminar-to-turbulent transition point and captures real gas effects, but the CFD data encompasses

more gridpoints and has a larger Mach range. Both sets of data must be reconciled in developing an aerodynamic database.

5.3 Development of an Analytic Pressure Model

Development of an analytic model that yields a pressure value given Mach number, flight angles, and tap location is critical to developing a method to obtain these flight parameters from pressure tap measurements. Oftentimes, a multidimensional interpolation is conducted to determine the flight parameters given a series of pressure measurements at a given time. This method has some limitations to take into account:

- Solutions for parameters may be incorrect given a sparse database
- Multiple solutions may exist for a given set of pressures
- Does not force physical constraints (symmetry is not required)
- Cannot resolve multiple data sources with discrepancies
- Multidimensional interpolation is computationally expensive

Developing an analytic model as a function of tap location and flight parameters using CFD data, then “anchoring” this model to wind tunnel data, which is sparse in clock angle, ϕ , resolves the two sets of data and yields a model that allows the determination of flight parameters given a series of pressure transducer data. The available wind tunnel data was assumed to more accurately depict the surface pressure values across the CM forebody, particularly around the stagnation point.

The CFD pressure coefficient varies with respect to clock angle given a fixed Mach number, total angle of attack, and cone angle. The data indicates a sinusoidal relationship

between pressure coefficient and clock angle. A series in cosine was used to model the distribution with the appropriate number of terms in the series carefully selected. Too few terms may not capture all of the variation in the data with respect to clock angle, but too many terms may introduce high-frequency artifacts that do not represent the data. Based on the number of maxima and minima in the pressure coefficient data with respect to clock angle, a three-term series in cosine was found to be sufficient.

$$C_P = a_1 + a_2 \cos \phi + a_3 \cos 2\phi \quad (5.1)$$

This series minimizes the error and is easily solved for the coefficients.

Figure 5.5: Comparison of series with different numbers of coefficients

Increasing the number of terms to four does not noticeably improve the residuals. For the range of available Mach numbers and angles of attack, the error is small, with a maximum error of approximately 3%.

Figure 5.6: Percent difference of CFD 3-term analytical model

The coefficients for this series vary with Mach number, total angle of attack, and cone angle. Further study of the coefficients with respect to total angle of attack indicates a quadratic relationship between the coefficients and total angle of attack. New coefficients to this quadratic equation are a function of cone angle and Mach number.

Figure 5.7: Quadratic relationship between angle of attack and 3-term model coefficients

The full analytic model for pressure coefficient as a function of tap location, Mach number, and total angle of attack can now be written, where the coefficients are functions of Mach number and cone angle.

$$C_P = c_1 + \alpha_T (c_2 + \alpha_T c_3) + \alpha_T (c_4 + \alpha_T c_5) \cos \phi + \alpha_T (c_6 + \alpha_T c_7) \cos 2\phi \quad (5.2)$$

This equation ensures symmetric (concentric about the symmetric axis) pressure distributions at 0-degrees total angle of attack. Errors associated with this model for a range of Mach number were within 3.5% of the pressure coefficient at high Mach numbers. This

error varies between 1% and 2% at lower Mach numbers.

Figure 5.8: Percent difference of CFD 7-term analytical model

The wind tunnel tests and CFD simulations only varied angle of attack, which the analytical model treats as total angle of attack. Sideslip angles are not likely to be zero, and must be included in the model. Because the analytic model is continuous in clock angle, the angle of attack and sideslip angle were represented as a total angle of attack and an additional rotational term, ψ , which is added to the clock angle of the pressure tap.

$$\alpha_T = \arccos(\cos \alpha \cos \beta) \quad \alpha = \arcsin(\cos \psi \sin \alpha_T)$$

$$\psi = \arctan\left(\frac{\cos \alpha \sin \beta}{\sin \alpha}\right) \quad \beta = \arctan\left(\frac{\sin \alpha_T \sin \psi}{\cos \alpha_T}\right)$$

$$C_P = c_1 + \alpha_T (c_2 + \alpha_T c_3) + \alpha_T (c_4 + \alpha_T c_5) \cos(\phi + \psi) + \alpha_T (c_6 + \alpha_T c_7) \cos 2(\phi + \psi) \quad (5.3)$$

It was necessary to develop the analytic model using the CFD data because the wind tunnel data is sparse in clock angle; there are only three available data points, four using symmetry arguments. The coefficients obtained using the CFD data were corrected to anchor the model to the wind tunnel data points while maintaining the same functional form. The correction terms are simply partial derivatives of the analytic model with respect to the coefficients.

$$\begin{Bmatrix} c_1 & \cdots & c_7 \end{Bmatrix} = \begin{Bmatrix} c_1 & \cdots & c_7 \end{Bmatrix}_{CFD} + \begin{bmatrix} \frac{\delta C_{P_{model}}}{c_1} & \cdots & \frac{\delta C_{P_{model}}}{c_7} \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{Bmatrix} C_{P_{WT}} - C_{P_{CFD}} \\ \vdots \end{Bmatrix} \quad (5.4)$$

The model is now anchored to the wind tunnel data points while maintaining the functional form of the CFD data.

Figure 5.9: “Anchored” analytical model compared to “unanchored” model and wind tunnel datapoints at cone angle of 19.45-degrees

Note that the available wind tunnel data does not exceed Mach 2.5. The coefficients obtained using the CFD data beyond this Mach number remain unchanged, with the average of the coefficients obtained using CFD data and the wind-tunnel-anchored coefficients taken at Mach 2.5; effectively the CFD-based coefficients corrected by half of the obtained correction term at Mach 2.5.

