ABSTRACT

JOSHI, AMEYA CHANDRAKANT. Batch Sedimentation in an Impulsively Heated System. (Under the direction of Dr. Thomas Ward.)

Suspensions of nearly mono-disperse spheres, subjected to impulsive-constant temperature heating from below, are studied in a batch sedimentation process. Experiments are performed on suspensions with a range of concentrations and different temperatures, to analyze the effects of varying temperatures on the rate of settling, shock formation and shock velocities. CCD imaging is used to observe and study the settling phenomenon. The Kynch theory of sedimentation (Trans. of Far. Soc., 1952) explains shock formation in low concentrations suspensions and states that particle speed is completely determined by the local density only. We extend this study to include both heating and high concentration suspensions. Higher concentration suspensions (greater than 20%) are characterized by a single shock. Two distinct shocks are observed for all the experiments involving heating from bottom with an exception for the non-heated setup where only one shock is observed. The experiments suggest that other models may be more appropriate for high concentration studies. Shock merge times and shock settling times for different temperatures and concentration as a measure of the speed of settling have been looked at in detail. The energy equation is solved for, using a finite difference approach to simulate the temperature profiles in the settling suspension. The jump condition between the solid-liquid interface (temperature shock) and the transient temperature analysis have been studied in detail. Results suggest that, high concentration suspensions characterized by a single concentration shock, settle down as a column with a linear temperature profile. This suggests that the thermal conductivity is uniform throughout indicating a constant concentration.

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Batch Sedimentation in an Impulsively Heated System

by
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DEDICATION

I dedicate this work to my parents, my brother and my sister-in-law, for their endless love and support. I would also like to thank my friends and room mates, here at NC State who have been like my family, creating a home away from home. Without the support and love of all these people I would not have been the person I am today.

BIOGRAPHY

Ameya Chandrakant Joshi was born on 17 February 1985 in Pune, Maharashtra, India. From his early childhood, he had a strong passion towards machines. His inquisitiveness led to him opening up his toys just to get to know how they worked, inviting the wrath of his parents. This interest and passion towards machines, grew over his schooling years and developed towards a career in Mechanical Engineering. He completed his Bachelor's in Mechanical Engineering from Pune University, Pune, India in 2007. By the time he graduated he had secured a job in Cognizant Technology Solutions, India, Pvt. Ltd. However after joining the corporate world, he realized that there was much more to gain from a higher degree. In August 2008, he resigned from the company and started pursuing his Masters degree in Mechanical Engineering at the North Carolina State University.

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Chapter 1

Introduction

Batch sedimentation is a solid-liquid separation process, in which particles separate out from the suspending fluid due to buoyancy effects. Particle laden suspensions have always been of great interest due to their presence in applications like the transport of slurry [1], separation of oil from tar sands using steam assisted gravity drainage (SAGD) technique [2] and also in the chemical industry [3]. The sedimentation process though, it appears fundamental is one of the defining problems in the SAGD process, used to separate out sand from tar-sands.

SAGD is an enhanced oil recovery technology to separate sand from heavy crude oil. Tar sands are a viscous mass analogous to peanut butter which cannot be pumped using centrifugal or reciprocating pumps. The process of separation of sand from tar-sands to get crude involves pumping of superheated steam into the well to reduce the viscosity. The viscosity of the oil decreases due to heating by the superheated steam, allowing migration of sand to the bottom of the well. Another well is then drilled into the reservoir to pump out the oil and condensed steam mixture. To reduce the residence time and to aid separation natural gas is sometimes pumped in the well and then ignited to raise steam temperatures. However to get energy in the form of crude some other form of energy needs to be expended to generate steam. For obvious economic reasons the energy spent to generate steam should be much lower as compared to the energy gained from the reservoir. This calls for the need to understand the exact nature

of the fundamental process of settling under the effect of heat. The effect of gravity on the size of particles, the concentration and the physical properties of suspending fluid all play an important role in determining the rate of settling of particles and allied phenomenon. Studying these has always been of great importance to the scientific community as the sedimentation process find applications in most of the industrial processes. Fig. 1.1 shows the SAGD process.

Figure 1.1: Steam assisted Gravity Drainage (SAGD)(Image courtesy:www.sagd.net)

1.1 Literature Review

Studies on batch sedimentation have been carried out in detail, however most of them have been theoretical or based on numerical analysis. G. J. Kynch is recognized as the pioneer in the study of batch sedimentation and was the first to propose a theory. In his paper Kynch [4] uses the method of characteristics to develop a relationship between the speed of fall of a particle in a dispersion and the local fluid properties. Considering the suspension as continuum he represents the continuity equation as a function of the local concentration ϕ and the settling velocity of the solid phase. The concentration ϕ is defined as the volume of the glass particles by the total volume ($V_l + V_s$) where V_l and V_s is the volume of the liquid and the solid phase respectively. A change in the particle concentration ϕ over a small but finite change in height is characterized as a shock. Kynch in his paper discussed three different modes of settling shown in Fig. 1.2

In Fig. 1.2 we clearly see two shocks $S1$ and $S2$ in all the three modes of settling. The only difference being the presence of concentration gradients. Fig. 1.2a shows three distinct zones separated by the two shocks are observed, with the clear liquid region above $S1$ and a region with constant concentration (equal to the initial concentration ϕ_0) between $S1$ and $S2$ and, the concentration approaching maximum packing below $S2$. Fig. 1.2b also shows two shocks but with a concentration gradient below $S2$ after which the concentration approaches the maximum packing limit. Fig. 1.2c shows concentration gradient in between the shocks closer to the region where the concentration approaches maximum packing. However all these modes of settling have been defined for very low to moderate concentration suspensions.

Since the work of Kynch there have been few modifications of the existing theory to accommodate particle sedimentation at large concentration where it is difficult to determine concentration gradient information. Indeed, it may be possible that particles sediment without gradients at large mixed concentrations since in the limit $\phi(x, t = 0) \rightarrow \phi_{max}$ (where ϕ_{max} is the maximum packing limit) we must have $\phi(x, t) \approx \phi_{max}$ which suggest that $\frac{d\phi(x, t)}{dx} = \frac{d\phi_{max}}{dx} = 0$ in the particle rich region. This suggest that there are no gradients in ϕ to leading order

Figure 1.2: Modes of settling described by Kynch in his paper, MS-1 Showing two distinct shock, MS-2 Showing 2 shocks but with a concentration gradient and MS-3 showing only one shock.

in this limit and the particles settle by collapse, that is $\phi = \phi(t)$. Again, this idea is difficult to prove because optical or spectrographic techniques for measuring concentration gradients in a dense suspension is difficult. Brenner et al. [5] study the effect of diffusivity in on the speed of the settling fronts and extend this to dense suspensions. Ward et al. [6] observed that with increasing concentration of the suspension the flocculation period changes even at constant temperature. They propose a correlation for the initial settling height and the ultimate settling height of the sediment. The formation of shocks and the settling velocities during batch sedi-

mentation have been analyzed in detail by Rao et al. [7]. These studies however, deal on with low concentration suspensions only, and at a constant ambient temperature. Fig. 1.3 shows the

Figure 1.3: Schematic for batch sedimentation of uniformly mixed suspension. The system is characterized by the formation of two concentration shocks (top and bottom)

schematic of the sedimentation process. The sudden change in concentration is visible to the naked eye. This is also termed as a concentration or a kinematic shock, but will be referred to as a shock henceforth. The system shown is characterized by the formation of two shocks. However at higher concentrations only one shock is formed. As the particles in the suspension settle down concentration gradients can form along the height of the settling column. For systems with a single shock, we propose that heat transfer will be distinct from a system where two shocks occur, since two-shocks suggest concentration gradients that would modify the flux terms in the standard energy equation, where the flux is given by $q = k(\phi)\nabla T$, for this system. The thermal conductivity, $k(\phi)$ will have a markedly different form if the sedimentation were

governed by concentration gradients (denoted by two-shocks) where $k(\phi(x, t))$, versus a single shock, where $k(\phi(t)) = k(t)$. Solving the full system of equations for the mass, momentum and energy conservation, i.e. with particle sedimentation would require novel numerical techniques and is outside of the scope of this study.

1.2 Problem Motivation & Numerical model

We propose to experimentally study the effect of the addition of energy by impulse heating of a viscous suspension, on the shock formation and allied properties in detail using standard optical techniques. We solve only the energy equation in the experimental domain to simulate the temperature profiles. This will enable us to understand the nature of particle movement and the flocculation periods in the viscous suspension and the underlying effects at different temperatures. The authors acknowledge that there are more sophisticated optical techniques such as NMR and X-ray, but believe that relevant qualitative and quantitative data may be collected with our experimental setup which is described in detail in the following sections. The fundamental equation governing particle sedimentation in viscous suspensions can be summarized as below.

$$\frac{dV_{particles}}{dt} + U_{shock} \cdot A = 0$$

Where,

$V_{particles}$ = the volume of the solid particles.

U_{shock} = Velocity of the moving shock.

Dividing through by V_{total}

$$\frac{d\phi}{dt} + U_{shock} \cdot \left(\frac{A}{V_{total}} \right) = 0 \tag{1.1}$$

Now, the shock velocity U_{shock}

$$U_{shock} = \frac{dh}{dt}$$

where, h is the displacement of the shock and $\frac{A}{V_{total}} = \frac{1}{h_{max}}$

$$\frac{d\phi}{dt} + \frac{1}{h_{max}} \cdot \frac{dh}{dt} = 0 \quad (1.2)$$

integrating Eq. 1.2 we get,

$$\frac{\phi(t)}{\phi_{max}} = \frac{-1}{h_{max}} \cdot \frac{h(t)}{\phi_{max}} + c \quad (1.3)$$

using the boundary conditions, at $t = 0$, $h(t) = h_{max}$ and $\phi(t) = \phi_0$

where, ϕ_0 = initial concentration.

$$c = \frac{\phi_0 + 1}{\phi_{max}}$$

Eq. 1.3 becomes

$$\begin{aligned} \frac{\phi(t)}{\phi_{max}} &= \frac{-1}{h_{max}} \cdot \frac{h(t)}{\phi_{max}} + \frac{\phi_0 + 1}{\phi_{max}} \\ \phi(t) &= \phi_0 + 1 - \frac{h(t)}{h_{max}} \end{aligned} \quad (1.4)$$

The parameter $h(t)$ i.e. the shock displacement in Eq. 1.4 needs to be determined. $h(t)$ is a function of time and hence can be referred to as the shock velocity. This shock velocity defines and determines the rate of change of concentration below the shock. The formation of concentration gradients creates temperature gradients in the suspension.

The theory suggested by Kynch [4], proposes a correlation for the velocity of the shock given by

$$v = \left(1 - \frac{\phi}{\phi_0}\right) \cdot u$$

where, u is the well-known Stokes velocity for single particle falling in a fluid.

