ABSTRACT

SUBRAMANIAN, VIJAY. A Viscoplastic Model with Rate-Dependent Hardening for Asphalt Concrete in Compression. (Under the direction of Dr. Murthy Guddati and Dr. Y. Richard Kim).

This dissertation presents a new type of uniaxial viscoplastic rate model based on viscoelastic convolution integrals for explaining the behavior of asphalt concrete in compression under repeated loading. Triaxial compression cyclic tests carried out for long rest periods, with different loading times and two different pulse shapes, square and haversine by researchers at North Carolina State University were used in developing the model. These tests demonstrate that the evolution of permanent deformation depends on load history. This history-dependent behavior is not captured accurately by some of the existing Perzyna-type viscoplastic models in which permanent deformation evolution depends on the current values of stress and viscoplastic strain. Therefore, in this study, viscoelastic-like integrals were used in the rate model to capture the effect of history.

As a simplification, and to better understand some aspects of the rate model, an incremental model was derived from the rate model by making assumptions of steady state conditions and slow hardening growth. This incremental model is capable of capturing material behavior observed from the cyclic tests mentioned above thereby verifying the rate model.

The proposed uniaxial viscoplasticity rate model is applicable to compressive creep and recovery experiments at 54°C with 1) several hundreds of cycles of loading including the secondary creep region, 2) haversine loading shapes at three different peak deviatoric stress levels, 620 kPa, 827 kPa, and 1034 kPa, and square loading shapes at 827 kPa peak deviatoric stress, and 3) long rest periods that allow complete viscoelastic recovery. A notable advantage of the model is the ability to predict the primary creep region behavior without the need for a separate model. Compared to many of the existing viscoplasticity models, the proposed uniaxial viscoplasticity has a relatively simple model form with a

physical mechanical analog that can be used to intuitively understand the model behavior. Yet, unlike the existing viscoplasticity models, the model has been successful in capturing the behavior of asphalt concrete in compression under a large number of cycles of creep and recovery loading with long rest period.

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A Viscoplastic Model with Rate-Dependent Hardening for Asphalt Concrete in Compression.

by
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DEDICATION

To my loving mother and my caring brother.

BIOGRAPHY

Vijay Subramanian spent most of his life in Chennai, India. He studied Civil Engineering at the Indian Institutes of Technology, Madras (IITM), where he developed a keen interest in Structural Engineering. After graduating from IITM at the top of his undergraduate class in 2002, he went on to do a masters in Civil Engineering in the field of Structural Reliability at the University of Waterloo, Canada. During this period, he discovered a growing interest in the field of Computational Mechanics that led to beginning a Doctor of Philosophy program in Civil Engineering in the field of Mechanics at North Carolina State University, Raleigh, USA in 2004 under the direction of Dr. Murthy Guddati. He earned his doctorate degree in 2011.

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Chapter 1 Introduction

1.1 Background

Asphalt concrete is a man-made composite material that exhibits complex time-dependent recovery and permanent deformation behavior. Asphalt concrete pavements are plagued by two major distresses: rutting and fatigue cracking. Rutting, i.e., local deformation and flow around the wheel load in asphalt concrete at elevated temperatures under cyclic loading, is considered a major distress in pavements. Although rutting in the asphalt layer does not constitute structural failure, it is nevertheless a serviceability failure owing to the compromise of safety when motorists drive on rutted pavements. Understanding rutting would help in the design of roads that can resist rutting over the service life of the pavement. The first step in understanding rutting involves understanding the material behavior of asphalt concrete in compression. To this end, the main focus of this research is to present a new uniaxial compression model for asphalt concrete at high temperatures and for repeated cyclic loading with rest periods.

1.2 State of the Art in Compression Modeling

The state of permanent deformation (viscoplastic) compression modeling of asphalt concrete is still in its formative stages. Uzan (1996) and Schapery (1999) suggested a phenomenological uniaxial viscoplastic model that employed functions of current permanent strain and applied stress. This type of model is found to predict the behavior of asphalt concrete in tension and compression under monotonic loading (Chehab et al., 2003 and Gibson et al., 2003). However the model is unable to accurately capture secondary region behavior in repeated creep and recovery tests. Taking a cue from the field of metal plasticity, many researchers have worked on Perzyna type models with yield and hardening functions but have had limited success in predicting the material behavior under repeated creep and recovery tests (Erkens, 2002, Gibson, 2006, and Dessouky et al., 2006 for example). It will be shown subsequently using experimental data that some of the model forms of the Perzyna-type are not suitable for explaining the behavior of a large number of repeated creep and

recovery cycles in compression for asphalt concrete. One of the main problems with these models is that the effect of load history on permanent deformation is not taken into account properly. Levenberg (2009) proposed a viscoplastic model using a Perzyna-type model form with hardening to predict permanent deformation in asphalt concrete in tension and compression with stress reversals. Although the model works for creep and recovery tests, it is limited to small strains (up to 1000 microstrains in compression). Yun (2008) proposed a model that blends some of the classical ideas in plasticity, such as yield surface and Perzyna-type flow, with the concept of having the yield surface evolve as a function of load history. This model has some success predicting the material behavior under certain kinds of repeated creep and recovery loads, but is not as successful when applied to other kinds of repeated creep and recovery loads (square-shaped and haversine-shaped). Thus, there exists a need for a viscoplastic model that can explain the behavior of asphalt concrete in compression for realistic cycles of loading.

1.3 General Objective

The general objective of this research is to propose a viscoplastic rate model for asphalt concrete in compression for creep and recovery loading.

1.4 Outline

Chapter 2 begins with an introduction of the basics behind asphalt concrete modeling and discusses the current viscoplasticity models for asphalt concrete in compression. The chapter concludes with a discussion on the limitations of the current models and gives the specific objectives for viscoplastic modeling of asphalt concrete in compression. Chapter 3 deals with the lessons learned from compressive creep and recovery experimental data and outlines the key aspects of asphalt concrete behavior in compression that have to be captured. Chapter 4 introduces the rate model for viscoplastic behavior of asphalt concrete in compression and goes into the details of the model. Chapter 5 covers an incremental model obtained as an approximation of the rate model for compression that can be used to obtain estimates of some of the rate model parameters to serve as an input to characterization of the rate model

parameters using optimization. Chapter 6 reveals details of the characterization process of the rate model, verification of the model, validation of the model, and sensitivity analysis of model parameters. Chapter 7 concludes the dissertation by listing contributions of the research and proposes future work related to viscoplastic modeling of asphalt concrete.

Chapter 2 Background and Literature Review

This chapter lays down some of the background needed to understand permanent deformation modeling of asphalt concrete in compression and also gives a brief review of work done by other researchers in the area of permanent deformation modeling of asphalt concrete subject to compressive loading.

2.1 Preliminaries

Asphalt concrete is a material that exhibits a rate dependent response for both recoverable and irrecoverable parts of deformation. That is, the response to an input depends on the input history and the rate of change of inputs. Most rate dependent materials also exhibit a strong temperature-based response. A creep and recovery test is a standard test that may be used to understand the behavior of time dependent materials. Figure 2.1(a) shows a creep and recovery test using a square stress loading pulse, σ , applied for time, t_l , followed by a long rest period, t_r . Figure 2.1(b) shows a typical total strain response for asphalt concrete. At the end of the long rest period, some of the deformation remains unrecovered and is called permanent deformation or viscoplastic deformation, ϵ_{vp} . The recoverable part of the deformation is called viscoelastic deformation, ϵ_{ve} and is modeled using viscoelasticity theories. The permanent deformation behavior of asphalt concrete has traditionally been modeled using viscoplasticity theories that are based on metal and polymer behavior.

2.1.1 Linear Viscoelasticity

The theory of linear viscoelasticity is applicable to the behavior of asphalt concrete at low stress and strain levels where the response to an input remains linear with respect to the input. The constitutive law for viscoelasticity may be stated as

$$\begin{aligned}\varepsilon(t) &= \int_{0^-}^t D(t-\tau) \frac{d\sigma}{d\tau} d\tau, \\ \sigma(t) &= \int_{0^-}^t E(t-\tau) \frac{d\varepsilon}{d\tau} d\tau,\end{aligned}\tag{2.1}$$

where $D(t)$ is the creep compliance and $E(t)$ is the relaxation modulus. The integration is performed from just before the start of loading as signified by $\tau = 0^-$. The integrals listed in (2.1) are called convolution integrals.

Figure 2.1 Creep and recovery test for asphalt concrete: a) stress history, and b) strain response.

The creep and relaxation functions are related as

$$\int_{0^-}^t D(t-\tau) \frac{dE(\tau)}{d\tau} d\tau = \int_{0^-}^t E(t-\tau) \frac{dD(\tau)}{d\tau} d\tau = 1.\tag{2.2}$$

Applying Laplace transforms to (2.2) (using s as the transformation variable) yields

$$(s\bar{E}(s))(s\bar{D}(s)) = \tilde{E}(s)\tilde{D}(s) = 1,\tag{2.3}$$

where $\bar{E}$ and $\bar{D}$ are the Laplace transforms of the relaxation and creep functions respectively, and $\tilde{E}$ and $\tilde{D}$ are the Carson transforms of the relaxation and creep functions respectively. The Laplace transform of a function, $f(t)$, is given by

$$\bar{f}(s) = \int_0^{\infty} f(t) \exp(-st) dt.\tag{2.4}$$

Equation (2.3) is the basis of the elastic-viscoelastic correspondence principle that draws parallels between elastic and viscoelastic materials. The creep and relaxation functions are typically modeled using mechanical analogs made up of spring and dashpot (viscous) elements. The Generalized Maxwell model, which is made up of Maxwell elements in parallel, can be used to model the relaxation function. Similarly, the Generalized Voigt model, which is made up of Voigt elements in series with each other and with a Maxwell element, can be used to model the creep function. A Maxwell model is made up of a spring and dashpot in series, while a Voigt model is made up of a spring and dashpot in parallel. Figure 4.3 shows a schematic of the Generalized Maxwell model and the Generalized Voigt model.