The pressure transducers generate pressure data in engineering units, not pressure coefficients. Pressure is normalized using the dynamic and static pressures and the pressure

coefficient. The analytic model was rewritten to reflect this.

$$C_P = \frac{P - P_\infty}{q_\infty} = \frac{P - P_\infty}{\frac{1}{2}\rho V_\infty^2} \rightarrow P = \frac{1}{2}C_P\rho_\infty V_\infty^2 + P_\infty \quad (5.5)$$

V_∞ can be rewritten in terms of known parameters.

$$\begin{aligned} V_\infty &= Ma \\ a &= \sqrt{\gamma RT_\infty} \end{aligned} \rightarrow V_\infty = M\sqrt{\gamma RT_\infty}$$

$$\begin{aligned} P &= \frac{1}{2}C_P\rho_\infty M^2\gamma RT_\infty + P_\infty \\ P_\infty &= \rho_\infty RT_\infty \end{aligned} \rightarrow P = \frac{1}{2}C_P M^2\gamma P_\infty + P_\infty$$

The analytical model is now a function of only two atmospheric parameters, atmospheric pressure and the atmospheric ratio of specific heats.

$$P = \frac{P_\infty}{2} (2 + \gamma C_P M^2) \quad (5.6)$$

The analytic model as derived is based on wind tunnel data, while deriving its functional form from CFD data. Symmetry constraints were maintained and the coefficients may be easily interpolated over Mach number, as cone angle is a known and fixed parameter for the pressure transducers.

5.4 Pressure Tap Placement

A heuristic approach to determine the optimal tap locations that ensure the maximum amount of information is obtained with minimal uncertainty or error was pursued using experimental and simulated data. The accuracy of estimated flight parameters via pressure transducer data may be maximized while minimizing errors by increasing the

distance between symmetrically-opposed taps about the axis of symmetry. The difference between these pressures should be zero at 0-degrees angle of attack and a positive or negative value based on a non-zero angle of attack, where the angle is in the same plane as the two tap and the symmetric axis. By this argument, taps should be placed as far apart as possible from each other, each one on the edge of the heatshield-cone shoulder. Uncertainties associated with Mach number and flight angle determination based on these pressures would likely be high as the flowfield in the shoulder region of the CM is quite turbulent.

Stagnation point location is a key factor in determining pressure tap locations. The cone angle, λ , of the maximum pressure coefficient is a well-defined function in angle of attack.

Figure 5.10: Stagnation point cone angle over a range of angles of attack at varying Mach numbers

This location is weakly sensitive to Mach number variation. The supersonic and subsonic design trim angles of attack of the CEV-PTM are 21.46-degrees and 12.21-degrees [23]. These values correspond to Mach numbers of 2.5 and 0.7. For the pressure transducers to measure the stagnation pressure during the aeroballistic test, pressure taps should be placed at a cone angle of 12.59-degrees.

Optimal tap locations as defined by the wind tunnel data may differ from those determined using CFD. As CFD data was used in the aerodynamic database, it must be included when determining tap placement. Constraining tap locations to cone angles

in which wind tunnel and CFD pressure coefficient data are within 10% of each other sufficiently addresses this issue.

Figure 5.11: Comparison of CFD and wind tunnel Orion CM data on vertical and horizontal planes

This agreement is maintained up to a maximum cone angle of 19.45-degrees. This is the largest cone angle at which pressure taps may be placed while maintaining agreement between CFD and wind tunnel data sources, which implies that turbulent flow field effects at the shoulder are minimized.

A cruciform arrangement of pressure taps is the optimal configuration to minimize

potential data biasing and maintain pairs of symmetrically-opposed pressure taps. By placing pressure taps at cone angles of 0, 12.59 and 19.45-degrees, a cruciform pattern may be constructed on the aeroballistic CEV-PTM forebody that collects optimal pressure data for use in determining flight parameters while maintaining agreement between aerodynamic table data sources.

Chapter 6

Parameter Estimation

The analytic pressure model may be used to compute the flight parameters, Mach number, angle of attack, and sideslip angle. A variety of approaches were investigated that all focused on iterative solutions to a standard matrix problem for a series of unknowns.

6.1 Least Squares Approach

Given pressure data, the least squares method can be used to determine the value of the parameters at a point in time

$$\begin{Bmatrix} M \\ \alpha \\ \beta \end{Bmatrix}_{n+1} = \begin{Bmatrix} M \\ \alpha \\ \beta \end{Bmatrix}_n + (B^T B)^{-1} B^T \{y\}$$

where B is the matrix of partial derivatives of pressure with respect to the flight parameters and y is a vector of residuals [24].

$$B = \begin{bmatrix} \left(\frac{\delta P}{\delta M}\right)_{Tap\ 1} & \left(\frac{\delta P}{\delta \alpha}\right)_{Tap\ 1} & \left(\frac{\delta P}{\delta \beta}\right)_{Tap\ 1} \\ \left(\frac{\delta P}{\delta M}\right)_{Tap\ 2} & \left(\frac{\delta P}{\delta \alpha}\right)_{Tap\ 2} & \left(\frac{\delta P}{\delta \beta}\right)_{Tap\ 2} \\ \vdots & \vdots & \vdots \\ \left(\frac{\delta P}{\delta M}\right)_{Tap\ i} & \left(\frac{\delta P}{\delta \alpha}\right)_{Tap\ i} & \left(\frac{\delta P}{\delta \beta}\right)_{Tap\ i} \end{bmatrix}$$

$$\{y\} = \begin{Bmatrix} P_{measured_{Tap\ 1}} - P_{computed_{Tap\ 1}} \\ P_{measured_{Tap\ 2}} - P_{computed_{Tap\ 2}} \\ \vdots \\ P_{measured_{Tap\ i}} - P_{computed_{Tap\ i}} \end{Bmatrix}$$

As the model is linear in two of the three parameters, the necessary elements of the matrix of partial derivatives of pressure with respect to angle of attack and sideslip angle may be calculated analytically. A central finite difference was used to determine the partial derivative of pressure with respect to Mach number.

This method works by finding the optimal combination of parameters in the 3-dimensional space that minimizes the residual pressure. The feasibility of the solution is improved by using the previous closest local minimum as the initial value for the next time step. This does not take into account the effects of real-world phenomena that are readily observable in measured data: scale factor, bias, and noise effects.