However this expression is valid only for low concentration suspensions. At higher concentrations the validity of this expression has not been verified. As the initial concentration of the suspension approaches maximum packing the velocity will depend upon the concentration and, in our system also on the impulse heating temperature.

A simple experimental setup was built to study particle dynamics and shock movement. Experiments involve the use of image analysis techniques to monitor particle fronts. Images were recorded at specific time intervals and then analyzed using an in-house Matlab code. A correlation for the shock velocity as a function of concentration and temperature is then proposed using the experimental data. The velocity is then substituted in Eq. 1.4 to solve for the concentration in the suspension. This solves the continuity or the mass transfer part of the problem. The energy part is then solved for by using the data and the parameters obtained from the solution of the continuity equation. Temperature profiles were simulated by solving the energy equation for which an in-house code was developed. Results obtained from the experiments and those obtained from the simulation have been presented separately and compared for temperature values.

This might give us an idea as regarding the development of temperature gradients in the SAGD process and might help us to further improve it, enhance the separation rates and reduce residence time.

Chapter 2

Experiments

2.1 The Setup

The setup consists of a borosilicate beaker, a CCD camera and thermocouples giving a digital read out. CCD imaging was used to study and analyze the shock locations formed during settling of an initially uniform suspension of glass particles ($\rho_s = 2.5 \frac{g}{cm^3}$) in silicone oil ($\rho_L = 0.94 \frac{g}{cm^3}$). The viscous solution was placed in a borosilicate glass beaker, which sat on a hot plate. A constant-temperature-automatic-cut off type hot plate was used to heat the suspension. It was desirable that the fluid viscosity remains stable. The physical properties of the fluid and the particles are summarized in Table 2.1. Three sets of concentrations of initial well mixed solutions of 20%, 40% and 50% were used for the experiments, considering a maximum packing factor of 0.60 which was measured using similar materials [8]. The initially well mixed solution was allowed to settle down and the settling fronts or shocks were recorded. External lighting was employed to illuminate the glass particles. The images were analyzed using MATLAB to study the different properties of the sedimentation phenomenon such as the time for initial formation of the shock(s), the shock velocities and the shock location at different time instances. The fluid used for the experiments was a 1000 cSt silicone oil (Clearco Products Company) mixed with Grade 9 GSB glass particles (Ceroglass Technologies Inc.) having average diameter of 300

Table 2.1: Parameter Chart

Parameter	Value
Density of Silicone oil (ρ_L) at 25 °C	940 $\frac{kg}{m^3}$
Kinematic Viscosity of Silicone oil (ν_L) at 25 °C	1000 cSt
Flash point of Silicone oil (T_{flash})	315 °C
Average Diameter (d) of Glass Particles	300 μm
Specific Gravity (ρ_s) of Glass Particles	2.45 to 2.50 $\frac{g}{cm^3}$

μm . The particles are rigid glass beads 95% of which are spherical and have the same size. Experiments were carried out when the suspension was maintained at room temperature of 25 °C and also impulsively heated from the bottom at temperatures of 100 °C, 150 °C, 200 °C and 250 °C. Images were recorded at regular time intervals of 3 seconds, 5 seconds or 10 seconds depending upon the concentration and the heating temperature. Measurements were made at pixel level of the images taken over the course of the experiment. A thermocouple probe was placed in the suspension just below the top interface to measure the steady state temperature. External lighting was used to ensure uniform light distribution over the suspension.

2.2 Procedure

Silicone oil and the glass particles exhibit fairly stable characteristics over the range of the temperature over which the experiments were carried out as stated in the manufacturers catalog. The suspension was made uniform and homogeneous, by vigorous stirring without entraining any air. The hot plate was heated to the required temperature until such time that its temperature remained uniform at the selected values. The borosilicate glass beaker containing the suspension was then placed on the hot plate with the thermocouple probe being placed just below the suspension-air interface. The glass particles (sediment) were allowed to settle down completely, carefully noting the time for the experiment to reach completion and ensuring that the fluid above the bottom interface was clear and free of particles. Fig. 2.1 shows the schematic of the experimental setup.

Figure 2.1: Schematic of experimental setup and region of interest used in the analysis

2.3 Operating Parameters

For each setup the ambient temperature was measured to be 25 °C. The experiment was run till such a time that when there are no particles floating in the solution. A dry run was conducted each time to measure the approximate amount of time for the particles to settle down, and also to ensure uniformity. It took approximately 8 to 12 hours for the particles to settle down completely, with variation depending upon the impulse heating temperature and the initial concentration. A temperature range of ambient to 250 °C was selected, based on the breakdown temperature of the oil, which could affect settling rates. The viscosity changes are specified to be linear by the manufacturer. For the selected temperature range the change in the viscosity of silicone oil is in the range of 1000 cSt at 25 °C to 100 cSt at 250 °C. Though

the viscosity change is large the fluid remains stable and in the safe operating range specified by the manufacturer.

2.4 Experimental Analysis Technique

In our analysis we use MATLAB to analyze the images taken by the camera. The code analyzes the images for particle motion using pixel values and calculates motion associated parameters such as shock movement, velocity, time for shock formation and time required for the shock to move down after formation. In the code each image is read as a 1040 x 1392 matrix with light intensity (pixel values) for each location. Pixel values have a range from zero to 255; 255 representing the highest intensity and zero indicating complete darkness or zero intensity. All calculations in our analysis are based on average pixel values measured along the width of the glass beaker. Each horizontal level is represented by an average pixel value which corresponds to the average concentration of the suspension at that level. Fig. 2.1 shows the region of interest (in gray) selected by specifying the matrix indices at the limits shown by arrows, such that each successive row index specifies the next level of in the suspension. The pixel values are averaged horizontally along the fluid level to report one value for the concentration for that vertical location. This was done for all fluid levels in the region of interest. Our analysis is based on the averaged values of concentration (pixel values) reporting changes in the value at different time instances, i.e. in different images, as the particles migrate downward. The entire system was placed in front of black colored construction paper. Hence when the illuminated glass particles settle down the clear fluid appears dark to the code due to the black background and the code can clearly distinguish between the clear fluid and the suspended particles.

The total height of the fluid h (refer to Fig. 1.3 in Chapter 1) in the beaker was monitored. A negligible change was observed in the volume of the suspension during the course of the experiment and hence the net change could safely be neglected. The in-house MATLAB code locates the shock(s) and calculates the velocity at which the particle fronts move. The selection

of the region of interest (see Fig. 2.1) gives us one more advantage, it reduces the optical distortion generated due to the curved nature of the glass beaker. This eliminates the need to use a square glass tank which requires considerable manufacturing effort. For the analysis the following assumptions were made -

1. The concentration at any horizontal layer is equal to the average concentration along the same layer.
2. Only one dimensional migration of particles has been considered.
3. There is no mass transfer between the solid and the liquid phase, and both the phases are incompressible.
4. Wall effects and optical distortions have been minimized by selecting appropriate regions for analysis, and hence may be neglected.

However the procedure still has an inherent limitation of changing light conditions for each setup. Changes in the external lighting creates difficulty in direct comparison between two experiments. To counter this a simple scaling technique has been developed for scaling both the height i.e. the shock locations, and also the concentration. All the heights used in our analysis have been scaled by the maximum height h i.e. the height of the air fluid interface. h_1 is the position of the top shock from the bottom surface and h_2 is the position of the bottom shock from the bottom surface when visible. For accurate comparison the shock location plots have been scaled in a unique way by keeping the final shock location as the reference point for comparison. Another limitation that comes up when analyzing high concentration suspensions is, the shadow effect of the suspended particles. For accurate location of the shock the shock interface needs to be well illuminated. However at higher concentrations the particles at the top cast a shadow on the particles below giving erroneous results from the code. This has been corrected by employing a small optical correction in the code.

2.5 Numerical Analysis

The data obtained from the image analysis was used to estimate the velocity of the top shock, for which a correlation is proposed. The proposed correlation was then used in the numerical analysis to calculate the shock locations and hence the concentration gradients.

The one dimensional energy equation was solved for in the domain using finite difference approximations to simulate the temperature gradients. The energy equation and other allied parameters were non-dimensionalized to scale all the data on a scale from 0 to 1. The central difference scheme which is second order accurate in space was used. 4-stage Runge Kutta method was employed for time integration to control the accumulation of errors, due to the small time step required for the analysis. The time for the shocks to form and also settle down was analyzed using the code for image analysis which was then used to calculate the required parameters for the numerical analysis. This eliminates the need to solve for the concentration i.e. the mass conservation equation and which reduces the complexity of the code and the analysis. Concentration gradients formed have been looked into experimentally and the data obtained used for the numerical analysis.

Results have been compiled and analyzed in separate sections for the image analysis and for the numerical analysis, the latter being dependant on the former for transport parameters. Following which the results have been discussed in detail explaining parameter interdependencies.

Chapter 3

Results from Image Analysis

Our analysis consists of two parts -

1. Estimation of fluid parameters - concentration gradients, settling speeds and particle migration time using image analysis.
2. Numerical analysis which solves for the energy equation in the experimental domain - for which fluid parameters obtained from the image analysis were used to estimate transport phenomenon without solving for the continuity equation.

Results first discuss the fluid parameters obtained from the image analysis and the MatLAB code written for the same. Separate sections for qualitative results explaining the nature of the problem and quantitative analysis giving actual numerical parameters have been presented for better understanding. Following this results for the numerical analysis have been presented with direct comparisons between the experimental values and the values obtained from the simulations for temperature profiles and probe temperatures.

3.1 Qualitative results

3.1.1 Images

Fig. 3.2 shows the different stages in the settling process for an initially uniform concentration, $\phi = 20\%$, and with plate temperature of 150 °C. Each successive image in Figs. 3.2(a)-(f) represents a later time instant. The last image, Fig. 3.2(f), is at the latest time instant, when all the noise has settled down and one clear shock is visible. The heavier particles settle down initially to the bottom of the beaker with the local concentration in that region approaching the maximum packing limit. Just above this layer, or front, is the well mixed suspension having a concentration which is at the initial average concentration, followed by a region of clear liquid (oil) with no particles in it (refer to Fig. 3.2(c)). At the phase interfaces, discontinuities in concentration and velocity are observed. These two discontinuities or shocks approach each other and then merge to form a single shock (Fig. 3.2(f)) after sedimentation is complete. The concentration below the final shock corresponds to maximum packing. Fig. 3.1(a)-(f) shows the settling stages from a concentration $\phi = 50\%$ at a plate temperature of 150 °C. A one-to-one comparison of individual images in Fig. 3.2(a)-(f) and Fig. 3.1(a)-(f) shows the distinct difference in the number of shocks formed with our optical technique. The particles settle down as one column resulting in only two distinct regions, a clear zone and a concentration zone where the concentration ϕ approaches maximum packing. The experiments are performed using a high

Table 3.1: Number of shocks observed for given data set

Plate temperature	Concentration		
	20%	40%	50%
Room Temp	1	1	1
100 °C	2	1	1
150 °C	2	2	1
200 °C	2	2	1
250 °C	2	2	1

viscosity silicone oil (1000 cSt) and glass beads ($300\text{ }\mu\text{m}$) to generate a nearly mono-disperse mixture. The system is initially well mixed by stirring in a glass beaker over a long period of time. Then the mixture is placed on a hot plate and images of the sedimentation are recorded using a CCD camera. Qualitative observations of the sedimentation suggest the presence of either one or two concentration shocks, depending on the initial concentration and hot plate temperature. As the plate temperature is increased the top shock is observed to sediment over a shorter elapsed time period. At the largest temperatures used in the experiments the sedimentation rate of the top shock actually increases relative to the other experiments, suggesting that there is a optimal operating temperature.