2.1.2 Viscoplasticity

Rate dependent permanent deformation is typically modeled using a Perzyna type viscoplasticity model such as

$$\dot{\epsilon}_{ij}^{vp} = \Gamma \langle f \rangle \frac{\partial F}{\partial \sigma_{ij}}, \quad (2.5)$$

where $\dot{\epsilon}_{ij}^{vp}$ is the viscoplastic strain rate tensor, f is an overstress function that is related to a yield surface, and F is a viscoplastic potential function that is used to give the direction of increase of the viscoplastic strain tensor. $\langle \cdot \rangle$ is the Macaulay bracket and is defined as

$$\begin{aligned} \langle h \rangle &= h, \text{ for } h > 0, \\ \langle h \rangle &= 0, \text{ for } h \leq 0. \end{aligned} \quad (2.6)$$

This construct ensures that the viscoplastic strain will not reduce. f is a measure of the distance of the applied stress from the yield surface. As the distance increases the driving force for viscoplastic deformation also increases. When the applied stress lies within or on the yield surface no viscoplastic deformation occurs. The functions f and F may typically be functions of one or more of the following: stress, σ ; strain, ϵ ; and quantities related to stress and/or strain histories. Hardening of the material under loading can be modeled using expansion of the yield surface (isotropic hardening) or translation of yield surface (kinematic

hardening). Different models are obtained by considering variations in f and F . The more common models assume $F = f$ (associative plasticity) with the overstress function based on standard yield functions like von Mises, Drucker-Prager, or Mohr-Coulomb yield functions. The Perzyna type viscoplasticity models have been successfully applied to metal and soil plasticity problems and are the dominant class of viscoplasticity models used in asphalt concrete. In the next section, the viscoplasticity models used for asphalt concrete will be discussed.

2.2 Viscoplasticity Models for Asphalt Concrete

2.2.1 A Simple Phenomenological Model

A simple uniaxial phenomenological model was suggested by Uzan (1996) and Schapery (1999) for characterizing permanent deformation behavior and is given by

$$\dot{\epsilon}_{vp} = \frac{g(\sigma)}{h(\epsilon_{vp})}, \quad (2.7)$$

where g and h are functions of stress, σ , and viscoplastic strain, ϵ_{vp} , respectively. This model was used to characterize the behavior of asphalt concrete in compression subject to monotonic strain rate tests by Gibson et al. (2003) by assuming the functions g and h to be of the form

$$\begin{aligned} h(\epsilon_{vp}) &= A\epsilon_{vp}^p, \\ g(\sigma) &= B\sigma^q, \end{aligned} \quad (2.8)$$

where A, B, p, q are model parameters. While the model had great success in modeling the behavior of asphalt concrete under monotonic loading, it was unable to accurately capture the secondary region behavior in creep and recovery tests. The reason for this may be explained by observing that the model is a special case of the form

$$\dot{\epsilon}_{vp} = k(\sigma, \epsilon_{vp}), \quad (2.9)$$

where k is a function of the stress, σ , and viscoplastic strain, ε_{vp} . In Chapter 3, it will be shown that viscoplastic models of the form $\dot{\varepsilon}_{vp} = k(\sigma, \varepsilon_{vp})$ may not be suitable for modeling compressive behavior of asphalt concrete under a large number of creep and recovery cycles.

2.2.2 Viscoplasticity Models based on Perzyna Model

2.2.2.1 Erkens (2002)

Erkens (2002) proposed a rate dependent plasticity model based on the Perzyna form for both tension and compression for asphalt concrete. The yield surface was based on the Hierarchical Single Surface (HiSS) model developed by Desai (Desai, 2001). The model was calibrated for a variety of monotonic loading rates in compression and for multiple temperatures. However, it is primarily a model for monotonic behavior and has not been applied for creep and recovery tests.

2.2.2.2 Gibson (2006)

Gibson (2006) proposed a three-dimensional compression model for asphalt concrete using the Perzyna form and based on the work of Erkens (2002). The model is given by

$$\begin{aligned}\dot{\varepsilon}_{ij}^{vp} &= \Gamma \left\langle f(F) \right\rangle \frac{\partial F}{\partial \sigma_{ij}}, \\ F &= J_2 - \left(\gamma (I_1 + R(\xi))^2 - \alpha(\xi) (I_1 + R(\xi))^n \right), \\ R(\xi) &= R_0 + R_A \xi^{k_2}, \\ \alpha(\xi) &= \alpha_0 \exp(\xi k_1), \\ \xi &= \int d\varepsilon_1^{vp} + \int d\varepsilon_2^{vp} + \int d\varepsilon_3^{vp}, \\ f(F) &= A \left(\frac{G(F)}{G_0(F)} - 1 \right)^N,\end{aligned}\tag{2.10}$$

where $\gamma, n, k_1, k_2, R_0, R_A, \alpha_0, A$, and N are the model parameters, F is the yield function with hardening, ξ is the viscoplastic strain trajectory, $G(F)$ is the distance of the applied stress point $(\sigma_1, \sigma_2, \sigma_3)$ in principal axes space from the hydrostatic line along the direction of

$\frac{\partial F}{\partial \sigma_i}$, and $G_0(F)$ is the distance of the flow surface, F , in principal axes space from the hydrostatic line along the direction of $\frac{\partial F}{\partial \sigma_i}$. I_1 and J_2 are the first invariant of stress and the second invariant of deviatoric stress respectively. In principal axes space, the two invariants may be written as

$$\begin{aligned} I_1 &= \sigma_1 + \sigma_2 + \sigma_3, \\ J_2 &= \frac{1}{6} \left((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right). \end{aligned} \quad (2.11)$$

The model was calibrated using one set of creep and recovery tests and validated using another set of creep and recovery tests. While the validation results were generally good, there was significant variability in some of the validation tests (as much as 40 % deviation of some replicates from the mean) that called the quality of model predictions into question. Also the testing protocol featured load times and stress levels designed to significantly accelerate the test so as produce strains of the order of 1 % - 2 % within 9 load cycles. In reality, such combinations of load times and load levels are not encountered in the field and the model may be said to be applicable more for the prediction of creep and recovery tests for a short number of load cycles.

The only source of load history dependence for the model comes from the viscoplastic strain trajectory, ξ . Under uniaxial (1-D) conditions and assuming that the stress is always compressive (does not cycle between tension and compression), the viscoplastic strain trajectory, ξ , at any given time, becomes the uniaxial viscoplastic strain, ε_{vp} , at that time (that is, not a function of the history of viscoplastic strain). Thus all functions in the model become functions of the current values of uniaxial viscoplastic strain, ε_{vp} , and the uniaxial stress, σ , (that is, not a function of the histories of viscoplastic strain and stress). Thus, although the model form is quite complicated and has many parameters, for the uniaxial compression case, it can be reduced to the form of

$$\dot{\epsilon}_{vp} = k(\sigma, \epsilon_{vp}), \quad (2.12)$$

which was encountered earlier in connection with the model phenomenological model proposed by Gibson et al (2003). Hence the model by Gibson (2006) may also not be suitable for modeling compressive behavior of asphalt concrete under a large number of creep and recovery cycles.

2.2.2.3 Dessouky et al. (2006)

Dessouky et al. (2006) proposed a viscoplastic model of the Perzyna-type with an extended Drucker-Prager yield surface for both tension and compression for asphalt concrete. The model also attempts to capture the effect of aggregate microstructure and damage progression on the material behavior. The model is given by

$$\begin{aligned} \dot{\epsilon}_{ij}^{vp} &= \Gamma \langle \phi(f) \rangle \frac{\partial g}{\partial \sigma_{ij}}, \\ \phi(f) &= f^N, \\ f &= \tau(\sigma_{ij}, \bar{\epsilon}_{vp}) - \alpha I(\sigma_{ij}, \bar{\epsilon}_{vp}, d, \Delta) - \kappa(\bar{\epsilon}_{vp}), \\ g &= \tau(\sigma_{ij}, \bar{\epsilon}_{vp}) - \beta I(\sigma_{ij}, \bar{\epsilon}_{vp}, d, \Delta), \\ \dot{\bar{\epsilon}}_{vp} &= \mu(\beta) \sqrt{\dot{\epsilon}_{ij}^{vp} \dot{\epsilon}_{ij}^{vp}}, \end{aligned} \quad (2.13)$$

where N, d, Δ, α , and β are material parameters. $\bar{\epsilon}_{vp}$ is the effective viscoplastic strain. This model was calibrated for monotonic tension and compression tests. However, the performance of the model remains unknown for creep and recovery tests. Under uniaxial (1-D) conditions and assuming that the loading is always compressive, it can be seen that the effective viscoplastic strain depends only on the current value of viscoplastic strain, ϵ_{vp} , and not on the history of viscoplastic strain. Using arguments similar to the ones used for the model proposed by Gibson (2006), it can be concluded that the model proposed by Dessouky et al. (2006) is also of the form, $\dot{\epsilon}_{vp} = k(\sigma, \epsilon_{vp})$. Hence the model proposed by Dessouky et al. (2006) may also not be suitable for modeling compressive behavior of asphalt concrete under a large number of creep and recovery cycles.