Scale factor is a proportionality constant that is multiplied by the actual, unknowable measurement. Bias is an additional term that is a fraction of the rated full-scale limit of the sensor. Noise is a random contribution to the data signal in the time domain that has a mean of zero and a standard deviation that is a fraction of the rated full-scale limit of the sensor. These components end up shifting the true sensor output into the measured values and can be expressed as a linear equation for each tap.

$$P_{measured} = sP_{actual} + b + noise \quad (6.1)$$

Thousands of datapoints may be generated from an aeroballistic flight test in the region of interest, in this case for the range of Mach numbers between barrel exit and 0.5. Solving for the “local” flight parameters at each time is a computationally simple operation, while solving for the “global” parameters of scale factor and bias requires much more memory and unnecessary matrix calculations. Available memory rapidly becomes a constraining factor when manipulating matrices that possess hundreds of thousands of elements. A back-substitution method was employed to work around this issue.

Consider two sets of nine pressure measurements (taken from the CEV-PTM pressure tap cruciform), corresponding to timestamps t_1 and t_2 . This corresponds to 9 individual scale factors and biases and 2 pairs of 3 flight parameters.

$$X_0 = \left\{ s_{Tap1} \quad s_{Tap2} \quad \cdots \quad s_{Tap9} \quad b_{Tap1} \quad b_{Tap2} \quad \cdots \quad b_{Tap9} \right\}^T$$

$$X_1 = \begin{Bmatrix} M_{t_1} \\ \alpha_{t_1} \\ \beta_{t_1} \end{Bmatrix} \quad X_2 = \begin{Bmatrix} M_{t_2} \\ \alpha_{t_2} \\ \beta_{t_2} \end{Bmatrix}$$

The parameters of interest, 24 in all, may be written as a single vector.

$$X = \begin{Bmatrix} X_0 \\ X_1 \\ X_2 \end{Bmatrix}$$

For a single batch least squares solution, sensitivity matrices and residual vectors must

be generated.

$$A_i = \begin{bmatrix} \left(\frac{\delta P}{\delta s_{Tap1}}\right)_{Tap1} & \cdots & \left(\frac{\delta P}{\delta s_{Tap9}}\right)_{Tap1} & \left(\frac{\delta P}{\delta b_{Tap1}}\right)_{Tap1} & \cdots & \left(\frac{\delta P}{\delta b_{Tap9}}\right)_{Tap1} \\ \left(\frac{\delta P}{\delta s_{Tap1}}\right)_{Tap2} & \cdots & \left(\frac{\delta P}{\delta s_{Tap9}}\right)_{Tap2} & \left(\frac{\delta P}{\delta b_{Tap1}}\right)_{Tap2} & \cdots & \left(\frac{\delta P}{\delta b_{Tap9}}\right)_{Tap2} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \left(\frac{\delta P}{\delta s_{Tap1}}\right)_{Tap9} & \cdots & \left(\frac{\delta P}{\delta s_{Tap9}}\right)_{Tap9} & \left(\frac{\delta P}{\delta b_{Tap1}}\right)_{Tap9} & \cdots & \left(\frac{\delta P}{\delta b_{Tap9}}\right)_{Tap9} \end{bmatrix}$$

$$B_i = \begin{bmatrix} \left(\frac{\delta P}{\delta M}\right)_{Tap1} & \left(\frac{\delta P}{\delta \alpha}\right)_{Tap1} & \left(\frac{\delta P}{\delta \beta}\right)_{Tap1} \\ \left(\frac{\delta P}{\delta M}\right)_{Tap2} & \left(\frac{\delta P}{\delta \alpha}\right)_{Tap2} & \left(\frac{\delta P}{\delta \beta}\right)_{Tap2} \\ \vdots & \vdots & \vdots \\ \left(\frac{\delta P}{\delta M}\right)_{Tap i} & \left(\frac{\delta P}{\delta \alpha}\right)_{Tap i} & \left(\frac{\delta P}{\delta \beta}\right)_{Tap i} \end{bmatrix}_i$$

$$y_i = \begin{bmatrix} P_{measured_{Tap1}} - P_{computed_{Tap1}} \\ P_{measured_{Tap2}} - P_{computed_{Tap2}} \\ \vdots \\ P_{measured_{Tap i}} - P_{computed_{Tap i}} \end{bmatrix}_i$$

A_i and B_i are 9 by 18 and 9 by 3 matrices of partial derivatives corresponding to the bias and scale factor parameters and the flight parameters, respectively. y_i is the 18 by 1 vector of pressure residuals. These matrices and vectors may be used to construct a system-level (flight parameters at each time step and scale factors and biases) sensitivity matrix and residual vector.

$$J = \begin{bmatrix} A_1 & B_1 & 0 \\ A_2 & 0 & B_2 \end{bmatrix}$$

$$y = \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix}$$

This leads to an expression for the least squares solution.

$$(J^T J) \delta X = J^T y \quad (6.2)$$

δX is the vector of correction terms on each of the parameters. Note that J is rank-deficient unless three sets of data are used. As an accumulative method is being developed, two sets of data will be sufficient for this derivation.

These matrices may now be evaluated and expanded.

$$J^T J = \begin{bmatrix} A_1^T A_1 + A_2^T A_2 & A_1^T B_1 & A_2^T B_2 \\ B_1^T A_1 & B_1^T B_1 & 0 \\ B_2^T A_2 & 0 & B_2^T B_2 \end{bmatrix}$$

$$J^T y = \begin{Bmatrix} A_1^T y_1 + A_2^T y_2 \\ B_1^T y_1 \\ B_2^T y_2 \end{Bmatrix}$$

$$(A_1^T A_1 + A_2^T A_2) \delta X_0 + A_1^T B_1 \delta X_1 + A_2^T B_2 \delta X_2 = A_1^T y_1 + A_2^T y_2 \quad (6.3)$$

$$B_1^T A_1 \delta X_0 + B_1^T B_1 \delta X_1 = B_1^T y_1 \quad (6.4)$$

$$B_2^T A_2 \delta X_0 + B_2^T B_2 \delta X_2 = B_2^T y_2 \quad (6.5)$$

Equations 6.4 and 6.5 may be rearranged, resulting in a two equations that relate the residuals and sensitivity matrices to correction terms on the scale factors and biases.