Quantitative measurements are performed by analyzing the images. These measurements include the shock location as a function of time, the top shock velocity, and the time required for the impulsively heated system to reach a steady state temperature. A correlation for the top shock velocity as a function of the temperature and concentration is proposed for the range of temperature where the top shock velocity is seen to decrease with temperature.

In the future it will be beneficial to perform additional experiments varying the particle size to determine their effects on the top shock velocity. Also it may be useful to perform experiments at temperatures that are greater than the ones discussed in this manuscript to see if the observed trends continue at the higher temperature. It also may be interesting to observe the origins of the sedimentation rate decrease through particle tracking, possibly utilizing laser fluorescence technology.

Figure 3.1: Stages in settling of a viscous suspension at $\phi = 20\%$ and 150° (actual images), arrows indicate the two shocks observed. Elapsed times are (a) $\Delta t = 0\text{s}$ (b) $\Delta t = 675\text{s}$ (c) $\Delta t = 1311\text{s}$ (d) $\Delta t = 1584\text{s}$ (e) $\Delta t = 2166\text{s}$ (f) $\Delta t = 8500\text{s}$

Figure 3.2: Stages in settling of a viscous suspension at $\phi = 50\%$ and 150° (actual images), arrows indicate the only shock observed. Elapsed times are (a) $\Delta t = 0\text{s}$ (b) $\Delta t = 1410\text{s}$ (c) $\Delta t = 3030\text{s}$ (d) $\Delta t = 4480\text{s}$ (e) $\Delta t = 5780\text{s}$ (f) $\Delta t = 9500\text{s}$

3.1.2 Number of shocks vs. concentration and temperature

The number of shocks is determined by analyzing the images over the complete time to settle t_m . Any observation of two shock, even at late times in the experiment, is denoted as a two shock experiment. The formation of two shocks is clearly seen for few sets of results while only one shock is observed for the remaining ones. The number of shocks formed depends upon the concentration and also the impulse temperature. As stated earlier three representative concentrations of 20%, 40% and 50% have been selected for analysis. Table 3.1 states the number of shocks observed in all conducted experiments. With increasing concentration the number of shocks observed reduces to 1 as compared to the two shocks observed for the 20% concentration. For the 40% case at 150 °C the bottom shock forms at later instant, just before the top shock reaches the final location. The number of shocks formed is critical to the concentration gradients formed in the suspension during settling. For the two shock phenomenon the concentration in between is not uniform and calls for a different approach as compared to the single shock phenomenon.

3.2 Quantitative Analysis

Fig. 3.3 shows a direct comparison between the image and the concentration plot generated by the code. The vertical axis of the plot corresponds to the suspension level in the experiment and the horizontal axis corresponds to the scaled concentration values in the suspension. Solid, black horizontal lines indicate the top and the bottom shock in the actual image. These two lines which are shown running through the concentration plot indicate a sudden change in the concentration at the points which we term as a shock. This image gives us an actual understanding of the concentration plots generated by the code. The figure shows the shock location for time $t = 1300$ s the curve with the diamond markers($\blacklozenge$). In the plot the concentration above the top shock is shown to be zero even though the actual image shows some particles

Figure 3.3: Comparison of actual images with scaled plot generated by Matlab code for $\phi = 20\%$

still in suspension which we term as noise. A direct comparison of the concentration plot in Fig 3.3 and Fig. 1.2 in Chapter. 1 shows that MS-3 in Fig. 1.2c matches with the plot generated by the image analysis. This is at a concentration of $\phi = 20\%$. Hence we can justify that the results obtained from our image analysis match with those in the Kynch theory. The code written for the image analysis locates the shocks based on the pixel values in each image. To avoid introduction of this noise in results generated by the code the pixel values below a specific threshold value have been neglected to calculate the shock locations. Doing so clears the noise in the data.

Fig. 3.4 (a) and (b) show the local concentrations in the fluid for $\phi = 20\%$ at room temperature and $250\text{ }^{\circ}\text{C}$ respectively at different time instances. Only one discontinuity is observed in the case of the 20% concentration at room temperature (no impulsive heating), as compared to two discontinuities observed in the heated case at $250\text{ }^{\circ}\text{C}$ for the same concentration. The local concentration plots for the 40% setting and the 50% setting are similar to those of the 20% setting with no heat, with an exception for the 40% case. For effective comparison the plots in Fig. 3.4 have been scaled. Concentration was scaled using the maximum packing at the bottom and the height scaled by the total height of the settling column.

Fig. 3.5 (a) and (b) show the location of the top and the bottom shock as function of time at two representative temperatures of $25\text{ }^{\circ}\text{C}$ and $150\text{ }^{\circ}\text{C}$. Solid lines indicate top shock and dotted lines indicate the bottom shock. For the unheated case (plate temperature equal to the room temperature) the bottom shock fails to form. The suspended particles settle down as one single column forming only one observable shock at the top. The horizontal line at level 1 is the location of the merged shock. Data has been scaled, to precisely have the same final shock location for comparison. In the 20% and 40% setting the top shock forms immediately and starts accelerating downwards. For the 50% setting the top shock takes a longer time to form and then translate. Both the shocks take longer time to form with increasing concentration and temperature. At plate temperature of $150\text{ }^{\circ}\text{C}$ a significant change in settling times is observed. For lower concentrations two shocks are observed when heated impulsively at temperatures

about 50 to 60 °C above the ambient.

Table 3.2: Time required for surface temperature to reach steady state and the shocks to merge or for a single observable shock the time required to settle down

Plate temperature	Steady state time(s)			Shock merge time(s)		
	20%	40%	50%	20%	40%	50%
room temp	-	-	-	2308	9850	19980
100 °C	4975	10860	8620	2135	8110	10850
150 °C	4900	7410	7130	1890	6915	8700
200 °C	4854	9220	6100	1414	4010	8040
250 °C	4729	10040	7060	2214	5045	9510

3.3 Discussion

The plots shown in Fig. 3.4(a) and (b) show the local concentration in the liquid. The figures show the location of the shock for both cases, one shock for the 20% concentration no heat setting and two shocks for the 20% with 250 °C impulse heating temperature. The data in Table 3.1 tells us that the number of shocks formed is a function of both, the concentration and the impulse temperature. As the plate temperature is increased the resultant motion of the top shock is composed of two contributions, that is, the downward motion due to gravity and the build up of suspended particles at the bottom. For the two shock case the settling column is divided into three distinct regions (refer Fig. 1.3 and Fig. 3.2). The top region or layer comprises of clear fluid, the intermediate region of suspension with the concentration values which are intermediate to the initial well mixed values and the maximum packing. The bottom region is the sediment with concentration approaching maximum packing. Fig. 3.5(a) and (b) gives a comparison of the shock location in the settling column as a function of time for the selected concentrations. The solid lines indicate the top shock and the dotted lines indicate the bottom shock, with the markers representing different concentrations. Fig. 3.4(a) for the room temperature setting, where the plate temperature is equal to the room temperature, the

Figure 3.4: Local concentration in the settling column (a) plate temperature = 25 °C (b) plate temperature = 250 °C for $\phi = 20\%$ with elapsed times as indicated in the legend. (a) shows the system with a single shock where there is a rapid change in pixel intensity over a small distance $\frac{h_{actual}}{h}$. (b) shows a two shock system where there are two shocks that appear at later times.

Figure 3.5: Averaged shock location plot (a) plate temperature = 25 °C (b) plate temperature = 150 °C. —■— = 20% top shock, —●— = 40% top shock, —▼— = 50% top shock, ...■... = 20% bottom shock, ...●... = 40% bottom shock, ...▼... = 50% bottom shock

formation of a bottom shock is not observed. The particles settle down as a column forming only one shock at the top forming only two regions in the settling suspension.

A direct comparison of the plots in Fig. 3.4(a) and 3.4(b) shows the distinct patterns of settling at the two temperatures of 25 °C and 150 °C. The 20% experiment shows the formation of a distinct bottom shock, that makes a late appearance for the 40% experiment. The bottom shock fails to form for concentration above 40%. The time required for the top shock to form increases with increasing concentration. The figures also show that the rate of downward motion of the top shock is almost the same for all the concentrations. The slopes for the curves denoting the top shock were measured and were found to be in a range of 10% to 15% of each other with the exception of 50% concentration at 250 °C. The time required for the shocks to merge or for the single shock to settle down to its final location is dependent on the top shock formation time only as the rates of settling of the top shock are almost equal.

Fig. 3.6(a) shows the time required for the shocks to merge and settle down to the final location for cases in which two shocks are observed. For the setups in which only one shock is observed the time required for the shock to settle down to the final position has been measured. Data in Table 3.2 shows that as the plate temperature increases the time required for the suspended particles to settle down decreases. But at temperature 250 °C the time required for the shocks to merge or settle down increases as compared to the time for 200 °C. Fig. 3.6(a) shows a semilog plot of the shock merge time data in Table 3.2. The plot clearly shows a threshold value beyond which heating of the suspension hinders the settling of suspended particles. A further increase in the impulse heating temperature slows down the particles. In our set of experiments this critical value is between 200 °C and 250 °C., independent of concentration.

Table 3.2 and Fig. 3.6(b) give the values and the trends for the time required for the setup to reach to a steady state temperature. The value for the time required to reach steady state

Figure 3.6: a.) Time required for the temperature in the vicinity of the free surface to reach a steady temperature. b.) The time required for the top shock, h_1 , to settle

increases for the 40% setup and then decreases for the 50% setup. The time required to reach steady state increases as the plate temperature is increases. The increase in temperature creates a temperature gradient in the fluid possibly creating convection currents due to buoyancy and/or viscous effects. From this result we postulate that convective heating may be less effective in this process as compared to conduction, since it leads to increase in the time required for the setup to reach steady state. We also note that the 40% concentration experiment has a transition from 1 to 2 observed shocks as the impulse heating temperature increases. This seems to correspond with the sudden increase in the time to reach steady state temperature t_{ss} shown in Figure 3.6(b).