2.2.2.4 Levenberg (2009)

Levenberg (2009) proposed a three-dimensional viscoplastic model based on the Perzyna framework for cyclic compression and tension tests for asphalt concrete. The model is also capable of handling tension-compression stress reversals. The uniaxial version of the model is given by

$$\begin{aligned}\dot{\epsilon}_{vp} &= \left\langle \frac{f}{K} \right\rangle^n \frac{\text{sgn}(\sigma)}{t_*}, \\ f &= (\sigma - x)\sigma, \\ \dot{x} &= H \left(1 + \frac{x}{D} \right) \dot{\epsilon}_{vp},\end{aligned}\tag{2.14}$$

where K, n, H , and D are model parameters, t_* is a reference time that maintains dimensional consistency, and x is a stress-like quantity that controls hardening. The drawback of the model is that it is limited to predicting creep and recovery tests with only a few load cycles and very low permanent strain levels (the maximum strain level encountered in the tests was 0.06 % in compression). Given the combination of being restricted to modeling tests with very low strain levels and a few cycles of load application, it is not clear if this model is applicable for modeling the behavior of asphalt concrete in compression subjected to a large number of load cycles.

2.2.2.5 Yun (2008)

Recognizing the need to augment the Perzyna type models with hardening functions that have a stronger dependence on load history, Yun (2008) proposed the following uniaxial model for compressive loading for asphalt concrete:

$$\begin{aligned}\dot{\epsilon}_{vp} &= \left\langle \frac{\sigma - G}{D} \right\rangle^m \text{sgn}(\sigma - G), \\ \dot{G} &= g(\dot{\sigma}, \sigma, G),\end{aligned}\tag{2.15}$$

where D and m are model parameters, and G is a yield surface that uses viscoelastic-like integrals to capture hardening and softening effects. Owing to the extremely complex nature

of the function, G , details are not provided here and can be found in Yun (2008). The model assumes that rest periods soften the material, while loading hardens the material. The model was fit using square shaped creep and recovery tests with several hundreds of cycles of loading. The fits are generally good; however the model does not explain the behavior of other kinds of creep and recovery tests well, such as square shaped loads with the same load time, and creep and recovery tests with haversine loading with the same load time. However, of all the models presented, this model comes closest in addressing the needs for viscoplastic modeling in asphalt concrete.

2.2.3 A Model Based on Mechanical Analogs

Drescher et al. (2010) proposed a model for total deformation of asphalt concrete in both tension and compression based on a mechanical analog. The mechanical analog consists of a Generalized Maxwell model in parallel with an elastoplastic element (spring in series with a slip device that yields (slips) beyond a certain stress). Hardening was introduced into the model through an increase in the yield threshold as a function of plastic strains. Energy principles were used to extend the uniaxial mechanical analog model to three dimensions. The model is not a dedicated viscoplastic model; instead the strain predictions include both viscoelastic and viscoplastic strain components. The model appears to be in a preliminary stage of development and no model fits to experimental data have been shown for either monotonic tests, or creep and recovery tests. The model however shares some common elements with the model being proposed in this research.

2.3 Limitations of Existing Viscoplastic Models for Asphalt Concrete for Cyclic Compressive Creep and Recovery Tests

As seen from the review of existing viscoplastic models for asphalt concrete, the best models to date are either applicable to monotonic compression tests or compressive creep and recovery tests limited to either a handful of cycles, or low strain levels. In addition, many of these models are unintuitive and lack physical explanation. The learning curve for many of these models can be steep, and getting an intuitive understanding for how these models

would behave under certain kinds of tests would be difficult given the complex nature of the models. Most of the models are also calibrated using a blackbox-like optimization scheme that further distances the user from gaining a physical understanding of the model.

2.4 Objectives for Viscoplasticity Modeling of Asphalt Concrete

The primary objective of the research is to propose a uniaxial viscoplasticity rate model in compression that is applicable to experiments with: 1) realistic number of load cycles (at least several hundreds of cycles, with the ability to model the secondary creep region), 2) realistic loading shapes (haversine loading) and load levels, and 3) realistic rest periods between load pulses, i.e. long enough to relax the material based on truck frequency. The secondary objectives are to 1) propose a model that is intuitive to understand, and 2) propose an incremental model based on the rate model that can be used to better understand the behavior of the rate model. The success of the modeling effort would make rutting performance evaluations possible when the model is integrated into structural finite element models.

Chapter 3 Experimental Observations

The key to a successful rate model for asphalt concrete in compression lies in understanding its behavior under repeated creep and recovery loading. This chapter briefly describes various experiments used in modeling and presents the key experimental observations that lead to model development. All experiments used in the current model development were performed by fellow doctoral students Yun (2008) and Choi (Hot-Mix Asphalt Performance-Related Specifications Based on Viscoelastoplastic Continuum Damage (VEPCD) Models, 2010 and 2011) at North Carolina State University.

3.1 Experimental Details

A variety of tests were performed to better understand the material behavior. Repetitive creep and recovery tests were performed at 54°C for different load shapes (square and haversine), peak deviatoric stress, load times, and rest periods. All tests used in this research are repeated creep and recovery tests with the confining pressure set at 140 kPa (20.3 psi). The specific tests used in understanding material behavior, model characterization, and model validation will be discussed at the appropriate point in the dissertation.

With the exception of the section dealing with the effect of peak deviatoric stress, the experimental observations in this chapter are based on tests with square and haversine load shapes with a peak deviatoric stress of 827 kPa (120 psi). During the loading period, a haversine load shape is defined as $\sigma(t) = \sigma_0 \sin^2(\pi t/t_l)$, where σ_0 is the peak stress, and t_l is the loading time. Each applied load was followed by a rest period. Three different loading times, 0.1, 0.4 and 1.6 s, were applied. Two different rest periods, 0.9 and 100 s, were applied as a short and a long rest period, respectively. The long rest period is associated with complete recovery of viscoelastic strains, thereby allowing the measurement of permanent strain. Two different kinds of load shapes were tested: square-shaped loads (denoted as SQ hereinafter), and haversine-shaped loads (denoted as HV hereinafter). Therefore, a total of 12 loading conditions were used in testing: three loading times, two rest periods, and two load

shapes. The following terminology is used hereinafter to denote loading conditions: 0.1 s-100 s SQ indicates that a square-shaped pulse is applied for 0.1 s followed by a 100 s rest period. In the case of a short rest period, since a 0.9 s rest period is inadequate for complete viscoelastic strain recovery, the test incorporates a block of such loads followed by a long rest period of 200 seconds to measure the permanent strain accumulated at the end of the block. A new block would then follow. Table 3.1 shows the blocks used for both SQ and HV tests with short rest periods. More details on experimental setup may be found in Yun (2008).

Table 3.1 Loading Matrix for Short Rest Period Tests

Loading Condition	Cycles in Loading Block	Rest Period after Loading Block (s)	Number of Block Repetitions
0.1 s-0.9 s	40	200	2
	80	200	1
	160	200	1
	320	200	4
	640	200	8
0.4 s-0.9 s	10	200	2
	20	200	1
	40	200	1
	80	200	4
	160	200	8
1.6 s-0.9 s	5	200	2
	10	200	1
	20	200	4
	40	200	8

It is important to draw a distinction between the HV tests performed in this study and typical TRLPD (Triaxial Repeated Load Permanent Deformation) or Flow Number (FN) tests. In the TRLPD or FN tests, the rest period is fixed at 0.9 s (assuming a 0.1 s loading time). Because 0.9 s is not enough time to allow complete viscoelastic recovery, the strains at the end of the 0.9 s rest period include some viscoelastic unrecovered strains. On the other hand, the HV tests performed here provide a 200 s rest period between blocks, and only the

strains measured at the end of these blocks are used in the analysis. Hence, the strains obtained from such a protocol are true permanent strains.