$$\delta X_1 = (B_1^T B_1)^{-1} B_1^T y_1 - (B_1^T B_1)^{-1} B_1^T A_1 \delta X_0 \quad (6.6)$$

$$\delta X_2 = (B_2^T B_2)^{-1} B_2^T y_2 - (B_2^T B_2)^{-1} B_2^T A_2 \delta X_0 \quad (6.7)$$

Note that the first term in each equation is the least squares solution when bias and scale factor are omitted, $\delta \tilde{X}_i$. Substituting δX_1 and δX_2 into 6.3 and rearranging,

$$\begin{aligned} (A_1^T A_1 + A_2^T A_2) \delta X_0 + A_1^T B_1 \left(\delta \tilde{X}_1 - (B_1^T B_1)^{-1} B_1^T A_1 \delta X_0 \right) \\ + A_2^T B_2 \left(\delta \tilde{X}_2 - (B_2^T B_2)^{-1} B_2^T A_2 \delta X_0 \right) = A_1^T y_1 + A_2^T y_2 \end{aligned}$$

$$\begin{aligned} \left(A_1^T A_1 - A_1^T B_1 (B_1^T B_1)^{-1} B_1^T A_1 + A_2^T A_2 - A_2^T B_2 (B_2^T B_2)^{-1} B_2^T A_2 \right) \delta X_0 \\ = A_1^T y_1 - A_1^T B_1 \delta \tilde{X}_1 + A_2^T y_2 - A_2^T B_2 \delta \tilde{X}_2 \quad (6.8) \end{aligned}$$

This is rank-deficient for two sets of data, but by evaluating the terms through this elimination phase it is evident that the correction term for the “global” parameters may be determined by accumulating terms calculated at each “local” loop, essentially performing a back-substitution on the matrix.

$$\sum_{t_1}^{t_N} \left(A_t^T A_t - A_t^T B_t (B_t^T B_t)^{-1} B_t^T A_t \right) \delta X_0 = \sum_{t_1}^{t_N} \left(A_t^T y_t - A_t^T B_t \delta \tilde{X}_t \right)$$

$$\delta X_0 = \left(\sum_{t_1}^{t_N} \left(A_t^T A_t - A_t^T B_t (B_t^T B_t)^{-1} B_t^T A_t \right) \right)^{-1} \sum_{t_1}^{t_N} \left(A_t^T y_t - A_t^T B_t \delta \tilde{X}_t \right) \quad (6.9)$$

6.2 Minimum Variance Approach with A Priori

Oftentimes an a priori estimate of a set of parameters to be estimated is available. Such information may include specified parameter values and associated uncertainties. Uncertainties in measurements are typically known. This information may be used to implement a minimum variance approach that incorporates this a priori knowledge [25].

$$(J^T \Gamma_\epsilon^{-1} J + \Gamma_\mu^{-1}) \delta X = J^T \Gamma_\epsilon^{-1} y + \Gamma_\mu^{-1} (\mu - X) \quad (6.10)$$

In this equation, μ is the a priori estimate of the parameters. Γ_μ is the a priori estimate of the covariance matrix and is assumed to be diagonal.

$$\Gamma_\mu^{-1} = \begin{bmatrix} \sigma_M^2 & 0 & 0 \\ 0 & \sigma_\alpha^2 & 0 \\ 0 & 0 & \sigma_\beta^2 \end{bmatrix}^{-1}$$

Γ_ϵ^{-1} is the the inverse of the variance of the pressure measurements. Equation 6.10 may be simplified by distributing this variance term across the matrices.

$$\Gamma_\epsilon^{-1} = \Gamma_\epsilon^{-1/2} \Gamma_\epsilon^{-1/2}$$

$$\hat{J} = \Gamma_\epsilon^{-1/2} J, \quad \hat{y} = \Gamma_\epsilon^{-1/2} y$$

$$\left(\hat{J}^T \hat{J} + \Gamma_\mu^{-1} \right) \delta X = \hat{J}^T \hat{y} + \Gamma_\mu^{-1} (\mu - X) \quad (6.11)$$

As demonstrated earlier, a back-substitution method may address memory concerns associated with the computationally expensive matrix manipulations required to simultaneously solve for the scale factors, biases, and flight parameters. Applying this method to the minimum variance with a priori approach is fairly straight-forward. Consider data obtained from a 9-tap cruciform. As before, J is the sensitivity matrix associated with both “global” and “local” parameters and y is the vector of residuals. Both are constructed from pressure measurements taken at two times, and both are normalized by the known pressure variance. The a priori covariance matrix, Γ_μ^{-1} and the vector of estimates, μ , are augmented with the a priori estimates of the scale factors and biases.

$$\mu = \begin{Bmatrix} \mu_0 \\ \mu_1 \\ \mu_2 \end{Bmatrix}$$

$$\Gamma_\mu^{-1} = \begin{bmatrix} \sigma_{\mu_{X_0}}^2 & 0 & 0 \\ 0 & \sigma_{\mu_{X_1}}^2 & 0 \\ 0 & 0 & \sigma_{\mu_{X_2}}^2 \end{bmatrix}^{-1}$$

As before, the matrices may be evaluated and expanded.

$$\left(\hat{A}_1^T \hat{A}_1 + \hat{A}_2^T \hat{A}_2 + \sigma_{\mu_{X_0}}^{-2}\right) \delta X_0 + \hat{A}_1^T \hat{B}_1 \delta X_1 + \hat{A}_2^T \hat{B}_2 \delta X_2 = \hat{A}_1^T y_1 + \hat{A}_2^T y_2 + \sigma_{\mu_{X_0}}^{-2} (\mu_0 - X_0) \quad (6.12)$$

$$\hat{B}_1^T \hat{A}_1 \delta X_0 + \left(\hat{B}_1^T \hat{B}_1 + \sigma_{\mu_{X_1}}^{-2}\right) \delta X_1 = \hat{B}_1^T \hat{y}_1 + \sigma_{\mu_{X_1}}^{-2} (\mu_1 - X_1) \quad (6.13)$$

$$\hat{B}_2^T \hat{A}_2 \delta X_0 + \left(\hat{B}_2^T \hat{B}_2 + \sigma_{\mu_{X_2}}^{-2}\right) \delta X_2 = \hat{B}_2^T \hat{y}_2 + \sigma_{\mu_{X_2}}^{-2} (\mu_2 - X_2) \quad (6.14)$$