The slowing down of particle sedimentation is attributed to vertical convection. As the suspension heats up the local density of the fluid at the bottom of the tank decreases. This less dense fluid moves up taking particles along with it. This buoyant fluid, with particles hinders, the suspended particles moving down under the effect of gravity and decreasing viscosity due to heating.

3.4 Correlation model for settling velocity

A correlation analogous to the one used by Richardson and Zaki [9] is proposed. In Fig. 3.7 we plot the settling velocities against the scaled concentrations. Each curve represents a specific temperature at which the suspension was analyzed. The slopes and the intercepts for the curves were measured and the average slope for all the curves calculated. The intercepts were plotted against temperature and the equation for the curve calculated. This was used to include the temperature effect in to basic form of the Richardson and Zaki co-relation.

$$U_{shock} = 9.768 \times 10^{-6} \left(1 - \frac{\phi}{\phi_{max}}\right)^{\alpha} . e^{\beta T} \quad (3.1)$$

Figure 3.7: $\log(\frac{H}{U_{\infty}})$ against $\log(1 - \frac{\phi}{\phi_{max}})$. The plots appear to be linear for the three highest temperatures 150 °C, 200 °C and 250 °C with clear nonlinear behavior at the lowest temperature

where, $\alpha = 1.73492$, $\beta = 0.0067363$. The equation is valid for the range of temperatures ($100\text{ }^{\circ}\text{C} < T < 200\text{ }^{\circ}\text{C}$) and concentrations between ($0.20 < \phi < 0.50$) to within an error of $\pm 10\%$ based on our data. The shock settling velocity for different concentrations and temperatures can be plotted and the optimum value selected. The constant 9.768×10^{-6} includes the terminal velocity [10] based on the Stokes velocity for a single particle setting in a fluid.

In the following sections we simulate the temperature profile in the settling suspension by solving for the energy equation. Special boundary conditions were needed to be developed for the free surface and also the moving interface. Solutions have been proposed only for the

one shock phenomenon. Formation of the bottom shock is not observed in high concentration suspension and hence the results proposed in the following section are more relevant to the single shock problem.

Chapter 4

Numerical Analysis

This section deals with the simulation of temperature profiles in the settling suspension. The energy equation was solved in a one-dimensional domain to simulate the temperature profiles. The results obtained from the numerical simulation were then compared with actual experimental data. A detailed error analysis has then been presented in the next section to explain the reasons for the introduction of errors in the results.

4.1 Governing equation

$$\rho.C_p \frac{\partial T}{\partial t} = K_a \frac{\partial^2 T}{\partial x^2} \quad (4.1)$$

Eq. 4.1 is the generic form of the law of conservation of energy in one-dimension where, the symbols have their usual meanings. We intend to solve for this non-linear equation using a finite difference approximation for both the spatial and the temporal derivative.

4.2 Boundary conditions

Fig. 4.1 shows the complete setup of the boundary conditions for the one shock problem and the expected temperature profile. At the bottom of the plate we have the Dirichlet Boundary condition, where the temperature is equal to the plate temperature set during the experiment.

$$T(x = 0) = T_{plate} \quad (4.2)$$

Figure 4.1: Boundary conditions (a) The initial and boundary conditions. (b) Expected temperature profile in the one-dimensional domain for a sing shock problem.

At the free surface a unique method was used to develop an expression for the boundary condition. We use the steady state assumption for developing the boundary condition. For steady state the time derivative becomes zero i.e. $\frac{\partial T}{\partial t} = 0$.

This reduces Eq. 4.1 to

$$\frac{\partial^2 T}{\partial x^2} = 0$$

integrating,

$$T(x) = c_1 \cdot x + c_2$$

Using the Dirichlet Boundary condition specified by Eq. 4.2, we get $c_2 = T_{plate}$ and above equation becomes,

$$T(x) = c_1 \cdot x + T_{plate} \quad (4.3)$$

At the free surface using the Newton's law of cooling, the heat flux q is given by

$$q = hA(\Delta T)$$

$$q = hA(T_s - T_\infty) \quad (4.4)$$

where, T_s is the interface temperature

In the immediate vicinity of the free surface, heat is transferred to air by conduction in a very small region. Using the Fourier's Law for heat conduction in this region

$$q = -K_a A \frac{\partial T}{\partial x} \quad (4.5)$$

where, K_a is the thermal conductivity of air.

The heat crossing the free surface to the air through conduction should, be equal to the heat carried away by the air by convection. therefore, from Eq. 4.4 and Eq. 4.5

$$-K_a A \frac{\partial T}{\partial x} = hA(T_s - T_\infty)$$

Using the backward difference approximation for the spatial derivative at the interface.

$$-K_a \frac{\Delta T}{\Delta X} = h(T_s - T_\infty)$$

$$-K_a(T_s - T_{s-1}) = h(T_s - T_\infty)$$

$$-(T_s - T_{s-1}) = \frac{h\Delta x}{K_a} \cdot (T_s - T_\infty)$$

by definition $\frac{h\Delta x}{K_a} = Nu_x$, the Nusselt Number for air. hence,

$$(1 + Nu_x) \cdot T_s = T_{s-1} + Nu_x \cdot T_\infty$$

$$T_s = \frac{T_{s-1} + Nu_x \cdot T_\infty}{1 + Nu_x} \quad (4.6)$$

Figure 4.2: Backward difference approximation at the interface

Eq. 4.6 gives the expression for the surface temperature and can be calculated independently. Fig. 4.2 gives the schematic of the finite difference approximation for the free surface.

At the shock location, there is a discontinuity in the temperature profile. This discontinuity needs to be modeled separately to include the effect of the shock movement. At the solid-liquid interface, even though there exists a jump condition, flux will always be conserved. Using this theory,

$$-\tilde{K} \left[\frac{\partial T}{\partial x} \right]_{susp} = -K_l \left[\frac{\partial T}{\partial x} \right]_l$$

where, $\tilde{K}$ is the thermal conductivity of the suspension below the interface and K_l is the thermal conductivity of the liquid above the interface. Below the interface, the concentration of the suspension ϕ is a function of time. As the particles settle down the concentration changes with time. The thermal conductivity $\tilde{K}$ of suspension below the interface is a function of the

concentration ϕ , making the thermal conductivity a function of time. Hence $\tilde{K} = K(t)$.

$$\left(\frac{K_t}{K_l}\right) \cdot \frac{\partial T}{\partial x} = \frac{\partial T}{\partial x} \quad (4.7)$$

Eq. 4.1 is the governing equation for our problem with Eq. 4.2, Eq 4.6 and Eq. 4.7 as the boundary conditions. To summarize

$$\rho.C_p \frac{\partial T}{\partial t} = K_a \frac{\partial^2 T}{\partial x^2}$$

with boundary conditions,

$$\begin{aligned} T(x=0) &= T_{plate} \\ T_s &= \frac{T_{s-1} + Nu_x.T_\infty}{1 + Nu_x} \\ \left(\frac{K_t}{K_l}\right) \cdot \frac{\partial T}{\partial x} &= \frac{\partial T}{\partial x} \end{aligned}$$

As stated earlier the above equation was discretized using finite difference approximations. However direct coding and solving for actual values requires the use of a small time step. The central difference approximation for the spatial derivatives is highly unstable, requiring a small time step. The stability bounds for explicit integration of the one-dimensional energy equation are $0 \leq \nu \leq 0.5$ where, ν is the Courant number. For details of the stability analysis please refer to Appendix. A.1.

4.3 Development of Functional parameters

Referring to Eq. 1.4 discussed in Chapter. 1 expressions for functional parameters such as the time dependant thermal conductivity $K(t)$, the shock displacement, the time required for the top shock to settle down completely and the final location of the top shock need to be developed and discussed.

The location of the top shock as a function of time is expressed as

$$h(t) = h_{max} - v_1(\phi, T).t$$

where, h_{max} is the total height of the suspension and v_1 is the velocity of the top shock discussed in section 3.4 in Chapter. 3. The time required for the top shock to settle down denoted by t_{end} , and is determined as below. When the top shock settles down completely and there is no further migration of particles the concentration ϕ_t below the shock equal ϕ_{max} . From above expression

$$h_{t_{end}} = h_{max} - v_1(\phi, T).t_{end}$$

.

Hence Eq.Eq. 1.4 becomes,

$$\phi_{max} = \phi_0 + \frac{v_1(\phi, T).t_{end}}{h_{max}}$$

giving, t_{end} as

$$t_{end} = \frac{(\phi_{max} - \phi_0).h_{max}}{v_1(\phi, T)} \quad (4.8)$$

The thermal conductivity and the specific heat are evaluated as

$$K(t) = K_l \left(1 - \frac{\phi(t)}{\phi_{max}} \right) + K_s \left(\frac{\phi(t)}{\phi_{max}} \right)$$

$$C_p(t) = C_{pl} \left(1 - \frac{\phi(t)}{\phi_{max}} \right) + C_{ps} \left(\frac{\phi(t)}{\phi_{max}} \right)$$

therefore,

$$\frac{\partial \theta}{\partial t^*} = \frac{K^*}{Pe} \cdot \frac{\partial^2 \theta}{\partial x^{*2}}$$

4.4 Non-Dimensionalization

The governing equation, the boundary conditions and the functional parameters were all non-dimensionalized. For the detailed procedure and the non-dimensional parameters please refer to Appendix A.2. Following is the complete set of equations that were coded in MatLab to simulate the temperature.

$$\frac{\partial \theta}{\partial t^*} = \frac{K^*.K_l.\phi_{max}}{\rho.C_p.v_1.h_{max}} \cdot \frac{\partial^2 \theta}{\partial x^{*2}}$$

the parameter formed by the terms on the right hand side of the above equation is called as the Peclet number.

$$Pe = \frac{\rho C_p v_1 h_{max}}{K_l \phi_{max}}$$

At the free surface the boundary condition in the non-dimensional form is

$$\frac{\theta_s - \theta_{s-1}}{\Delta x} = -Nu.\theta_s$$

and the interface condition

$$K^* \left[\frac{\partial \theta}{\partial x^*} \right]_s = \left[\frac{\partial \theta}{\partial x^*} \right]_l$$

where,

$$K^* = \frac{K(t)}{K_l}$$

When the shock settles down a clear liquid zone is formed above it. The non-dimensionalization for this region gives a slightly different equation as the scaling parameter used for the analysis was the thermal conductivity of the liquid region K_l . Hence the equation becomes

$$\frac{\partial \theta}{\partial t^*} = \frac{1}{Pe} \cdot \frac{\partial^2 \theta}{\partial x^{*2}}$$

The above set of equations was then solved simultaneously for both the clear region and the liquid-particle region in Matlab using a finite difference method.