3.2 Trends in Permanent Strain Evolution

Using the available test data, a few important observations can be made based on the permanent strain, $\varepsilon_{vp}(N)$, at the end of the load pulse, the pulse number, N , at which point the strain is measured and the incremental permanent strain in a pulse, i.e., $\Delta\varepsilon_{vp}(N) = \varepsilon_{vp}(N) - \varepsilon_{vp}(N-1)$. These observations include the following:

1. For long rest periods (>100 s), in general, $\Delta\varepsilon_{vp} = A_1 N^{-\beta}$, where A_1 and β are parameters. β is typically between 0.75 and 0.95 for the test conditions used. A_1 depends on the test condition. These trends are shown in Figure 3.1(a) for the SQ tests and in Figure 3.1(b) for the HV tests.
2. For short rest periods (0.9 s), in general, $\varepsilon_{vp} = A_2 + B_2 \log(N)$, i.e., permanent strain follows a semi-log dependence with respect to N . A_2 and B_2 depend on the test condition. Note that this equation can be written in the earlier power law form for long rest periods with $\beta = 1$. That is, differentiation of the semi-log form yields $d\varepsilon_{vp}/dN = B_2/N$. For a large enough N , the local permanent strain rate in a pulse would be approximately equal to the average permanent strain in a pulse, which can be written as $d\varepsilon_{vp}/dN \approx \Delta\varepsilon_{vp}/\Delta N$. Setting $\Delta N = 1$ (because the permanent strain increment is over a single pulse) provides $\Delta\varepsilon_{vp} \approx B_2/N$. This form is of the same type as the power law form for long rest periods, but with $\beta = 1$. In short, the rest period affects the power, β . These trends are shown in Figure 3.1(c) for the SQ tests and in Figure 3.1(d) for the HV tests. The only tests that do not seem to follow the semi-log rule are the SQ 1.6 s-0.9 s tests.
3. Shorter rest periods make the material appear harder, whereas longer ones tend to make the material appear easier to deform. For example, using two SQ tests with 1.6 s load time and different rest periods of 0.9 s and 100 s, the test with the long rest period will show a significantly higher permanent deformation as a function of pulse number, N , than the test

with a short rest period. The only exception to this occurrence is the HV test with 1.6 s load time, where the 0.9 s rest period test has higher deformation on average than the 100 s rest period test.

Note that the observation regarding short rest periods seems to contradict well-established models for the secondary region from FN tests wherein the permanent strain versus number of loading cycles relationship is linear in log-log scale (i.e., the power law in arithmetic scale) (Advanced Research Associates, 2004). However, as indicated earlier, the ‘permanent’ strains measured in a FN test include some recoverable viscoelastic components owing to the lack of adequate rest for complete relaxation. It is believed that the addition of viscoelastic component changes the apparent evolution of permanent strain, making a semi-log type behavior appear like a power law.

In addition, a unifying characteristic of the test data mentioned is that they reveal the inherent hardening of the material with increasing number of load repetitions. That is, with each additional load pulse of the same shape and characteristics, the incremental deformation keeps reducing. This is indicated by the plots of Figure 3.1(a) and Figure 3.1(b) for long rest period tests. The same can be inferred for the short rest period tests by noting that the permanent strain evolutions for these tests follow a semi-logarithmic trend as a function of pulse number as explained earlier. The hardening occurs only in compressive loading and is due to the effect of internal microstructure (interaction of aggregates with asphalt binder). Any model for asphalt concrete in compression will need to account for hardening correctly in order to be successful.

Figure 3.1 Power law behavior of permanent strains for: a) 100 s rest period SQ Tests, b) 100 s rest period HV tests, c) 0.9 s rest period SQ tests, and d) 0.9 s rest period HV tests.

3.3 Effect of Load History on Permanent Strain Evolution

Experiments conducted at NC State University can be used to demonstrate that certain traditional viscoplasticity forms cannot explain the behavior observed in asphalt concrete. Consider a simple version of the Perzyna type law (uniaxial case):

$$\dot{\epsilon}_{vp} = \Gamma \langle f(\sigma) \rangle \frac{dF(\sigma)}{d\sigma}, \quad (3.1)$$

where Γ is a fluidity parameter like viscosity; $f(\sigma)$ is an overstress function that governs the rate of permanent strain growth; $F(\sigma)$ is the yield surface and $\langle \cdot \rangle$ is the Macaulay bracket. Note that all functions depend only on stress, σ in this case and are not dependent on the history of stress, σ , or strain, ϵ_{vp} . Equation (3.1) can in fact be considered a simplified form of a model proposed elsewhere (Uzan, 1996 and Schapery, 1999). Their model is of the form

$$\dot{\epsilon}_{vp} = \frac{g(\sigma)}{h(\epsilon_{vp})}, \quad (3.2)$$

where g and h are functions of stress, σ , and viscoplastic strain, ϵ_{vp} , respectively. For such a model, the viscoplastic strain rate depends only on the current values of stress and viscoplastic strain. Equations (3.1) and (3.2) can in turn be considered as special cases of the form

$$\dot{\epsilon}_{vp} = k(\sigma, \epsilon_{vp}) \quad (3.3)$$

that was encountered in Chapter 2. It will be shown that such a model form may not be suitable for modeling the behavior of asphalt concrete subject to a large number of compressive creep and recovery cycles.

Tests conducted by Yun (2008) show that the strain rate depends not only on the current values of stress and viscoplastic strain, but possibly on the past history of the loading. Two kinds of tests are used to prove this idea: the SQ tests discussed earlier and a Variable Load

Time test (SQVT) that applies a set of pulses with load times 0.05, 0.1, 0.2, 0.4, 1.6 and 6.4 s, with each pulse followed by a 200 s rest. At the end of the 6.4 s loading, the cycle of different loading times repeats. For the SQ tests, each pulse is followed by a 200 s rest period. Both tests are conducted at a confinement (hydrostatic) pressure of 140 kPa with a peak deviatoric stress of 827 kPa. The incremental strains for the pulses are extracted and plotted against the viscoplastic strain in Figure 3.2. For a given load time, the average permanent strain rate in a pulse is equal to $\Delta\epsilon_{vp}/t_l$, where t_l is the load time. For large enough permanent strains, the average strain rate would be quite close to the strain rates at various points in the load pulse. Because the stresses are the same in each SQ and SQVT test, the incremental permanent strains for a particular load time in the SQVT test, say 0.1 s, should coincide with that of the 0.1 s SQ test at the same level of permanent strain, if Equation (3.3) is valid. Figure 3.2 shows that this phenomenon occurs only for 1.6 s load time and does not happen for 0.1 s load time. The behavior with the 0.4 s load time is somewhere between the two. These data show that history may affect the evolution of permanent strains and that a state variable may be needed that can track the past loading history applied on the material. Models for viscoplasticity that reduce to (3.3) may not be suitable for compressive creep and recovery loading for asphalt concrete.

3.3.1 Current Viscoplasticity Models Revisited

As seen in the previous section, models of the form $\dot{\epsilon}_{vp} = k(\sigma, \epsilon_{vp})$ may not be suitable for compressive creep and recovery loading for asphalt concrete. In Chapter 2, it was shown that the models proposed by Uzan (1996), Schapery (1999), Gibson et al (2003), Gibson (2006) and Dessouky et al (2006) reduce to the form $\dot{\epsilon}_{vp} = k(\sigma, \epsilon_{vp})$ in the case of uniaxial compression-only loading. Hence these models may not perform well for cyclic creep and recovery in compression. The models proposed by Levenberg (2009) and Yun (2008) do not reduce to the form $\dot{\epsilon}_{vp} = k(\sigma, \epsilon_{vp})$, but have limitations when applied to creep and recovery loading in compression as discussed in Chapter 2.

Figure 3.2 Plot of permanent strain vs. incremental permanent strain in each pulse for SQ and SQVT tests.

3.4 Effect of Peak Deviatoric Stress

The behavior of asphalt concrete in compression is strongly dependent on the applied peak stress. This can be illustrated using tests conducted by Choi at North Carolina State University. Creep and recovery tests with haversine load shape and long rest periods (100 s) were conducted for three different load times; 0.1 s, 0.4 s, and 1.6 s, and for three different peak deviatoric stress levels; 620 kPa, 827 kPa, and 1034 kPa. The results from these tests are shown in Figure 3.3 to Figure 3.5. A 25 % reduction of peak stress from 827 kPa to 620 kPa results in a permanent strain evolution that is within 25 % on average. In the case of 0.1 s-100 s HV, the replicates for the 620 kPa test condition seems to overlap with those of the 827 kPa test condition, however, the average of the replicates for the 827 kPa test condition is higher than the average of the replicates for the 620 kPa test condition. A 25 % increase in peak stress from 827 kPa to 1034 kPa, however, results in a 100 % increase in permanent

strain. This shows that asphalt concrete behaves in a highly nonlinear fashion for stress levels higher than 827 kPa. A model for asphalt concrete in compression would need to handle the nonlinear behavior with respect to stresses.

Figure 3.3 Plot of permanent strain vs pulse number for 0.1 s-100 s HV tests for three peak deviatoric stress levels: 620 kPa, 827 kPa, and 1034 kPa.

Figure 3.4 Plot of permanent strain vs pulse number for 0.4 s-100 s HV tests for three peak deviatoric stress levels: 620 kPa, 827 kPa, and 1034 kPa.

Figure 3.5 Plot of permanent strain vs pulse number for 1.6 s-100 s HV tests for three peak deviatoric stress levels: 620 kPa, 827 kPa, and 1034 kPa.

3.5 Summary

Thus in summary, one can learn the following things from the experimental data:

1. Asphalt concrete under compression exhibits hardening behavior that significantly depends on the load history applied.
2. For long rest periods, the incremental permanent strains seem to follow a power-law dependence as a function of the pulse number.
3. For short rest periods, the permanent strains seem to follow a semi-log dependence as a function of the pulse number.
4. Viscoplasticity models that reduce to the form $\dot{\epsilon}_{vp} = k(\sigma, \epsilon_{vp})$ may not be suitable for modeling cyclic compression behavior of asphalt concrete.
5. Stress level seems to have a highly nonlinear effect on asphalt concrete in compression.