Equations 6.13 and 6.14 may be rearranged, resulting in a two equations that relate the residuals and sensitivity matrices to correction terms on the scale factors and biases.

$$\delta X_1 = \left(\hat{B}_1^T \hat{B}_1 + \sigma_{\mu_{X_1}}^{-2}\right)^{-1} \left(\hat{B}_1^T \hat{y}_1 + \sigma_{\mu_{X_1}}^{-2} (\mu_1 - X_1)\right) - \left(\hat{B}_1^T \hat{B}_1 + \sigma_{\mu_{X_1}}^{-2}\right)^{-1} \hat{B}_1^T \hat{A}_1 \delta X_0 \quad (6.15)$$

$$\delta X_2 = \left(\hat{B}_2^T \hat{B}_2 + \sigma_{\mu_{X_2}}^{-2}\right)^{-1} \left(\hat{B}_2^T \hat{y}_2 + \sigma_{\mu_{X_2}}^{-2} (\mu_2 - X_2)\right) - \left(\hat{B}_2^T \hat{B}_2 + \sigma_{\mu_{X_2}}^{-2}\right)^{-1} \hat{B}_2^T \hat{A}_2 \delta X_0 \quad (6.16)$$

Note that the first term in each equation is the minimum variance solution when bias and scale factor are omitted, $\delta \tilde{X}_i$. Substituting δX_1 and δX_2 into 6.12 and rearranging,

$$\begin{aligned} &\left(\hat{A}_1^T \hat{A}_1 + \hat{A}_2^T \hat{A}_2 + \sigma_{\mu_{X_0}}^{-2}\right) \delta X_0 + \hat{A}_1^T \hat{B}_1 \left(\delta \tilde{X}_1 - \left(\hat{B}_1^T \hat{B}_1 + \sigma_{\mu_{X_1}}^{-2}\right)^{-1} \hat{B}_1^T \hat{A}_1 \delta X_0\right) \\ &+ \hat{A}_2^T \hat{B}_2 \left(\delta \tilde{X}_2 - \left(\hat{B}_2^T \hat{B}_2 + \sigma_{\mu_{X_2}}^{-2}\right)^{-1} \hat{B}_2^T \hat{A}_2 \delta X_0\right) = \hat{A}_1^T \hat{y}_1 + \hat{A}_2^T \hat{y}_2 + \sigma_{\mu_{X_0}}^{-2} (\mu_0 - X_0) \end{aligned}$$

$$\begin{aligned}
& \left(\hat{A}_1^T \hat{A}_1 - \hat{A}_1^T \hat{B}_1 \left(\hat{B}_1^T \hat{B}_1 + \sigma_{\mu_{X_1}}^{-2} \right)^{-1} \hat{B}_1^T \hat{A}_1 + \hat{A}_2^T \hat{B}_2 \right) \delta X_0 \\
& - \left(\hat{A}_2^T \hat{B}_2 \left(\hat{B}_2^T \hat{B}_2 + \sigma_{\mu_{X_2}}^{-2} \right)^{-1} \hat{B}_2^T \hat{A}_2 + \sigma_{\mu_{X_0}}^{-2} \right) \delta X_0 \\
& = \hat{A}_1^T \hat{y}_1 - \hat{A}_1^T \hat{B}_1 \delta \tilde{X}_1 + \hat{A}_2^T \hat{y}_2 - \hat{A}_2^T \hat{B}_2 \delta \tilde{X}_2 + \sigma_{\mu_{X_0}}^{-2} (\mu_0 - X_0) \quad (6.17)
\end{aligned}$$

The inclusion of a priori terms prevents this solution from being rank-deficient despite having only two sets of data. As before, the correction term for the “global” parameters may be determined by accumulating terms calculated at each “local” loop.

$$\begin{aligned}
& \left(\sum_{t_1}^{t_N} \left(\hat{A}_t^T \hat{A}_t - \hat{A}_t^T \hat{B}_t \left(\hat{B}_t^T \hat{B}_t + \sigma_{\mu_{X_t}}^{-2} \right)^{-1} \hat{B}_t^T \hat{A}_t \right) + \sigma_{\mu_{X_0}}^{-2} \right) \delta X_0 \\
& = \left(\sum_{t_1}^{t_N} \left(\hat{A}_t^T \hat{y}_t - \hat{A}_t^T \hat{B}_t \delta \tilde{X}_t \right) \right) + \sigma_{\mu_{X_0}}^{-2} (\mu_0 - X_0)
\end{aligned}$$

$$\begin{aligned}
\delta X_0 = & \left(\sum_{t_1}^{t_N} \left(\hat{A}_t^T \hat{A}_t - \hat{A}_t^T \hat{B}_t \left(\hat{B}_t^T \hat{B}_t + \sigma_{\mu_{X_t}}^{-2} \right)^{-1} \hat{B}_t^T \hat{A}_t \right) + \sigma_{\mu_{X_0}}^{-2} \right)^{-1} \\
& \left(\left(\sum_{t_1}^{t_N} \left(\hat{A}_t^T \hat{y}_t - \hat{A}_t^T \hat{B}_t \delta \tilde{X}_t \right) \right) + \sigma_{\mu_{X_0}}^{-2} (\mu_0 - X_0) \right) \quad (6.18)
\end{aligned}$$

Note that as uncertainties increase (“looser” a priori terms), this expression becomes identical to the least squares expression.

6.3 Method Validation

To test the parameter estimation software, simulated pressure data were generated. A trajectory of the lifting-body CEV-PTM was generated using POST to determine the ranges of the on-board sensors. POST, the Program to Optimize Simulated Trajectories, is a software package used to simulate and optimize complex systems in 3 or 6 degrees-of-freedom [26]. This same trajectory was used to generate pressure data using the analytic pressure model.