In the following section results obtained from the solution of the energy equation discussed above have been presented. Results have been compared with actual experimental values followed by the error analysis followed by conclusions.

Chapter 5

Results of Numerical Analysis

5.1 Transient Probe Temperature Analysis

The numerical analysis presents results in the form of temperature plots. The thermocouple probe shown in Fig. 2.1 in Chapter. 2 is used for transient temperature mapping just below the free surface. This temperature was plotted as a function of time. Actual probe locations in the images were estimated from the image analysis. The code developed for the image analysis reads in each image as a matrix, whose elements correspond to the pixel numbers. The probe is exactly located using these pixel numbers or matrix elements. The probe location is critical to getting accurate results as, the instant the concentration shock moves across the probe changes, with the position of the probe. Any deviation from the actual location will result in erroneous results.

Fig. 5.1a and b shows a comparison between the experimental temperature values measured by the probe and the temperature values obtained from the simulation. The red curve denotes the values measured by the probe in the actual experiment and the black curve denoting the values obtained from the numerical simulation. A cursory look at the plots shows that both the curves are in good agreement of each other. The code captures the shock at the correct temperature but at a later time instant.

Fig. 5.2 shows the probe temperature comparison for a plate temperature of 250 °C for 50% concentration. The simulation results show a large deviation from the experimental values.

Figure 5.1: Transient temperature comparison (a) plate temperature = 150 °C (b) plate temperature = 200 °C for $\phi = 50\%$ Both figures show a shock in the temperature profile at the instant when the concentration shock moves past the probe.

Figure 5.2: Transient temperature comparison at plate temperature = 250 °C for $\phi = 50\%$

The actual values show a plateau where the temperature remains constant after the shock has passed the probe. Then it again shows a rise and then remains constant.

Fig. 5.3 shows the probe temperature comparison for actual and simulation values for an initial concentration of $\phi = 40\%$ at a plate temperature of $200\text{ }^{\circ}\text{C}$. The simulation results show a large deviation from the actual experimental values. The experiment shows two shocks as stated in Table. 3.1 in Chapter. 3. Similar results are obtained for the 20% case at all temperatures.

Figure 5.3: Transient temperature comparison at plate temperature = $200\text{ }^{\circ}\text{C}$ for $\phi = 40\%$

5.2 Temperature profiles

The following section shows the temperature profiles obtained from the numerical analysis at the end of the experiment when the top shock has just settled down or when the bottom shock

and top shock have just merged together. It is practically difficult to measure the temperature in the suspension, to generate a temperature profile showing temperature gradients. For comparison the surface temperature measured using a thermocouple placed just below the surface is, compared with the results from the numerical simulation. The temperature values at the two boundaries are compared. Though this comparison may not be completely accurate, a comparable probe temperature value will suggest that this method provides a good first cut estimate of the temperatures in the suspension.

The results from the numerical simulation are compared with the experimental values for two important parameters.

1. The transient probe temperatures.
2. The final shock location or the displacement of the top shock from the free liquid surface.

Table. 5.1 shows the comparison between the actual and simulated probe temperatures at the selected initial concentrations and plate temperatures. Table. 5.2 presents the actual and simulated shock displacement values for the selected initial concentrations. The final shock location depends only on the concentration and not on the plate temperature.

Table 5.1: Probe temperature comparison

Plate temperature	Probe temperature					
	20%		40%		50%	
	actual	simulated	actual	simulated	actual	simulated
100 °C	51 °C	28.36 °C	58 °C	57.56 °C	49 °C	53.47 °C
150 °C	65 °C	31.45 °C	67 °C	49.77 °C	61 °C	59.77 °C
200 °C	97 °C	42.88 °C	79 °C	58.42 °C	72 °C	77.44 °C
250 °C	121 °C	59.25 °C	107 °C	76.52 °C	87 °C	85.43 °C

Figure 5.4: Temperature profile in the suspension when the shock has settled down at plate temperatures a.)100 °C and b.)150 °C for $\phi = 50\%$

Figure 5.5: Temperature profile in the suspension when the shock has settled down at plate temperatures a.)200 °C and b.)250 °C for $\phi = 50\%$

Figure 5.6: Temperature profile in the suspension when the shock has settled down = 100 °C for $\phi = 40\%$

Table 5.2: Comparison of shock displacement values

Concentration	Actual measured displacement (m)	Value from Simulation (m)
20%	0.02255	0.02526
40%	0.0635	0.06152
50%	0.08255	0.08054

5.3 Discussion

The results in Table. 5.1 show good agreement of the simulation results with the experimental values, at concentrations closer to the maximum packing. Table. 3.1 gives the number of shocks that are observed at each selected concentration and plate temperature. A direct comparison

between Table. 3.1 and the above results can be made. For cases with one observed shock the probe temperature results from the simulation show good agreement with experimental values. Results compare for an initial concentration of 50% at plate temperature 100 °C, 150 °C and 200 °C. and, also at 40% initial concentration at a plate temperature of 100 °C. This validates our theory, that at high concentrations particles settle down as column and only one shock is observed with no concentration gradients. If a concentration against time curve was plotted it will appear as a rectangle. As time passes the aspect ratio of the rectangle will change with the area remaining constant. Fig. 5.2 shows the probe temperature comparison for a plate temperature of 250 °C for 50% concentration. The simulation results show a large deviation from the experimental values. This can be attributed to the decreased top shock velocity reported in Section. 3.4 in Chapter. 3. The fluid properties change rapidly after 200 °C at which our velocity relation does not hold.

Table. 5.2 gives the displacement of the top shock from the free surface. Results show excellent agreement between actual and simulated values. The final shock location values have been calculated from the velocity correlation proposed in Section 3.4 of Chapter 3 by multiplying the velocity by the elapsed time.

A detailed error analysis presented in the the next chapter shows that predicted shock velocity values are within a range of $\pm 6\%$ for the above stated concentrations and plate temperatures indicating that the velocity correlation holds for high concentrations.

Chapter 6

Error Analysis

This section presents a detailed error analysis for the project. The probable causes of errors being introduced in the analysis are -

1. Optical distortion due to use of a cylindrical tank instead of a square one.
2. Use of different threshold pixel values used to locate shocks.
3. Accuracy of the thermocouples.

The use of a cylindrical tank creates an optical distortion when viewing in one plane. The easiest way to remove this is to use a square tank. However manufacturing a perfect square tank requires precision equipment. However to remove this distortion we have selected a region of interest which forms only a small arc, which when viewed from the front appears flat. This effectively removes the optical discontinuity.

To locate shocks, in each experiment a threshold value has been selected to distinguish between the clear fluid and the shock. This value is dynamically selected by the code according to the pixel values of the entire image. However this value changes with each setup. This is due to the changing light conditions with each setup. Errors which might be introduced in the calculations have been reduced by employing a simple scaling technique. The threshold pixel value which changes with the experiment, itself is used to scale all pixel values for that setup. This scales all values between 0 and 1. The thermocouples used are of p-type manufactured by

Fischer Scientific. The manufacturers report and accuracy of $\pm 2\%$ for each reading. Errors due to this cannot be eliminated.

6.1 Error Measurements

In this section the actual deviation of the proposed relation and the simulation from the actual values has been discussed.

Table 6.1: Comparison between Experimental and Calculated values for velocity at $\phi = 20\%$

Sr.No.	Temp	Expt. Value($\frac{m}{s}$)	Actual Value($\frac{m}{s}$)	Percent Error
1	25	5.73×10^{-6}	5.96×10^{-6}	-4.03
2	100	8.51×10^{-6}	9.87×10^{-6}	-4.23
3	150	1.24×10^{-5}	1.38×10^{-5}	-5.76
4	200	2.19×10^{-5}	1.94×10^{-5}	7.50
5	250	1.32×10^{-5}	2.71×10^{-5}	12.98

Table 6.2: Comparison between Experimental and Calculated values for velocity at $\phi = 40\%$

Sr.No.	Temp	Expt. Value($\frac{m}{s}$)	Actual Value($\frac{m}{s}$)	Percent Error
1	25	3.64×10^{-6}	3.92×10^{-6}	-7.69
2	100	3.40×10^{-6}	3.18×10^{-6}	4.55
3	150	4.81×10^{-6}	4.45×10^{-6}	10.47
4	200	7.36×10^{-6}	6.24×10^{-6}	9.28
5	250	6.73×10^{-6}	8.73×10^{-6}	10.03

In Table. 6.1, 6.2 and 6.3, a comparison of shock velocity values has been presented. The values for the single shock problem i.e. are in close agreement with those from the experimentally measured values. The deviation of the results from the experimentally measured values can be attributed to the following reasons.

1. Particles not being resolved clearly between adjacent pixels

Table 6.3: Comparison between Experimental and Calculated values for velocity at $\phi = 50\%$

Sr.No.	Temp	Expt. Value($\frac{m}{s}$)	Actual Value($\frac{m}{s}$)	Percent Error
1	25	5.66×10^{-7}	6.18×10^{-7}	-5.81
2	100	1.42×10^{-6}	1.02×10^{-6}	6.16
3	150	1.55×10^{-6}	1.43×10^{-6}	5.74
4	200	1.87×10^{-6}	2.01×10^{-6}	-6.48
5	250	8.97×10^{-7}	2.42×10^{-6}	16.26

2. A plate temperature of 250 °C shows decrease in the shock settling velocities as against an expected increase. This introduces a sudden change in velocity trends creating problems while curve fitting. The averaging of slopes and the intercepts changes the coefficients, when this decrease in velocity is taken into account, causing deviation in the results.

Taking into account all the errors our analysis provides a realistic approach to the estimation of the temperatures in a high concentration settling suspension. Results show that the theory shows good agreement between actual and the simulation results for 50% concentration at plate temperatures of 25 °C, 100 °C, 150 °C and 200 °C, and also for 40% at 100 °C. In all these cases the distinguishing factor is the number of shocks formed. Though the proposed top shock velocity correlation provides velocity estimates which, are in close agreement with the experimental values even for cases where two shocks are reported, the transient temperature analysis show large deviation for the 2 shock problem.

Chapter 7

Conclusions

In this manuscript we experimentally investigate the batch sedimentation of an impulsively heated system. The problem is analogous to many modern batch sedimentation processes used to separate oil from sand, where the solid volume fraction is typically at high values. In the limit of maximum packing the mixture becomes difficult to separate which is why energy, in the form of heat, is added to decrease the viscosity of the fluid portion of the mixture, hence increasing the sedimentation rate.

The experiments are performed using a high viscosity silicone oil (1000 cSt) and glass beads (300 μm) to generate a nearly mono-disperse mixture. The system is initially well mixed by stirring in a glass beaker over a long period of time. Then the mixture is placed on a hot plate and images of the sedimentation are recorded using a CCD camera. Qualitative observations of the sedimentation suggest the presence of either one or two concentration shocks, depending on the initial concentration and hot plate temperature. As the plate temperature is increased the top shock is observed to sediment over a shorter elapsed time period. At the largest temperatures used in the experiments the sedimentation rate of the top shock actually increases relative to the other experiments, suggesting that there is a optimal operating temperature.