Chapter 4 Rate Model Development

Starting with the trends from experimental data, the rate model went through numerous versions before converging on the final version used in this dissertation. It is quite conceivable that this final version may also undergo modifications in the future as the needs for modeling evolve. All the model forms attempted to capture the experimental data trends in some way. Herein follows a discussion of the latest version of the rate model, its physical aspects, model parameters and their effect on permanent strain evolution, and numerical implementation details. The chapter then concludes with a brief discussion of the history of model development.

4.1 Proposed Rate Model

Additive decomposition of strains is first assumed, i.e.,

$$\varepsilon(t) = \varepsilon_{ve}(t) + \varepsilon_{vp}(t), \quad (4.1)$$

where $\varepsilon(t)$ is the total strain and $\varepsilon_{ve}(t)$ is the viscoelastic (VE) strain. The proposed model focuses on modeling viscoplastic strains, $\varepsilon_{vp}(t)$ alone and hence does not provide a model form for viscoelastic strains, $\varepsilon_{ve}(t)$. The following rate model is proposed as a means to capture viscoplastic behavior of asphalt concrete in compression.

$$\begin{aligned} H(t) &= H(t_{ini}) + \left\langle \int_{t_{ini}}^t D_2(t-\tau) \frac{d\sigma(\tau)_{b,H}}{d\tau} d\tau \right\rangle, \\ \varepsilon_{vp}(t) &= \left\langle \int_{0^-}^t D_1(t-\tau) \frac{d\sigma(\tau)_{b,vp}}{d\tau} d\tau \right\rangle, \end{aligned} \quad (4.2)$$

During Loading:

$$\begin{aligned} \sigma(t)_{b,H} &= H_1 \sigma(t)^q, \\ \sigma(t)_{b,vp} &= \frac{(G_1 \sigma(t))^{p_1} + (G_2 \sigma(t))^{p_2}}{H(t)^\alpha}, \end{aligned}$$

where H is a state variable related to hardening in the material; t_{ini} is the point from which model prediction starts; $H(t_{ini})$ is the initial value of H , and serves two purposes: representation of initial hardening in the material, and numerical regularization of the integral; $\sigma_{b,H}$ is the back stress associated with hardening; $\sigma_{b,vp}$ is the back stress associated with permanent strain evolution; and α is a model parameter that governs the effect of hardening on permanent strain evolution. $D_1(t)$ and $D_2(t)$ are creep compliances similar to those used in viscoelasticity, but they are not related to the viscoelastic creep compliance. The parameters H_1, G_1, G_2, q, p_1 , and p_2 serve to capture the effect of stress level on permanent strain evolution. The brackets $\langle \rangle$ are called the Macaulay brackets and in the context of this model are defined as,

$$\begin{aligned} \frac{d}{dt}\langle f \rangle &= \dot{f}, \quad \dot{f} \geq 0, \\ \frac{d}{dt}\langle f \rangle &= 0, \quad \dot{f} < 0. \end{aligned} \tag{4.3}$$

Note that this is a different definition for the Macaulay brackets. This definition creates the back stress in the model. Essentially, the brackets ensure that the hardening function and the permanent strain function will never decrease even when the load reduces or becomes zero. In the case of the hardening function, an assumption is made that it does not decrease at any time during the loading history, and in the case of permanent strain, by definition it cannot decrease. Hereinafter, the convolution integral is represented as

$\left\langle \int_{-\infty}^t D(t-\tau) \frac{d\sigma_b}{d\tau} d\tau \right\rangle = \langle D * \sigma_b \rangle$ when convenient. It should be noted that there are two convolution integrals, $D * \sigma_b$, in the model (with $D(t)$ being either $D_1(t)$ or $D_2(t)$) and discussions related to the behavior of $D * \sigma_b$ in general are applicable to both integrals. Various aspects of the model and the motivation behind the parameters in the model will be explained subsequently in this chapter.

4.2 Understanding the Model

The physical representation of the integral is shown in Figure 4.1(a) where a VE model with a creep compliance of $D(t)$ is in parallel with a slip device. The back stress, σ_b , is the stress in the viscoelastic element and is equal to the applied loading, σ , when the material is being loaded and $D^*\sigma_b$ is increasing. When $D^*\sigma_b$ has a tendency to reduce due to unloading, the slip element locks to keep $D^*\sigma_b$ constant and allows the viscoelastic element to relax. When $D^*\sigma_b$ begins to increase again as a result of loading, the slip element allows the VE model to deform. In case the material involves a minimum stress threshold before undergoing permanent deformation, the applied stress, σ , may be replaced with $\langle \sigma - \sigma_y \rangle$, where σ_y is a constant stress below which no permanent deformation occurs.

4.2.1 Back Stress

The motivation for using back stress can be explained as follows. Physically, one can imagine that as compressive loading is applied on a specimen of asphalt concrete, the aggregates would begin to come closer, with the mastic (asphalt binder and fine particles) getting compressed. It is expected that load paths would form within the aggregate structure with continued loading and the aggregate structure would become denser thereby resisting further deformation and hence appearing harder to deform. Because the aggregate structure is significantly stiffer than the mastic at high temperatures, e.g. 54°C, the mastic may be unable to push the aggregates apart as the mastic recovers from compressive deformation during unloading. This situation may be likened to how the slip device in the model holds the strain in the VE model constant when there is a tendency for the strain across the slip device to decrease. Because the strain in the mastic is not allowed to recover, the stress in the mastic gradually starts to relax as a result of binder viscoelastic effects. In the model, this relaxation has the potential to affect the future evolution of deformation. No additional deformation is possible in the VE model until the applied stress exceeds the back stress in the VE model. Figure 4.1(b) summarizes these ideas for square load pulses with rest periods.

Figure 4.1 a) Mechanical analog equivalent of $\langle D^* \sigma_b \rangle$, b) Schematic of back stress and $\langle D^* \sigma_b \rangle$ evolution for square loading pulses with rest periods and c) Effective area of applied load as a function of rest period.

4.2.2 Effect of Rest Period

Rest periods play an interesting role in the mechanical model shown in Figure 4.1(a). When a long enough rest period is provided, the slope of the back stress becomes constant by the end of the rest period, and, depending on the nature of $D(t)$, the back stress can become zero given enough rest. Under such conditions, the full applied load contributes to the deformation in the subsequent pulse. If the rest period is shorter than the minimum rest period for complete relaxation, only a portion of the applied load in the next pulse will be able to produce deformation as shown in Figure 4.1(c). Thus the model has the potential to produce additional hardening in the two integrals, $\langle D * \sigma_b \rangle$ for short rest period tests. This effect was however found to be insufficient to produce all of the hardening observed in the experiments and the explicit hardening function H had to be incorporated into permanent strain evolution. Note also that the effect of rest period is seen only for short rest periods; for long rest periods, the explicit hardening function is necessary.

4.2.3 Steady State Conditions

For certain creep functions, $D(t)$ the application of repeated loads of the same shape and rest period leads to the development of steady state conditions. That is, the incremental growth in $\langle D * \sigma_b \rangle$ with each pulse becomes constant, and the back stress at the end of the rest period becomes the same for all subsequent pulses. This phenomenon occurs, for example, for generalized Maxwell models with the Maxwell elements in parallel (and no spring or dashpots in parallel) as well as for generalized Voigt models with the Voigt elements in parallel and a Maxwell element in series. A Maxwell element has a spring and dashpot in series whereas a Voigt element has a spring and dashpot in parallel. The number of pulses needed to reach steady state depends on the load time, rest period and the shape of $D(t)$. For a long enough rest period, t_r steady state is reached in the first pulse, as long as $E(t_r) = 0$, where $E(t)$ is the relaxation function corresponding to $D(t)$. Once steady state is reached,

the integral $\langle D^* \sigma_b \rangle$ will increase linearly. Note that the peak value of back stress associated with the hardening function, H , remains constant and hence the hardening function H , can reach steady state. Steady state will typically not be reached for the integral describing permanent strain evolution because the peak back stress (effective applied stress) for the integral steadily decreases in magnitude owing to hardening.

4.3 Model Aspects

Having gotten a physical understanding of the model, it would now be instructive to look at different components of the model and to understand their role in the model behavior. This section will focus on three aspects: the stress dependence part of the model, the creep functions used in the model, and the evolution of the hardening function.

4.3.1 Stress Dependence

All of the stress dependence functions in the model are based on power laws (the one for permanent strain evolution is a sum of two power laws). Based on the results from test data shown in Chapter 3 regarding the effect of stress level, power law functions seem to be a good choice for capturing the effect of stress. The power law function has two effects: modification of peak stress, and change of applied load shape. The change of load shape has some interesting consequences for the model and will be discussed in more detail.