Figure 6.1: POST-generated flight parameters and simulated pressure profile

Biases and noise were generated using a normally random distribution across the pressure taps with a standard deviation of 1% of the full scale pressure rating of the pressure transducers, 3450 kiloPascals (500psi). Scale factors were calculated in a similar manner at 1% of the “unknowable” pressure. This signal was filtered using the software tools developed for the analysis of the actual data to more closely match the actual solution method.

Figure 6.2: Residuals of filtered POST-based pressure profile

Note the relatively small amplitude of the sideslip angle signal as observed for the CEV-TM solution. The resulting small lateral pressure differential may result in difficulty resolving the sideslip angle and scale factor or bias values. A least squares (“loose” a

priori) solution condition was applied to the full 9-tap dataset, which further supported this assertion. “Local” and “global” iterations continued until convergence, defined as a relative error on the parameters of 0.25%.

Figure 6.3: Pressure residuals for 9-tap least squares solution

The pressure residuals are small relative to the signal noise of 1.5% of the full-scale rating of the pressure transducer, 3450 kiloPascals (500psi). This is as much as 40% to 50% of the pressure signal when the CEV is subsonic.

Figure 6.4: Mach number solution for 9-tap least squares solution

The relative error for the Mach number estimate is less than 1% for low Mach numbers, which is when the noise-to-signal ratio is greatest. The oscillations in the Rayleigh-Pitot estimate are due to the motion of the CEV and the stagnation point (maximum value seen by the pressure tap cruciform) jumping between pressure taps.

Figure 6.5: Angle of attack solution for 9-tap least squares solution

The flight angle estimates are more sensitive than the Mach number estimate because less information is required to determine Mach number. As a result, the relative error for the angle of attack estimate increases as Mach number decreases. This is due to the larger noise-to-signal ratio and the decreasing pressure differential along the vertical taps with decreasing Mach number.

Figure 6.6: Sideslip angle solution for 9-tap least squares solution

The poor solution for sideslip angle is likely due to the small lateral pressure differential. As there was no real boundary set on the solution via a priori conditions, the converged solution is not the same as the input data. A priori uncertainties were selected using expected angular rates and deceleration forces, and manufacturer data associated with pressure transducer fidelity.

Table 6.1: A priori uncertainties set for solution parameters

	Mach	α (degrees)	β (degrees)	Scale factor	bias	Pressure (Pascals)
σ	0.015	2.5	1.0	1.05	5000	2000

By applying “tighter” a priori conditions on the bias and scale factor uncertainties and on the flight parameters at each timestep, the solution was improved.

Figure 6.7: Pressure residuals for 9-tap minimum variance with a priori solution

The peak residuals are roughly twice as large as observed in the least squares case. This is because the solution corresponding to the minimum error is constrained by the a priori uncertainties. The residuals are still small when compared to the noise component of the signal (40% to 50% of the total pressure signal) at low Mach numbers.

Figure 6.8: Mach number solution for 9-tap minimum variance with a priori solution

The relative error of the Mach number estimate is less than 2.5% for low Mach numbers, which is acceptable considering the noise magnitude.

Figure 6.9: Angle of attack solution for 9-tap minimum variance with a priori solution

The angle of attack estimate is almost identical to the least squares case. This implies that angle of attack is well-determined and will be fairly insensitive to errors in estimates for other parameters, like sideslip angle.

The sideslip angle estimate has improved with the introduction of a priori uncertainties. Because the lateral pressure differential is small to begin with, the increasing noise-to-signal ratio at lower Mach numbers translates into an increased sideslip angle uncertainty.

Figure 6.10: Sideslip angle solution for 9-tap minimum variance with a priori solution

A comparison of the mean and standard deviation of the converged parameters and pressure residuals illustrates the improvement in the solution due to the “tighter” a priori values.

Table 6.2: Mean and standard deviation of estimated parameter residuals using least squares and minimum variance with a priori

	Mach	AoA (degrees)	Sideslip angle (degrees)
mean (LS)	0.002	0.502	1.003
standard deviation (LS)	-0.002	0.055	0.338
mean (MV)	0.008	0.527	0.325
standard deviation (MV)	-0.011	-0.313	-0.026

Table 6.3: Mean and standard deviation of pressure residuals using least squares and minimum variance with a priori

	Tap #1	Tap #2	Tap #3	Tap #4	Tap #5	Tap #6	Tap #7	Tap #8	Tap #9
μ_{LS}	40.9	78.0	26.5	18.2	45.6	78.2	-4.7	-50.1	48.2
σ_{LS}	590.7	458.8	516.4	576.1	487.2	540.9	574.7	574.0	589.7
μ_{MV}	-24.5	-15.0	-11.2	12.9	0.1	-39.2	165.6	129.1	-295.2
σ_{MV}	550.0	286.1	404.8	481.8	373.7	750.3	822.1	748.0	899.3

Note that converged pressures from taps 7 and 9, the inner set of lateral taps, are the most poorly determined, as expected. This is reflected in the converged solutions for the scale factors and biases. The “global” loops converged in 2 and 3 iterations for the least squares and minimum variance cases, respectively. A termination criteria of 2.5% of the pressure uncertainty was set for all cases.

Table 6.4: Comparison of converged scale factors and biases using least squares and minimum variance with a priori

Scale Factor	Tap #1	Tap #2	Tap #3	Tap #4	Tap #5	Tap #6	Tap #7	Tap #8	Tap #9
Actual	0.995	0.996	0.993	1.008	0.993	1.005	0.982	0.976	1.003
Least Squares	0.996	0.988	1.008	1.014	0.976	1.001	0.992	0.981	0.992
% Difference LS	0.03	0.79	1.54	0.65	1.65	0.47	1.05	0.48	1.05
Minimum Variance	0.999	0.994	0.996	1.015	0.995	1.006	0.986	0.983	1.004
% Difference MV	0.34	0.16	0.33	0.70	0.22	0.03	0.46	0.72	0.13
Bias	Tap #1	Tap #2	Tap #3	Tap #4	Tap #5	Tap #6	Tap #7	Tap #8	Tap #9
Actual	-5867.6	3438.8	382.2	4238.3	590.7	3100.5	28.8	9716.8	-5120.4
Least Squares	-3826.2	6896.4	-161.8	4269.4	6233.5	4460.7	2027.6	13163.0	-1502.6
% Difference LS	-34.8	100.5	142.3	0.7	955.3	43.9	6929.1	35.5	-70.7
Minimum Variance	-6717.5	4119.7	600.1	2478.1	996.1	3494.1	-1009.6	10418.4	-5665.1
% Difference MV	-14.5	19.8	57.0	41.5	68.6	12.7	3599.8	7.2	-10.6