Quantitative measurements are performed by analyzing the images. These measurements include the shock location as a function of time, the top shock velocity, and the time required

for the impulsively heated system to reach a steady state temperature. A correlation for the top shock velocity as a function of the temperature and concentration is proposed for the range of temperature where the top shock velocity is seen to decrease with temperature.

A numerical simulation is performed to generate temperature profiles in the fluid suspension is developed. Results show that data obtained from these simulations aligns with the experimental data with small errors. The correlation proposed for the top shock velocity in the image analysis is used to estimate the top shock displacement. A special jump condition was modeled at the fluid - solid interface to include the effect of the sudden change in the concentration. Results obtained show excellent agreement with the experimental values for the high concentration systems ($\phi > 40\%$). The numerical simulation successfully solves for the energy equation and provides the a realistic estimate of the temperatures in the settling suspensions. Tar-sands discussed in Chapter. 1 have concentrations which are close to the maximum packing limit. Our analysis of high concentration suspensions will definitely provide an insight into the actual mechanics of the settling phenomenon. The selection of an optimum temperature, based on our analysis might help increasing sand settling velocities and hence reduce the residence time between actual pumping of the steam and the time till the oil is pumped out.

In the future it will be beneficial to perform additional experiments varying the particle size to determine their effects on the top shock velocity. Also it may be useful to perform experiments at temperatures that are greater than the ones discussed in this manuscript to see if the observed trends continue at the higher temperature. It also may be interesting to observe the origins of the sedimentation rate decrease through particle tracking, possibly utilizing laser fluorescence technology.

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APPENDICES

Appendix A

Stability analysis and Non-dimensional Analysis

A.1 Stability analysis for explicit integration of one-dimensional energy equation

$$\frac{\partial T}{\partial t} - \alpha \frac{\partial^2 T}{\partial x^2} = 0$$

discretizing

$$\frac{U_i^{n+1} - U_i^n}{\Delta t} - \frac{\alpha}{\Delta x^2} \cdot [U_{i+1}^n - 2U_i^n + U_{i-1}^n] = 0$$

introducing the error term

$$\frac{\epsilon_i^{n+1} - \epsilon_i^n}{\Delta t} - \frac{\alpha}{\Delta x^2} \cdot [\epsilon_{i+1}^n - 2\epsilon_i^n + \epsilon_{i-1}^n] = 0$$

$$e^{a(t+\Delta t)} \cdot e^{ik_m x} - e^{at} e^{ik_m x} - \left(\frac{\alpha \Delta t}{\Delta x^2} \right) \cdot \left[e^{at} \cdot e^{ik_m(x+\Delta x)} - 2e^{at} \cdot e^{ik_m x} + e^{at} \cdot e^{ik_m(x-\Delta x)} \right] = 0$$

dividing throughout by $e^{at} \cdot e^{ik_m x}$

$$G - 1 - \nu \left[e^{ik_m \Delta x} - 2 + e^{-ik_m \Delta x} \right] = 0$$

where, G = amplification factor given by $\frac{\epsilon_i^{n+1}}{\epsilon_i^n}$

and ν is the CFL number given by $\left(\frac{\alpha \Delta t}{\Delta x^2} \right)$

$$G = 1 - \nu [\cos \theta + i \sin \theta + \cos \theta - i \sin \theta - 2]$$

$$G = 1 - \nu[2 \cos \theta - 2]$$

for stability the magnitude of the amplification factor should be less than one. i.e. $|G|^2 \leq 1$.
which give the stability condition as

$$\nu \leq \frac{1}{1 - \cos \theta}$$

giving the stability bounds as,

$$0 \leq \nu \leq \frac{1}{2}$$

A.2 Non-dimensional Analysis

We solve for the energy equation in the our domain,

$$\rho.C_p \frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial x^2} \tag{A.1}$$

Non-dimensionalizing Eq. A.1

$$\theta = \frac{T - T_\infty}{T_{plate} - T_\infty}$$

$$\rho.C_p \frac{\partial \theta}{\partial t} = K \frac{\partial^2 \theta}{\partial x^2}$$

The thermal conductivity of the suspension is defined as

$$K(t) = K_l \left(1 - \frac{\phi(t)}{\phi_{max}} \right) + K_s \left(\frac{\phi(t)}{\phi_{max}} \right)$$

Consider Eq. 1.4 in Chapter 1

$$\phi(t) = \phi_0 - \frac{h(t)}{h_{max}} + 1$$

where,

$$h(t) = h_{max} - U_{shock}(\phi, T).t$$

here, $h(t)$ is the displacement of the shock from the free liquid surface. U_{shock} is the top shock velocity, the expression for which has been derived in Section. 3.4 of Chapter. 3. The non-dimensionalized thermal conductivity is derived by scaling it using the thermal conductivity of the liquid (silicon oil in our experiments).

$$K^* = \frac{K(t)}{K_L} = \left(1 - \left(\frac{\phi_0 + \frac{c_0(\phi, T).t}{h_{max}}}{\phi_{max}}\right)\right) + \frac{K_s}{K_l} \cdot \left(\frac{\phi_0 + \frac{c_0(\phi, T).t}{h_{max}}}{\phi_{max}}\right) \quad (\text{A.2})$$

let $\beta = \frac{K_s}{K_L}$

we define non-dimensionless time as,

$$t^* = \frac{c_0(\phi, T)}{h_{max}}.t$$

the expression for the non-dimensional thermal conductivity becomes

$$K^* = 1 + (\beta - 1)(\alpha + t^*)$$

where, $\alpha = \frac{\phi_0}{\phi_{max}}$

the energy equation becomes,

$$\frac{\partial \theta}{\partial t} = \frac{K^*.K_L.\phi_{max}}{\rho.C_p.c_0.h_{max}} \left(\frac{\partial^2 \theta}{\partial x^2} \right)$$

in shorter format

$$\frac{\partial \theta}{\partial t} = Pe. \left(\frac{\partial^2 \theta}{\partial x^2} \right)$$

where

$$Pe = \frac{K^*.K_L.\phi_{max}}{\rho.C_p.c_0.h_{max}}$$

is the Peclet number.

Non-dimensional Boundary and interface conditions

The free surface boundary condition is given by

$$-K \frac{\partial T}{\partial x} = h.(T - T_\infty)$$

in terms of the non-dimensional temperature,

$$\frac{-K}{h_{max}} \cdot \frac{\partial \theta}{\partial x^*} = h \cdot \theta$$

so, at the surface point 's', using the theory explained in section. 4.2 of Chapter. 4,

$$\frac{\theta_s - \theta_{s-1}}{\Delta x^*} = -Nu \cdot \theta_s$$

At the liquid-particle interface, using the constant flux condition across the interface described in section. 4.2 of Chapter. 4,

$$K(t) \cdot \left(\frac{\partial \theta}{\partial x^*} \right) = K_l \cdot \left(\frac{\partial \theta}{\partial x^*} \right)$$

dividing throughout by K_l

$$\frac{K(t)}{K_l} \left(\frac{\partial \theta}{\partial x^*} \right)_s = \left(\frac{\partial \theta}{\partial x^*} \right)_l$$

from Equation. A.2

$$K^* \left(\frac{\partial \theta}{\partial x^*} \right)_s = \left(\frac{\partial \theta}{\partial x^*} \right)_l$$

Appendix B

Code

B.1 Image analysis

```
%*****%
%Main calling program                                     %
%*****%
% calculating final shock location based on image numbers 3500 to
5871

clear all
format long
start_image = 1501;
final_image = 1800;
%call to function mov_mak to calculate local concentrations
[loc_conc] = loc_con(start_image,final_image);

%Final merged shock location
[approx_max_shock_conc, approx_loc_shock] = max(loc_conc);
final_shock_loc = round(mean(approx_loc_shock));
clear start_image final_image loc_conc s1 s2 s3 s4 s5;
```

```

start_image = 1;
final_image = 2000;

% call to mov_mak to calculate the local concentrations for the
    first 2000
% images
[loc_conc ,img_movie] = loc_con(start_image ,final_image);

% Correction employed to remove the small slope in the first plot
% due to lighting limitations
[loc_conc_corrected ,corrtn] = correction(loc_conc ,start_image);

% call to conc_plots to plot relevant concentration plots
[loc_conc_scaled] = conc_plots(loc_conc_corrected ,loc_conc);

%call to shock_location to plot the shock location plot
[location_bottom_shock_corrected , location_top_shock]=
    shock_location...
        (loc_conc ,final_shock_loc);

% call to function time_plots to plot velocities
[top_shock_velocity ,bottom_shock_velocity ,time ,top_shock_avg1 ,...
    bottom_shock_avg1] = time_plots(location_top_shock ,...
    location_bottom_shock_corrected);

% call to function mov_mak to create movie
% mov_mak(img_movie);

```

```

%*****%

%Function to read the Images and then %
% calculate the Local concentrations %
%*****%

function [loc_conc ,img_movie]= loc_con (start_image ,final_image)

s1='x.BMP';
s2='xx.BMP';
s3='xxx.BMP';
s4='xxxx.BMP';

%*****%

%Region of Interest in the First Image selected manually

left_value=450;
right_value=850;
top_start=150;
bottom_end=900;
n=1;

%*****%

for i=start_image:1:final_image

    if (i<10 && i>=1)

        s5=strrep(s1 , 'x' ,num2str(i));

    elseif (i>10 && i<100)

        s5=strrep(s2 , 'xx' ,num2str(i));

    elseif (i>=100 && i<1000)

        s5=strrep(s3 , 'xxx' ,num2str(i));

    elseif (i>=1000)

```

```

        s5=strcmp(s4,'xxxx',num2str(i));
    end

    [a,b]=imread(s5,'bmp');
    image(a),colormap(b)
    % h = colormap(b);
    img_movie(n)=getframe;
    n=n+1;
    i
% Calculates the average local concentration based upon the pixel
values
% from the image
    for j=top_start:bottom_end
        loc_conc(j-top_start+1,(i+1-start_image))=sum(a(j,left_value
            :right...
            _value))/length(a(j,left_value:right_value));

    end
end

% Correction to remove the slope from the first plot caused due
lighting
% limitations
function[loc_conc_corrected,corrtn] = correction(loc_conc,
    start_image)

%***** Calculation of Correction*****