Recall that the stress dependence functions shown in (4.2) were $H_1 \sigma(t)^q$ for hardening evolution and $(G_1 \sigma(t))^{p_1} + (G_2 \sigma(t))^{p_2}$ for permanent strain evolution. The parameters, H_1 , G_1 and G_2 produce linear scaling of the loads. The powers, q , p_1 and p_2 are responsible for load shape changes. No load shape changes occur when the applied loading is a square shaped load. In the case of a haversine shaped load however, the powers produce a change in effective load shape. Normalized plots of the function $\sigma(t)^q$ are shown in Figure 4.2 as a function of the power q . $\sigma(t) = \sigma_{\max} \sin^2(\pi t/t_l)$, where $\sigma_{\max}$ is the peak stress and t_l is the loading time. When $q = 1$, the original haversine loading is recovered. For $q < 1$, the

modified load shape has a greater area than the haversine shape and begins to approach the square shape as q tends to zero. At $q = 0$, $\sigma(t)^q$ has a square shape with the following rule: $\sigma(t)^0 = 0$, when $\sigma(t) = 0$, $\sigma(t)^0 = 1$, otherwise. When $q > 1$, the modified load shape has a smaller area than the haversine shape and the apparent load time approaches zero as q tends to ∞ . These features may be used to control the evolution of hardening and permanent strain as needed.

Figure 4.2 Normalized plot of $(\sigma(t))^q$ as a function of q .

4.3.2 Creep Functions

The two creep functions, $D_1(t)$ and $D_2(t)$, are based on Generalized Voigt Models (Prony models). Such models have been used extensively for modeling viscoelasticity. Figure 4.3 shows the two kinds of mechanical analogs used to model viscoelasticity. The first shows the Generalized Maxwell model that is made up of Maxwell elements in parallel. The second

shows a Generalized Voigt model that is equivalent to the Generalized Maxwell model as there exists a one-to-one correspondence between the parameters of each when the responses are identical. The Generalized Voigt model has Voigt models in series with each other and with a Maxwell element.

A creep function may be modeled using a Generalized Voigt model with the creep function given as:

$$D(t) = D_0 + \sum_{i=1}^{n-1} D_i (1 - \exp(-t/\rho_i)) + D_n t, \quad (4.4)$$

where $(n-1)$ indicates the number of Voigt elements in series, D_0 and D_i are spring compliances (inverse of stiffness), and ρ_i are called creep retardation times. The retardation times are related to the spring compliance, D_i , and the dashpot viscosity, η_i , of each Voigt element as $\rho_i = D_i \eta_i$. The viscosity term, $D_n = D_0/\rho_0$, is associated with the linear term in the creep compliance and is responsible for long term flow under a creep load for the model.

A relaxation function may be modeled using a Generalized Maxwell model as follows:

$$E(t) = \sum_{i=1}^n E_i \exp(-t/\tau_i), \quad (4.5)$$

where n indicates the number of Maxwell elements in parallel (n may also be called the number of Prony terms), E_i is the spring stiffness for each Maxwell element, and τ_i is the relaxation time for each Maxwell element. The relaxation time is related to E_i and the dashpot viscosity, ν_i , of each Maxwell element as $\tau_i = \nu_i/E_i$. Since the creep function and the relaxation function for viscoelastic behavior are interrelated, given the parameters for the Generalized Maxwell model for relaxation, one can determine the parameters for the creep function of the same material in terms of the Generalized Voigt model. In this research, the viscoelastic element in Figure 4.1(a) is modeled as a Generalized Maxwell model and the integrals are calculated using the parameters of such a model. The parameters of a Generalized Maxwell model will hereinafter be denoted as:

$E = [E_1, E_2, \dots, E_n]$; $\tau = [\tau_1, \tau_2, \dots, \tau_n]$. The parameters of a Generalized Voigt model will hereinafter be denoted as: $D = [D_0, D_1, \dots, D_{n-1}]$; $\rho = [\rho_0, \rho_1, \dots, \rho_{n-1}]$.

Figure 4.3 Mechanical analogs for viscoelasticity: a) Generalized Maxwell model, b) Equivalent Generalized Voigt model in terms of spring compliances.

4.3.3 Hardening Evolution

The hardening function in the rate model is assumed to also follow a viscoelastic-like evolution. This stems from an assumption that the hardening and deformation processes may be time dependent and related to the same mechanisms. When the hardening function is modeled using a Generalized Maxwell model in parallel with a slip device, then steady state conditions may eventually be reached depending on the loading characteristics and the hardening function parameters. Once steady state is reached, the subsequent hardening function becomes linear in the number of pulses.

Figure 4.4 shows the evolution of the hardening function for a creep and recovery test with square pulses. In this figure, the hardening function values at the end of the load pulses are plotted. The load time is 0.4 s, the rest period is 0.9 s, and the peak stress is 1 kPa. The stress dependence parameters are $H_1 = 1$; $q = 1$, and the hardening function parameters are taken as $D = [0.05, 0.9] \text{ kPa}^{-1}$; $\rho = [5, 0.5] \text{ s}$. The simulation is done for 50 cycles. As can

be seen from the figure, the hardening function, $H(N)$ increases rapidly initially and eventually reaches steady state, at which point the increase in $H(N)$ with each additional pulse is the same and much smaller than the initial increase.

Figure 4.4 Example for evolution of hardening function at the end of square pulses with steady state being reached subsequently.

Figure 4.5 Example for evolution of hardening function as a function of time for square pulses with steady state being reached subsequently.

Figure 4.5 shows the hardening function, $H(N)$, as a function of time for the same simulation for the first few pulses. This reveals how the hardening function changes over the first few cycles as a function of loading and rest period. During the rest periods, in accordance with the model constraints, the hardening function remains constant. The hardening function increases only during loading and the increase in the hardening function can be seen to decrease with subsequent load pulses until reaching a constant value.

4.3.4 Effect of Hardening on Permanent Strain Evolution

The purpose of the hardening function is to retard the growth of permanent strain with subsequent cyclic compressive loading. This is achieved by considering the effective load,

$$\sigma(t)_{b,vp} = \frac{(G_1\sigma(t))^{p_1} + (G_2\sigma(t))^{p_2}}{H(t)^\alpha}, \text{ where the function } H(t)^\alpha \text{ scales the applied stress for}$$

permanent deformation (see equation (4.2)). Since $H(t)$ increases monotonically, $\sigma(t)_{b,vp}$ will decrease monotonically as long as the load shape and load amplitude remain constant. This will result in the incremental permanent strain in each pulse steadily decreasing, which is the effect we are targeting.

Figure 4.6 shows the evolution of the effective load $\sigma(t)_{b,vp}$ for a creep and recovery test with square pulses. The load time is 0.4 s, the rest period is 10 s, and the peak stress is 1 kPa. The stress dependence parameters are $G_1 = 1$; $p_1 = 1$; $G_2 = 0$; $p_2 = 1$, and the hardening function parameters are taken as $D = [0.05, 0.9] \text{ kPa}^{-1}$; $\rho = [5, 0.5] \text{ s}$. α is taken as 0.9. The simulation is done for 50 cycles. The monotonic reduction of peak effective load, $\sigma(t)_{b,vp}$, with time can be clearly seen. Figure 4.7 shows the same plot zoomed in over the first few cycles to show more detail of how the hardening function also changes the shape of the effective load slightly and this effect is greatest in the initial region and diminishes with subsequent pulses when the change in the hardening function over a pulse becomes small compared to the overall hardening function value.

Figure 4.6 Example for effective load $\sigma(t)_{b, vp} = \sigma(t)/H^\alpha$ as a function of time for square pulses.

Figure 4.7 Effective load $\sigma(t)_{b, vp} = \sigma(t)/H^\alpha$ as a function of time for square pulses zoomed in over the first few pulses to show detail.

4.4 Numerical Computation of Integrals

The convolution integrals listed in the rate model are evaluated numerically. While closed form solutions exist for computing the integral for square pulses, such solutions do not exist for haversine pulses and for the applied load subject to the stress dependence functions used in the model. In addition, the stress dependence functions also make it difficult to get closed form solutions. Hence all integrals are computed numerically. The numerical algorithm for

computing the integral, $I(t_{n+1}) = \left\langle \int_{-\infty}^{t_{n+1}} D(t-\tau) \frac{d\sigma_b}{d\tau} d\tau \right\rangle = \langle D * \sigma_b \rangle$, is given below.