The solutions for both scale factor and bias are improved by including the tighter a priori uncertainties on the parameters. Even though the percent differences for the biases seem quite large, the biases are on the same order of magnitude as the noise in the pressure signal. The scale factor has a more significant effect on the solution and

the percent differences are acceptable. By using a priori values, carefully selected based on the manufacturer-provided sensor specifications and noise observations, it should be possible to estimate the flight parameters to within 5% of the actual values. This error decreases as Mach number, and in turn dynamic pressure, increases.

Chapter 7

Post-Flight Analysis of CEV-PTM Data

Two different CEV-PTM configurations were fired in July 2008 at the U.S. Army Proving Grounds in Aberdeen, MD: a 5-tap cruciform which utilized the outermost set of pressure taps, and the full 9-tap cruciform. The 5-tap configuration was fired first; as the data obtained from the shot showed no immediately apparent anomalies or signal corruption, the 9-tap configuration was fired as well. Meteorological measurements were also collected, specifically, atmospheric pressure, relative humidity, and air temperature.

7.1 Data Preprocessing

Before flight angle and Mach number reconstruction could be carried out, the data had to be interpolated and filtered. ARL had already applied calibrations to the data and converted the digital bits into engineering units. Further processing involved performing a 7-point running mean on the data to remove bit noise, interpolating datapoints from

all sensors to the same timestamp, decimating the data to a 1000Hz sample rate, then filtering the data using a digital lowpass Butterworth filter to remove high frequency noise components. Sufficiently high cutoff frequencies were selected to avoid affecting observed vehicle dynamics [27].

Figure 7.1: Example of filtering output for 9-tap case, pressure tap #2

Each step of the filtering process is illustrated in this example figure. Note the noise distribution is identified and removed by the lowpass filter without removing the physically-attributable components of the signal.

The pressure data was plotted and studied with respect to time to make sure the observed signals seemed reasonable.

Figure 7.2: 5 and 9-tap pressure data collected from CEV-PTM shots

The pressure tap near the stagnation point should have the highest pressure throughout the timeframe of interest, the lateral tap pairs should be somewhat out of phase with each other, but of the same magnitude, and the tap farthest from the stagnation point should have the lowest pressure signal.

From these plots of the 5-tap and 9-tap cases immediate conclusions were drawn. The CEV-PTM was flying initially at a negative trim angle of attack of roughly negative 20-degrees, as the stagnation point seems to be in the vicinity of the #2 and #6 pressure taps. Also, it would appear that there was very little variation in sideslip angle and the amplitude of the signals seen in the lateral tap pairs is relatively low. Mach number was estimated by using the stagnation pressure (maximum observable pressure at each timestamp) and applying isentropic and normal shock relations.

Figure 7.3: Comparison of radar to pressure-based Mach number estimate (using isentropic and normal shock relations)

Initial comparisons to radar-based Mach number appear to confirm these estimates.

7.2 Parameter Estimation Assuming No Scale Factors or Biases

Initial conditions and a priori values were selected for both the 5 and 9-tap configurations based on the initial radar-based Mach number and the noise observed from the pressure transducers during static calibrations tests. Scale factors and biases were initially assumed to have no error and therefore “tight” (highly certain) a priori conditions were set for these values, which reduced the problem to solving only for the flight parameters at each time step (1 “global” iteration).

Pressure residuals were minimized, though noise becomes a larger component of the signal as time progresses and Mach number decreases.

Figure 7.4: Pressure residuals for 5-tap case

For comparison the POST-computed estimates for flight angle and Mach number were plotted along with the converged solution.

Figure 7.5: Converged Mach plot for 5-tap case

Mach number agrees very well with both the POST simulation, radar data, and isentropic/normal shock estimates. Note that for Mach number above 2.5, there appears to be an oscillation in the converged solution. This is likely due to the exclusive use of CFD in developing the aerodynamic database above Mach 2.5. This section of the database does not include the interactions captured during wind tunnel tests, which the rest of the database is based on.

Results for the 5-tap case appear to confirm the flight angle observations made from the pressure signal plots.

Figure 7.6: Flight angles for the 5-tap case

The trim angle of attack appears to be approximately negative 22-degrees until the CEV-PTM become subsonic, at which point the trim angle of attack becomes negative 10-degrees. Trim sideslip angle appears small, approximately 1-degree, and the magnitude of the oscillation increases as the test article transitions into a subsonic flight regime and dynamics begin to overcome aerodynamic forces.

The 9-tap case also showed very good agreement between the radar-based Mach number and the converged solution. As seen for the 5-tap case, pressure residuals were minimized, though noise becomes a larger component of the signal as time progresses and Mach number decreases.

Figure 7.7: Pressure residuals for 9-tap case

Figure 7.8: Converged Mach plot for 9-tap case

The trim angles of attack are negative 22-degrees (supersonic) and negative 15-degrees (subsonic), which are slightly different from the values obtained for the 5-tap case, but overall they follow a similar trend.

Figure 7.9: Flight angles for the 9-tap case

Unlike with the 5-tap solution, a significant drift is observed in the trim sideslip angle solution. To allow a direct comparison to the values obtained using the 5-tap data, the solution method was carried out on the same taps using the 9-tap data (tap #1, 2, 3, 4, and 5); the data obtained from the inner ring of four taps was omitted.