```



```

[m,n] = size(loc_conc);
loc_conc_max = max(max(loc_conc))
for i=1:1:m
    corrtm(i) = loc_conc_max - loc_conc(i,1);
end

%Adding the Correction to all the images
for i=1:n
    for j = 1:m
        loc_conc_corrected(j,i) = corrtm(j) + loc_conc(j,i);
    end
end

%*****%
% Corrected concentration scaled based on the maximum %
% and the minimum values for the given set. Scaling employed%
% to have concentrations based %
% on a scale of 0-1. %
% *****%

function[loc_conc_scaled] = conc_plots(loc_conc_corrected,loc_conc)

[m,n] = size(loc_conc_corrected);
M = max(max(loc_conc_corrected));
N = min(min(loc_conc_corrected));
for i=1:n
    for j=1:m

```

```

        loc_conc_scaled(j,i) = (loc_conc_corrected(j,i) - N) / (M -
                                N);
    end
end

%Concentration Plots
for i = 1:1:m
    height(i) = i*0.09;
end
for i = 1:1:m
    height_scaled(i) = height(i)/height(m);
end
[z,b] = size(loc_conc_scaled);
for j = 1:1:b
    for i = 1:1:z
        if loc_conc_scaled(i,j) < 0.3
            loc_conc_scaled(i,j) = 0.0;
        end
    end
end
end
figure(4), plot(loc_conc_scaled(1:20:end,1),height_scaled(1:20:end),
    '-.k','LineWidth',0.5,...
    'MarkerFaceColor','k',...
    'MarkerSize',5),title('scaled concentration plot')
ylabel('Scaled height of the column'),...
    xlabel('scaled concentration values');
hold on

```

```

plot(loc_conc_scaled(1:20:end,100),height_scaled(1:20:end),'-ko',',',
      LineWidth',0.5,...
      'MarkerFaceColor','k',...
      'MarkerSize',5),hold on
plot(loc_conc_scaled(1:20:end,150),height_scaled(1:20:end),'-kd',',',
      LineWidth',0.5,...
      'MarkerFaceColor','k',...
      'MarkerSize',5)
hold on
plot(loc_conc_scaled(1:20:end,200),height_scaled(1:20:end),'-.kx',',',
      LineWidth',0.5,...
      'MarkerFaceColor','k',...
      'MarkerSize',5)
hold on
plot(loc_conc_scaled(1:20:end,250),height_scaled(1:20:end),'-k<',',',
      LineWidth',0.5,...
      'MarkerFaceColor','k',...
      'MarkerSize',5)
hold on
plot(loc_conc_scaled(1:20:end,300),height_scaled(1:20:end),'-k+',',',
      LineWidth',0.5,...
      'MarkerFaceColor','k',...
      'MarkerSize',5)
hold on
plot(loc_conc_scaled(1:20:end,350),height_scaled(1:20:end),'-k^',',',
      LineWidth',0.5,...
      'MarkerFaceColor','k',...

```

```

    'MarkerSize',5)
hold on
plot(loc_conc_scaled(1:20:end,400),height_scaled(1:20:end),'-k*','
    LineWidth',0.5,...
    'MarkerFaceColor','k',...
    'MarkerSize',5)
hold on
plot(loc_conc_scaled(1:20:end,500),height_scaled(1:20:end),'-k>','
    LineWidth',0.5,...
    'MarkerFaceColor','k',...
    'MarkerSize',5)
hold on
plot(loc_conc_scaled(1:20:end,600),height_scaled(1:20:end),'-.kd','
    LineWidth',0.5,...
    'MarkerFaceColor','k',...
    'MarkerSize',5)
hold on
plot(loc_conc_scaled(1:20:end,700),height_scaled(1:20:end),' :kd','
    LineWidth',0.5,...
    'MarkerFaceColor','k',...
    'MarkerSize',5)
hold on
plot(loc_conc_scaled(1:20:end,1000),height_scaled(1:20:end),' :ko','
    LineWidth',0.5,...
    'MarkerFaceColor','k',...
    'MarkerSize',5)
hold on

```

```

plot(loc_conc_scaled(1:20:end,1300),height_scaled(1:20:end),'-kd',',
    LineWidth',0.5,...
    'MarkerFaceColor','k',...
    'MarkerSize',5)
hold on
plot(loc_conc_scaled(1:20:end,2000),height_scaled(1:20:end),'kp',',
    LineWidth',0.5,...
    'MarkerFaceColor','k',...
    'MarkerSize',5)
legend('time = 0','time = 1500s','time = 2250s','time = 3000s','time
    = 3750s','time = 4500s','time = 5250s',...
    'time = 6000s','time = 7500s','time = 9000s','time = 10500s',',
    time = 15000s','time = 19500s','time = 30000s')
fig=gcf;
set(findall(fig,'-property','FontSize'),'FontSize',15)
h_xlabel = get(gca, 'XLabel');
set(h_xlabel, 'FontSize',14);

h_ylabel = get(gca, 'YLabel');
set(h_ylabel, 'FontSize',14);

```

%

*****%

*% Calculation of shock location based on the value from the initial
call %*

% to mov_mak.m

%

%

******%*

```
function [ location_bottom_shock_corrected , location_top_shock ] =  
    shock_location ( loc_conc , final_shock_loc )
```

```
[ top_shock_conc , location_top_shock ] = max( loc_conc ( 1:1:  
    final_shock_loc , : ) ); % locating the top shock using the built in  
    max function
```

*% Correction to bottom shock to remove erroneous points due to
% non-formation of the bottom shock*

```
[ bottom_shock_conc , location_bottom_shock ] = max( loc_conc (   
    final_shock_loc : end , : ) ); % locating the bottom shock using the  
    built in max function
```

```
location_bottom_shock_corrected = location_bottom_shock +  
    final_shock_loc ;
```

```
[ merge , location ] = min( find( location_bottom_shock_corrected == (   
    location_top_shock + 2 ) ) ) % double check the value of the merged  
    shock location ( final shock )
```

%

******%*

```

%
% For averaging values please select suitable number of images
%
% changing the value at line number 9

```

%

%

```
function [top_shock_velocity , bottom_shock_velocity , time ,  
top_shock_avg1 , bottom_shock_avg1] = time_plots (location_top_shock ,  
location_bottom_shock_corrected)
```

```

% Vaue based
on the number of points required on the velocity plot%
time_interval = 5;

```

the experimental setup

images_analysed = 2000;

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```

    variable 'image_average_no '

time_step = image_average_no*time_interval;
                                %calculation of time step for the
                                given average number
time_length = images_analysed*time_interval;
                                %time for which the experiment was
                                conducted
time = 1:time_step:time_length;
                                %creation of the time
                                matrix
shock_array_length = images_analysed*time_interval / time_step;
                                %length of shock matrix based on above specified
                                variables

% Averaging the shock location values based on the number selected
in line
% 9

for i = 1:1:shock_array_length
    top_shock_avg(i) = sum(location_top_shock((image_average_no*i
    - ...      %averaging the shock locations nased on the
    average value specified
    (image_average_no-1):1:image_average_no*i))*0.09/
    image_average_no;          % in line number 9
    bottom_shock_avg(i) = sum(location_bottom_shock_corrected...
    (image_average_no*i-(image_average_no-1):1:image_average_no*

```



```

        i)) * 0.09 ...
    /image_average_no;
end
for i = 1:1:shock_array_length
    top_shock_avg1(i) = top_shock_avg(i)/top_shock_avg(
        shock_array_length);
    bottom_shock_avg1(i) = bottom_shock_avg(i)/bottom_shock_avg(
        shock_array_length);
end

% Average shock location plot
[t,n] = size(time);
end_time = time(1,n)
for i = 1:1:n
    time(1,i) = time(1,i)/time(1,n);
end
figure(9), plot(time,top_shock_avg1,'-ks','LineWidth',2,...
    'MarkerFaceColor','k',...
    'MarkerSize',5);
hold on
figure(9), plot(time,bottom_shock_avg1,'-ko','LineWidth',2,...
    'MarkerFaceColor','k',...
    'MarkerSize',5),xlabel('Time Instant scaled'),...
ylabel('Instantaneous location (mm) (origin is the top surface
    and the other end of location axis is the bottom)'),...
legend('Top Shock Location','Bottom Shock Location'),...
title('Averaged Location Plot');

```

```

% Calculation of shock velocities based on average shock location
% calculated above

for i = 1:1:shock_array_length
    if i ~= shock_array_length
        top_shock_velocity(i) = (top_shock_avg(i+1) - top_shock_avg(i))
            /...      %Velocity calculation based on the number specified
            in line 9
            time_step;

            %both for the bottom and the top shock
        bottom_shock_velocity(i) = (bottom_shock_avg(i+1) - ...
            bottom_shock_avg(i))/time_step;

    elseif i == shock_array_length

        top_shock_velocity(shock_array_length) = ...
            (top_shock_avg(shock_array_length) - top_shock_avg...
            (shock_array_length - 1))/time_step;

        bottom_shock_velocity(shock_array_length) = ...
            (bottom_shock_avg(shock_array_length)...
            - bottom_shock_avg(shock_array_length - 1))/time_step;

    end
end

```

% Plotting the Averaged Velocities

```

figure(8), plot(time,top_shock_velocity,'-ks','LineWidth',2,...
    'MarkerFaceColor','k',...
    'MarkerSize',7);

hold on

figure(8), plot(time,bottom_shock_velocity,'-ko','LineWidth',2,...
    'MarkerFaceColor','k',...
    'MarkerSize',7),xlabel('Time Instant'),...
ylabel('Instantaneous Velocity (mm/second)'),...
legend('Top Shock Velocity','Bottom Shock Velocity'),...
title('Velocity Plot'),grid('on');

```

%

******%*

% Function to remove the optical illusion(discontinuity in the top

%

% shock location)

%

%

******%*

```

function [location_top_shock,location_bottom_shock_corrected] =
    top_shock_correction(location_top_shock,

```

```

location_bottom_shock_corrected)

error_location = find(diff(location_top_shock)>=100)
                % finds the location of the discontinuities

break_loc_curve = min(error_location);
                %considers the smallest
                discontinuity

factor = location_top_shock(break_loc_curve+1)/...
                % Calculation for calculating scaling
                factor
                location_top_shock(break_loc_curve);

for i=1:1:break_loc_curve
    location_top_shock(i) = location_top_shock(i)*factor;
                % Scaling the top shock location and
                returning to function shock_location
    if location_bottom_shock_corrected(i) <=(location_top_shock(
        break_loc_curve+1)+2);
        location_bottom_shock_corrected(i) = NaN;
    end
end

[p,q] = size(location_top_shock);
for i = (break_loc_curve):1:(q-1)
    if location_top_shock(i+1) < location_top_shock(i)
        location_top_shock(i+1) = location_top_shock(i);

```

end

end

Appendix C

Code

C.1 For solving the energy equation