The back stress, σ_b , is not exactly the same as the applied stress, σ . Firstly, σ_b is the stress in the viscoelastic element in Figure 4.1(a) and that is different from stress, σ , applied on the entire mechanical analog. The entire history of σ is known, whereas σ_b depends on the load history and is calculated along with the integral, I . The applied load, σ , for the convolution integral may include stress dependence functions as well as scaling by hardening functions. The creep functions are described using Generalized Voigt model and the parameters are denoted as $D = [D_0, D_1, \dots, D_{m-1}]$; $\rho = [\rho_0, \rho_1, \dots, \rho_{m-1}]$, where $(m-1)$ is the number of Voigt elements in parallel as shown in Figure 4.3(b). The initial state assumptions are: $I(0) = 0$, $\sigma(0) = 0$, $\sigma_b(0) = 0$. $I(t_n)$, $\sigma(t_n)$, $\sigma(t_{n+1})$ and $\sigma_b(t_n)$ are assumed to be known. The following steps are involved in the calculation of $I(t_{n+1})$, $\sigma_b(t_{n+1})$:

Step 1: Calculate $I(t_{n+1})$ using $\Delta\sigma_{n+1} = \sigma(t_{n+1}) - \sigma_b(t_n)$; $\Delta t_{n+1} = t_{n+1} - t_n$;

$$\begin{aligned}
I(t_{n+1}) &= I_0(t_{n+1}) + \sum_{i=1}^{i=m-1} I_i(t_{n+1}) + I_m(t_{n+1}), \\
I_0(t_{n+1}) &= D_0 \sigma(t_{n+1}), \\
I_i(t_{n+1}) &= D_i (\sigma(t_{n+1}) - \sigma_i^R(t_{n+1})), \\
\sigma_i^R(t_{n+1}) &= \exp(-\Delta t_{n+1}/\rho_i) \sigma_i^R(t_n) + \frac{(1 - \exp(-\Delta t_{n+1}/\rho_i))}{\Delta t_{n+1}/\rho_i} \Delta \sigma_{n+1}, \\
I_m(t_{n+1}) &= I_m(t_n) + \frac{1}{2} \frac{D_0}{\rho_0} \Delta t_{n+1} \Delta \sigma(t_{n+1}),
\end{aligned} \tag{4.6}$$

where $I_0(t_{n+1})$ and $I_m(t_{n+1})$ are the strains in (or integral values associated with) the spring and the dashpot element in the Maxwell element in series with the Voigt elements, and $I_i(t_{n+1})$ is the strain in the i^{th} Voigt element with $i = 1..(m-1)$. $I_0(0) = 0, I_i(0) = 0, I_m(0) = 0$, and $\sigma_i^R(0) = 0$. $\sigma_i^R(t_{n+1})$ may be understood as the effective stress in the dashpot in the i^{th} Voigt element. This above set of equations is the standard state variable approach for numerically computing convolution integrals for viscoelastic materials assumed to follow the Generalized Voigt model.

Step 2: Check if $I(t_{n+1}) \geq I(t_n)$.

If yes:

$$\sigma_b(t_{n+1}) = \sigma_b(t_n) + \Delta \sigma(t_{n+1}). \tag{4.7}$$

If not:

$$\begin{aligned}
I(t_{n+1}) &= I(t_n), \\
\Delta \sigma(t_{n+1}) &= \frac{\left(I(t_{n+1}) - I_e(t_n) - I_m(t_n) + \sum_{i=1}^{i=m-1} D_i \exp(-\Delta t_{n+1}/\rho_i) \sigma_i^R(t_n) \right)}{\left(D_0 + \frac{D_0}{2\rho_0} \Delta t_{n+1} + \sum_{i=1}^{i=m-1} D_i \left(1 - \frac{(1 - \exp(-\Delta t_{n+1}/\rho_i))}{\Delta t_{n+1}/\rho_i} \right) \right)}, \\
I_e(t_n) &= \left(D_0 + \frac{D_0}{\rho_0} \Delta t_{n+1} + \sum_{i=1}^{i=m-1} D_i \right) \sigma(t_n), \\
\sigma_b(t_{n+1}) &= \sigma_b(t_n) + \Delta \sigma(t_{n+1}).
\end{aligned} \tag{4.8}$$

By applying the above algorithm one can calculate both $I(t_{n+1})$ and $\sigma_b(t_{n+1})$. During loading, $\sigma_b(t_{n+1}) = \sigma(t_{n+1})$, while during unloading, the two may be different.

4.5 History of Model Development

Having introduced many of the model's nuances, it is now appropriate to go into a discussion of the history of model development so as to aid future compression model development in asphalt concrete. The current rate model is but a step in a long series of attempts by the author to model the material behavior of asphalt concrete in compression. Given the scope of compression modeling and the ever-present need to expand model capabilities, it is likely that the current model form would be modified and/or augmented to serve wider roles. The future researcher may be aided by the knowledge of some of the salient model forms that were tried but were not successful. The discussion starts with a model based on an alternate creep function and then goes into a discussion of models with different ways of introducing hardening. Some of the models may not be introduced in chronological order. The stress dependence functions were added only for the current version of the model. Hence all of the models discussed herein involve the actual load history being used directly in the convolution integral. One aspect that is common to all models (including the current model) is the application of a power of α to the hardening function. The motivation for this is based on the experimentally observed law of $\Delta\epsilon_{vp} = A_1 N^{-\beta}$ (from the chapter on Experimental Observations). As shown earlier, under steady state conditions and repeated loads, the hardening function, $H(t)$, shows a linear behavior in terms of load pulse number, N . It is assumed that the $N^{-\beta}$ term is related to hardening and hence $H^{-\alpha}$ is used to scale applied load. For brevity, as stated earlier, the following short form will be used for the convolution integral: $\left\langle \int_{-\infty}^t D(t-\tau) \frac{d\sigma_b}{d\tau} d\tau \right\rangle = \langle D * \sigma_b \rangle$ as needed. For all of the models (unless otherwise

stated), the hardening function, $H(t)$ is computed in the same way as the current model:

$$H(t) = H(t_{ini}) + \left\langle \int_{t_{ini}}^t D_2(t-\tau) \frac{d\sigma_b}{d\tau} d\tau \right\rangle.$$

4.5.1 Power Law and Square Root based Creep Functions

While most of the models tried involved simple creep functions based on the Generalized Maxwell model or the Generalized Voigt model, a foray was made into trying out functions based on square roots and power laws. The power law creep function is:

$$D(t) = D_0 \left(1 + \left(\frac{t}{\rho} \right)^\beta \right). \quad (4.9)$$

When $\beta = 0.5$ the square root based creep function is obtained. These models have the advantage of being able to capture more diverse creep behavior with a smaller number of parameters than the creep functions based on the mechanical analogs. A significant advantage of using these creep functions is that the integral, $\langle D^* \sigma_b \rangle$, is of the form FN^γ when subjected to repeated loading, where F and γ are constants, and N is the pulse number. However, a significant disadvantage of the power law creep functions is that no closed form solutions exist for the integrals and the numerical computation of the integral, $\langle D^* \sigma_b \rangle$, is orders of magnitude more expensive than the numerical computation of integrals involving models based on mechanical analogs. The reason for this is that because of the power law forms accurate numerical approximations of the form $I(t_{n+1}) = f(\Delta t_{n+1})I(t_n) + g(\Delta t_{n+1}, \Delta \sigma_{n+1})$ are not available. As a result, the integral, $\langle D^* \sigma_b \rangle$, has to be recalculated for each time step and this makes numerical computation extremely expensive and the process scales poorly with an increase in time steps. Usage of power law based creep functions was eventually dropped in favor of mechanical analog based creep functions, when the latter functions were able to capture the behavior of long rest period repeated creep and recovery loading well.

4.5.2 Models with Alternate Hardening Functions

$$\text{a) } \varepsilon_{vp} = \left\langle \int_{-\infty}^t \frac{D(t-\tau)}{H^\alpha(\tau)} \frac{d\sigma_b}{d\tau} d\tau \right\rangle$$

Figuring out the right way for handling hardening in the convolution integral has been a long-standing problem in this research. The above model was one of the first attempts at introducing hardening into the convolution. In this model, the compliance reduces over time (material gets stiffer). Owing to the way the model is set up, this aspect results in residual stresses building up in the model with the unfortunate effect that permanent strains can continuously increase even when loads are completely removed following a loading period. Since this behavior does not make any physical sense, the model was abandoned.

$$\text{b) } \varepsilon_{vp} = \varepsilon_{vp}(t_{ini}) + \frac{1}{H^\alpha(\tau)} \left\langle \int_{t_{ini}}^t D(t-\tau) \frac{d\sigma_b}{d\tau} d\tau \right\rangle$$

Owing to the problems in having the hardening function inside the convolution integral it was decided to take the hardening function outside. It was believed that the initial permanent strain, $\varepsilon_{vp}(t_{ini})$, should not be scaled by hardening and hence was kept separate from the integral. Although appearing like a rational approach, the resulting model had a serious flaw: the model was not invariant with respect to the starting point in the test, t_{ini} , from which point onwards characterization was performed. Different sets of parameters would be obtained for different sets of t_{ini} and the differences between the parameter values would not be explained by experimental variability alone. This model was also abandoned and $\varepsilon_{vp}(t_{ini})$ was combined with the integral and scaled by hardening for the next model. This ensures that the model is invariable with respect to the starting point.

$$c) \ \varepsilon_{vp} = \frac{1}{H^\alpha(\tau)} \left\langle \varepsilon_{vp}(t_{ini}) + \int_{t_{ini}}^t D(t-\tau) \frac{d\sigma_b}{d\tau} d\tau \right\rangle$$

This model showed promise in capturing many of the material behaviors of interest, however, the main drawback is that it cannot guarantee that permanent strains will always increase or remain constant. While the hardening function, $H(t)$, and the integral, $\langle D^* \sigma_b \rangle$, will individually never decrease due to the constraints placed on them, permanent strain from the model will be guaranteed to always increase only when $\langle D^* \sigma_b \rangle$ increases at a faster rate than $H^\alpha(\tau)$. This requirement cannot always be met and is a significant disadvantage for the model.

$$d) \ \varepsilon_{vp} = \varepsilon_{vp}(t_{ini}) + \left\langle \int_{t_{ini}}^t D(t-\tau) \frac{d\sigma_b}{d\tau} d\tau \right\rangle$$

This model does not use any dedicated hardening function. Such a model is feasible only when the creep function is based on the power law as listed previously. When using creep functions based on the mechanical analogs, the evolution of $\langle D^* \sigma_b \rangle$ will not show the range of hardening observed in the material. Once steady state is reached, $\langle D^* \sigma_b \rangle$ will become linear and such a trend is not observed in the material behavior. This model is capable of explaining the behavior of a few test conditions but not all of them and the model could benefit from a dedicated hardening function despite having power law creep functions.