Figure 7.10: Mach number solution obtained using outer cruciform of 9-tap case

Figure 7.11: Flight angle solution obtained using outer cruciform of 9-tap case

Trim values and trend are similar to the 5-tap case, though the amplitudes different; this is not unexpected as these are results from two different test articles. These results imply either a significant shock-induced bias/scale factor in one of the inner set of taps (shock-induced as all pressure transducers read atmospheric pressure before firing), or a weighting effect that resulting from the lack of any significant lateral pressure differential, due to a small variation in sideslip angle, as seen in the method validation.

7.3 Parameter Estimation With Scale Factors and Biases

The a priori uncertainties on the scale factors and biases were loosened to the values used during method validation and applied to the 5 and 9-tap flight data.

Figure 7.12: 5-tap pressure residuals and Mach number solution with scale factor and bias estimates

Figure 7.13: 5-tap solution for flight angles with scale factor and bias estimates

Figure 7.14: 9-tap pressure residuals and Mach number solution with scale factor and bias estimates

Figure 7.15: 9-tap solution for flight angles with scale factor and bias estimates

Both sets of estimated parameters seem similar in both amplitudes and trends, though the pressure residuals are further minimized due to uncertainties applied to the scale factors and biases. As the standard deviation of the flight parameters is computed at each time step, they were compared to the defined a priori uncertainty values.

Figure 7.16: 9-tap standard deviations for flight parameters

The uncertainty in sideslip angle rapidly approaches the boundary set by the a priori standard deviation. This means, assuming no failure of the pressure transducers, that

- There is simply not enough lateral pressure differential to reliably compute sideslip angle

- Equally weighting pressure taps along the lateral line results in a solution where “weak” data sources (taps 7 and 9) distort the solution obtained using “strong” data sources (taps 3 and 5).

Based on the seemingly reasonable pressures output by the taps, it is unlikely that any of the taps failed in-flight or were faulty. Uncertainties on the pressure transducers were altered to effectively weight the “strong” data sources more than the “weak” data sources in obtaining a solution.

Table 7.1: Uncertainty of pressure transducers

	Tap #1	Tap #2	Tap #3	Tap #4	Tap #5	Tap #6	Tap #7	Tap #8	Tap #9
σ (Pascals)	2000	2000	2000	2000	2000	5000	10000	5000	10000

Figure 7.17: Pressure residuals for 9-tap case with weighted pressure uncertainties

The magnitude of the pressure residuals is the same despite the increased uncertainty.

Figure 7.18: 9-tap solution for Mach number with weighted pressure uncertainties

Figure 7.19: 9-tap solution for flight angles with weighted pressure uncertainties

Note that all of the the estimated parameters more closely resemble the results obtained for the 5-tap case and the simulations, while still minimizing the pressure residual. As before, the uncertainties were also computed.

Figure 7.20: 9-tap standard deviations for flight parameters with weighted pressure uncertainties

This confirms the weakness of the solution of sideslip angle due to the increased effective uncertainty of the lateral pressure differentials. The “global” loop completed in 2 iterations, converging on solutions for scale factors and biases.

Table 7.2: 9-tap scale factors and biases with weighted pressure uncertainties

	Tap #1	Tap #2	Tap #3	Tap #4	Tap #5	Tap #6	Tap #7	Tap #8	Tap #9
Scale Factor	0.995	0.996	0.993	1.008	0.993	1.005	0.982	0.976	1.003
Bias (Pascals)	-5867.6	3438.8	382.2	4238.3	590.7	3100.5	28.8	9716.8	-5120.4

Note that the biases are on the same order of magnitude as the noise in the pressure signal, and as a result the scale factor has a more significant effect on the solution. These estimated values are within the manufacturer specifications: scale factor of 0.1% and bias of 17,237 Pascals (2.5psi) [20].

7.4 CEV-PTM Aerodynamic Coefficients

Aerodynamic coefficients were computed for the 9-tap test article, using the solution case with estimated scale factors and biases and the weighed pressure uncertainties and the forces and torques computed using the accelerometer and angular rate gyroscope data.

Figure 7.21: Pitch and yaw rate time derivatives

Note the distinct shifts in dynamics corresponding to supersonic and subsonic Mach numbers. This is also seen in the coefficients.

Figure 7.22: Pitch and yaw rate time derivatives

As observed in the solution, there is increased uncertainty in lateral parameters, like sideslip angle. This resulted in an increased uncertainty in yaw moment coefficient computation. The yaw moment coefficient is weakly coupled to Mach number, and in turn dynamic pressure, unlike the pitch moment coefficient.

Table 7.3: Converged aerodynamic parameters for CEV-PTM

	α_{trim} (degrees)	β_{trim} (degrees)	C_{m0}	C_{n0}	$C_{m\alpha}$	$C_{n\beta}$	$C_{m\dot{\alpha}}$	$C_{n\dot{\beta}}$
1.1<M<2.5	-20.5	1.2	0.05	-6.89e-4	-0.17	0.16	-3.65e-6	-2.44e-4
M<0.9	-16.3	-0.5	0.13	-0.04	-0.18	0.18	-1.53e-4	-8.72e-5

The converged stability values imply both longitudinal and lateral static stability. The supersonic trim angle of attack is relatively close to the designed supersonic trim value, 22-degrees. The nonzero trim value of the sideslip angle implies lateral mass offsets. The moment arm necessary for such a trim value is within the measurement tolerance of the center of gravity location. The stability derivatives due to pitch and yaw velocities are quite small, though they have a greater effect on the pitch and yaw moments at higher Mach numbers as the time derivatives of angle of attack and sideslip angle are larger for high Mach numbers.

Chapter 8

Conclusions

Aeroballistic testing with on-board sensors, including pressure transducers, yields high fidelity data that may be used to determine aerodynamic parameters at relatively little cost. Future testing may benefit from some minor changes:

- Multiple 9-tap tests at repeatable orientations, some with equivalent angle of attack and sideslip angle magnitudes
- Documentation of in-bore orientation
- Higher exit velocity, and the incorporation of higher Mach numbers in the analytic database

At the time of this publication, independent efforts at both NASA and Arrow Tech to reconstruct the CEV-PTM trajectories and obtain aerodynamic coefficients had not been completed. Comparison between the two test shots and simulations indicates that the obtained values are at least reasonable.

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