```
clear all
close all
clc

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%phi = 20%; h_max = 0.074284 %
%phi = 50%; h_max = 0.1067 %
%phi = 40%; h_max = 0.093218 %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%parameters changing with setup
phi = 0.50;
h_max = 0.1067;
N = 100;
h = h_max/N;
T = zeros(N,1) ;
T_plate = 150;
T(1) = 1;
```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% constants
T_infinity = 27;
phi_max = 0.60;
my_alpha = phi/phi_max;
heat_trans_coeff = 5.7;
th_condu = 0.0614;
NU = heat_trans_coeff*h/th_condu;
K_s = 0.937;
K_l = 0.1590984;
my_beta = (K_s/K_l);
rho_s = 2475;
rho_l = 940;
Cp_s = 3516.91;
Cp_l = 1507.2;
c_l = 9.768*10^-6*((1 - (my_alpha))^1.73492)*exp(0.0067363*T_plate);
rho = rho_l*(1 - (my_alpha)) + rho_s*(my_alpha);
Cp = Cp_l*(1 - (my_alpha)) + Cp_s*(my_alpha);
count = 0;
Pe = (rho*Cp*h_max*c_l)/(K_l*phi_max);
t = 0;
dt = 0.05;
a = 0;
n=N;
t_end = (phi_max - phi)*h_max/(phi_max*c_l);%(1- my_alpha)/phi_max;
T_diff = T_plate - T_infinity;
t_shock_end = 8677.42*exp(-5.01872*(1 - phi/phi_max));

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% grid generation
for i = 1:1:N
    x(i) = a + i*h;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%initial condition
for i = 2:1:N
    T(i) = 0;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% t_star = dt;
for k = 0:dt:t_shock_end
    count = count+1
    t_tilda = (c_1*t/(phi_max*h_max));
    K_star = 1 + (my_beta - 1) *(my_alpha + t_tilda);
    [T] = solid_region(T,Pe,h,dt,K_star,NU,N,x);
    % figure(1), plot(x(1,1:1:N),T(1:1:N,1));
end
for k = 0:dt:t_end
    count = count+1
    if N==n || n <= 3
        if t == 0
            t_tilda = (c_1*dt/(phi_max*h_max));
        else
            t_tilda = (c_1*t/(phi_max*h_max));
        end
    end
end

```



```

end

K_star = 1 + (my_beta - 1) *(my_alpha + t_tilda);
[T] = solid_region(T,Pe,h,dt,K_star,NU,N,x);

else

t_tilda = (c_1*t/(phi_max*h_max));
K_star = 1 + (my_beta - 1) *(my_alpha + t_tilda);
[T] = liquid_region(T,Pe,h,K_star,NU,N,n,K_1,dt,x);
end

%t_star = (c_1*t/h_max);
h_t = c_1*t           % shock_movement
if h_t >= h
    n = ceil(h_t/h)
end

t= t+dt;
for i = 1:1:N
    T_actual(i) = T(i)*T_diff + T_infinity;
end

%    figure(2), plot(x(1,1:1:N),T_actual(1,1:1:N),'k','linewidth
',1.5);

%    xlabel('Height of column')
%    ylabel('Temperature in degrees centigrade')
%set(q,'Value',count)
%    M(count) = getframe(gcf);
end

```

```

% movie2avi(M, '40_100.avi', 'compression', 'cinepak')

figure(1), plot(x(1,1:1:N), T(1:1:N,1));
figure(2), plot(x(1,1:1:N), T_actual(1,1:1:N));
xlabel('Height of column')
ylabel('Temperature in degrees centigrade')

%*****%
%Code for solving the solid+liquid region part of the domain %
%*****%

function [T] = solid_region(T, Pe, h, dt, K_star, NU, N, x)
Tn1(1) = T(1);
Tn2(1) = T(1);
Tn3(1) = T(1);
Tn4(1) = T(1);

    for i = 2:1:N-1
        Tn1(i) = T(i) + (K_star*dt/Pe)*(T(i+1) - 2*T(i) + T(i-1));
    end
    Tn1(N) = (Tn1(N-1))/(1 + NU*h);
    for i = 2:1:N-1
        dT1(i) = (Tn1(i+1) - 2*Tn1(i) + Tn1(i-1));
    end
%*****%

    for i = 2:1:N-1
        Tn2(i) = T(i) + 0.5*(K_star*dt/Pe)*dT1(i);
    end
end

```

```

Tn2(N) = (Tn2(N-1))/(1 + NU*h);
for i = 2:1:N-1
    dT2(i) = (Tn2(i+1) - 2*Tn2(i) + Tn2(i-1));
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for i = 2:1:N-1
    Tn3(i) = T(i) + 0.5*(K_star*dt/Pe)*dT2(i);
end
Tn3(N) = (Tn3(N-1))/(1 + NU*h);
for i = 2:1:N-1
    dT3(i) = (Tn3(i+1) - 2*Tn3(i) + Tn3(i-1));
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for i = 2:1:N-1
    Tn4(i) = T(i) + (K_star*dt/Pe)*dT3(i);
end
Tn4(N) = (Tn4(N-1))/(1 + NU*h);
for i = 2:1:N-1
    dT4(i) = (Tn4(i+1) - 2*Tn4(i) + Tn4(i-1));
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for i = 2:1:N-1
    Tn(i) = T(i) + (1/6)*h*(dT1(i) + 2*dT2(i) + 2*dT3(i) + dT4(i)
    ));
end
Tn(N) = (Tn(N-1))/(1 + NU*h);

```

```

for i = 2:1:N
    T(i) = Tn(i);
    if T(i)>1
        figure(1),plot(x(1,1:1:N),T(1:1:N,1));
        break
    end
end

%    figure(1),plot(x(1,1:1:N),T(1:1:N,1));
%*****%
% code for solving the liquid region of the domain %
%*****%
function [T] = liquid_region(T,Pe,h,K_star,NU,N,n,K_l,dt,x)

fprintf(' \ntwo regions ')
K_l = 1;
Tn1(1) = T(1);
Tn2(1) = T(1);
Tn3(1) = T(1);
Tn4(1) = T(1);

for i = 2:1:N-n-1
    Tn1(i) = T(i) + (K_star*dt/Pe)*(T(i+1) - 2*T(i) + T(i-1));
end
Tn1(N-n+1) = T(N-n+1) + dt*(K_l/Pe)*(T(N-n+2) - (2*T(N-n+1)) + T(N-n
));
Tn1(N-n) = (1/(K_star+K_l))*(K_l*T(N-n+1) + K_star*T(N-n-1));
        % condition at the interface

```

```

for k = 2:1:(N-n+1)
    T(k) = Tn1(k);
                                                                    %
    updating values.
end
for i = (N-n+2):1:N-1
                                                                    % space
    loop for points aftr the discontinuity
    Tn1(i) = T(i) + dt*(K_l/Pe)*(T(i+1) - (2*T(i)) + T(i-1));
end
Tn1(N) = (Tn1(N-1))/(1 + NU*h);
                                                                    %
    boundary condition at the top interface

for i = 2:1:N-1
    dT1(i) = (Tn1(i+1) - 2*Tn1(i) + Tn1(i-1));
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for i = 2:1:N-n-1
    Tn2(i) = Tn1(i) + 0.5*(K_star*dt/Pe)*dT1(i);
end
Tn2(N-n+1) = Tn1(N-n+1) + dt*0.5*(K_l/Pe)*(Tn1(N-n+2) - (2*Tn1(N-n
+1)) + Tn1(N-n));
Tn1(N-n) = (1/(K_star+K_l))*(K_l*T(N-n+1) + K_star*T(N-n-1));
    % condition at the interface

for k = 2:1:(N-n+1)

```

```

Tn1(k) = Tn2(k);

                                                                    %
        updating values.
end
for i = (N-n+2):1:N-1

                                                                    % space
        loop for points aftr the discontinuity
        Tn2(i) = Tn1(i) + 0.5*dt*(K_l/Pe)*(Tn1(i+1) - (2*Tn1(i)) + Tn1(i
        -1));
end
Tn2(N) = (Tn2(N-1))/(1 + NU*h);

                                                                    %
        boundary condition at the top interface

for i = 2:1:N-1
    dT2(i) = (Tn2(i+1) - 2*Tn2(i) + Tn2(i-1));
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for i = 2:1:N-n-1
    Tn3(i) = Tn2(i) + 0.5*(K_star*dt/Pe)*dT2(i);
end
Tn3(N-n+1) = Tn2(N-n+1) + 0.5*dt*(K_l/Pe)*(Tn2(N-n+2) - (2*Tn2(N-n
+1)) + Tn2(N-n));
Tn3(N-n) = (1/(K_star+K_l))*(K_l*Tn2(N-n+1) + K_star*Tn2(N-n-1));

        % condition at the interface

for k = 2:1:(N-n+1)
    Tn2(k) = Tn3(k);

```

```

                                                                    %
        updating values.

    end
    for i = (N-n+2):1:N-1

                                                                    % space

        loop for points aftr the discontinuity

        Tn3(i) = Tn2(i) + 0.5*dt*(K_l/Pe)*(Tn2(i+1) - (2*Tn2(i)) + Tn2(i
            -1));

    end
    Tn3(N) = (Tn3(N-1))/(1 + NU*h);                                                                    %
        boundary condition at the top interface

    for i = 2:1:N-1
        dT3(i) = (Tn3(i+1) - 2*Tn3(i) + Tn3(i-1));
    end
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    for i = 2:1:N-n-1
        Tn4(i) = Tn3(i) + (K_star*dt/Pe)*dT3(i);
    end
    Tn4(N-n+1) = Tn3(N-n+1) + dt*(K_l/Pe)*(Tn3(N-n+2) - (2*Tn3(N-n+1)) +
        Tn3(N-n));
    Tn4(N-n) = (1/(K_star+K_l))*(K_l*Tn3(N-n+1) + K_star*Tn3(N-n-1));
        % condition at the interface

    for k = 2:1:(N-n+1)
        Tn3(k) = Tn4(k);

                                                                    %

```

```

        updating values .

end
for i = (N-n+2):1:N-1

                                                    % space

    loop for points after the discontinuity
        Tn4(i) = Tn3(i) + dt*(K_l/Pe)*(Tn3(i+1) - (2*Tn3(i)) + Tn3(i-1))

        ;
    end
    Tn4(N) = (Tn4(N-1))/(1 + NU*h);
                                                    %
        boundary condition at the top interface

for i = 2:1:N-1
    dT4(i) = (Tn4(i+1) - 2*Tn4(i) + Tn4(i-1));
end
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

for i = 2:1:N-1
    Tn(i) = T(i) + (1/6)*h*(dT1(i) + 2*dT2(i) + 2*dT3(i) + dT4(i));
end
Tn(N) = (Tn(N-1))/(1 + NU*h);

for i = 2:1:N
    T(i) = Tn(i);
    if T(i)>1
        figure(1),plot(x(1,1:1:N),T(1:1:N,1));
        break
    end
end

```


end

end

figure(1),**plot**(x(1,1:1:N),T(1:1:N,1));