Chapter 5 An Incremental Model Based on the Rate Model

There are strong incentives to simplify the rate model proposed earlier and minimize the computational cost for model characterization and prediction. Such a model is especially useful for obtaining trends for simulations and for providing an initial estimate of the parameter values used in the rate model. The initial estimates of some parameters can be an input for the optimization scheme used in the characterization of the rate model. This chapter will introduce additional tests used in the verification of the incremental model, discuss the reduction of the rate model into an incremental model, and show the capabilities of the incremental model. It should be understood that the incremental model is only an approximation of the rate model and does not replace it.

5.1 Experimental Data Used

Characterization of the incremental model is performed using the repeated creep and recovery tests with square and haversine load shapes discussed earlier. Only tests with 827 kPa (120 psi) peak deviatoric stress were used in the characterization and verification. In addition to using the tests described in Chapter 3, tests with composite loading histories were used to verify the model. In the composite loading condition tests, multiple load conditions follow one after the other. Table 5.1 shows the loading matrix of the composite loading condition tests. These composite tests are designated as CSQ or CHV. A given test includes only one kind of load shape. So, a CHV test does not involve any square load pulses and vice versa. The first group of each composite test uses the history outlined in Table 3.1, and the history of the rest of the groups is outlined in Table 5.1. For example, $(0.1 \text{ s}-0.9 \text{ s} \times 640) \times 10$ means that 640 pulses of 0.1 s-0.9 s is applied followed by a 200 s rest period and is repeated 10 times. As can be surmised from Table 3.1 and Table 5.1, blocks with progressively increasing load repetitions are required to produce appreciable increase in permanent strain.

Table 5.1 Loading Matrix of Composite Loading Condition Test

Specimen ID	Composite Loading Group Number				
	#1	#2	#3	#4	#5
CSQ (1)	0.1 s-0.9 s	(0.1 s-100 s) ×440	(0.1 s-0.9 s ×640)×10	-	-
CSQ (2)	0.1 s-0.9 s	(0.1 s-100 s) ×100	-	-	-
CSQ (3)	0.1 s-0.9 s	(1.6 s-100 s) ×300	-	-	-
CSQ (4)	0.1 s-0.9 s	(1.6 s-100 s) ×160	(0.1 s-0.9 s ×640)×6	-	-
CSQ (5)	0.4 s-0.9 s	(0.4 s-100 s) ×100	(0.4 s-0.9 s ×100)×1	(0.4 s-100 s) ×100	-
CSQ (6)	0.4 s-0.9 s	(0.4 s-100 s) ×100	(0.4 s-0.9 s ×100)×1	(0.4 s-100 s) ×100	-
CSQ (7)	0.4 s-0.9 s	(0.4 s-100 s) ×100	(0.4 s-0.9 s ×100)×1	(0.4 s-100 s) ×100	-
CSQ (8)	1.6 s-0.9 s	(1.6 s-100 s) ×300	(0.4 s-0.9 s ×160)×10	-	-
CSQ (9)	1.6 s-0.9 s	(1.6 s-100 s) ×300	(0.4 s-0.9 s ×160)×10	-	-
CSQ (10)	1.6 s-0.9 s	(1.6 s-100 s) ×300	(1.6 s-0.9 s ×60)×10	(1.6 s-0.9 s ×600)×1	-
CHV (1)	0.1 s-0.9 s	(0.1 s-100 s) ×320	-	-	-
CHV (2)	0.1 s-0.9 s	(1.6 s-100 s) ×320	(0.1 s-0.9 s ×640)×8	-	-
CHV (3)	0.1 s-0.9 s	(1.6 s-100 s) ×320	(0.1 s-0.9 s ×640)×8	-	-
CHV (4)	0.4 s-0.9 s	(0.4 s-100 s) ×100	(0.4 s-0.9 s ×100)×1	(0.4 s-100 s) ×50	-
CHV (5)	0.4 s-0.9 s	(0.4 s-100 s) ×100	(0.4 s-0.9 s ×100)×1	(0.4 s-100 s) ×50	-
CHV (6)	0.4 s-0.9 s	(0.4 s-100 s) ×100	(0.4 s-0.9 s ×100)×8	(0.4 s-0.9 s ×200)×8	-
CHV (7)	1.6 s-0.9 s	(1.6 s-100 s) ×200	(1.6 s-0.9 s ×40)×10	(0.4 s-100 s) ×200	(0.4 s-0.9 s ×160)×10
CHV (8)	1.6 s-0.9 s	(0.4 s-100 s) ×200	(0.4 s-0.9 s ×160)×10	(1.6 s-100 s) ×200	(1.6 s-0.9 s ×40)×10
CHV (9)	1.6 s-0.9 s	-	-	-	-

5.2 Reduction of Rate Model to Incremental Model

Consider the rate model without any stress dependence terms:

$$\begin{aligned} H(t) &= H(t_{ini}) + \left\langle \int_{t_{ini}}^t D_2(t-\tau) \frac{d\sigma_{b,H}(\tau)}{d\tau} d\tau \right\rangle, \\ \varepsilon_{vp}(t) &= \varepsilon_{vp}(t_{ini}) + \left\langle \int_{t_{ini}}^t D_1(t-\tau) \frac{d\sigma_{b,vp}(\tau)}{d\tau} d\tau \right\rangle, \end{aligned} \quad (5.1)$$

During Loading:

$$\sigma(t)_{b,H} = \sigma(t),$$

$$\sigma(t)_{b,vp} = \frac{\sigma(t)}{H(t)^\alpha},$$

where H is the hardening function; t_{ini} is the point from which model prediction starts; $H(t_{ini})$ is the initial value of H and is meant to represent initial hardening in the material; $\varepsilon_{vp}(t_{ini})$ is the initial value of permanent strain ε_{vp} ; $\sigma_{b,H}$ is the back stress associated with hardening and $\sigma_{b,vp}$ is the back stress associated with permanent strain evolution; σ is the applied loading, and α is a model parameter. $D_1(t)$ and $D_2(t)$ are creep compliances. Simplifying this model is not trivial owing to the presence of the hardening integral within the integral used to calculate permanent strains. Hence an approximation is first made to bring the hardening integral outside the integral used to compute permanent strain. This leads to:

$$\begin{aligned}
H(t) &= H(t_{ini}) + \left\langle \int_{t_{ini}}^t D_2(t-\tau) \frac{d\sigma_{b,H}(\tau)}{d\tau} d\tau \right\rangle, \\
\varepsilon_{vp}(t) &= \frac{\varepsilon_{vp}(t_{ini}) + \left\langle \int_{t_{ini}}^t D_1(t-\tau) \frac{d\sigma_{b,vp}(\tau)}{d\tau} d\tau \right\rangle}{H^\alpha(t)},
\end{aligned}
\tag{5.2}$$

During Loading:

$$\sigma(t)_{b,H} = \sigma(t),$$

$$\sigma(t)_{b,vp} = \sigma(t).$$

With the integrals separated out, one can now use steady state assumptions to derive the incremental model. Assuming that steady state conditions are reached, the increment in $\langle D_1 * \sigma_b \rangle$ at steady state for each pulse can be written as $I_{vp}(t_l, t_r, \sigma_0)$ and the increment in $\langle D_2 * \sigma_b \rangle$ can be written as $I_H(t_l, t_r, \sigma_0)$, where t_l is the loading time, t_r is the rest period, and σ_0 is the peak stress. The incremental model being derived is valid for any kind of shape, as long as the load shape and rest period are not changed during the test. It is for this reason that the stress dependence functions were dropped for the rate model in this section as they do not change the shape of the load pulses over the course of the test. Definitions for t_l and σ_0 may have to be defined appropriately depending on the load shape. For the steady state, (5.2) can be written as

$$\begin{aligned}
H(N) &= H(N_{ini}) + (N - N_{ini}) I_H(t_l, t_r, \sigma_0), \\
\varepsilon_{vp}(N) &= \frac{\varepsilon_{vp}^0(N_{ini}) + (N - N_{ini}) I_{vp}(t_l, t_r, \sigma_0)}{H^\alpha},
\end{aligned}
\tag{5.3}$$

where N_{ini} is any pulse number in the load history where the integral is under steady state conditions. It does not have to be the first pulse when the integral reaches the steady state. N is the current pulse number. Without any loss of generality, N_{ini} can be set to zero, in which case, $H(N_{ini})$ and $\varepsilon_{vp}^0(N_{ini})$ are assumed to be values extrapolated to $N_{ini} = 0$, and are written as H_{ini} and $\varepsilon_{vp,ini}^0$, respectively. This assumption makes the incremental model

applicable for the entire load history, however, for obvious reasons the predictions may not be good before the steady state conditions are reached. From (5.3) we have

$$\begin{aligned}
\varepsilon_{vp} &= \frac{A + BN}{(C + N)^\alpha}, \\
A &= \frac{\varepsilon_{vp,ini}^0}{\left(I_H(t_l, t_r, \sigma_0)\right)^\alpha}, \\
B &= \frac{I_{vp}(t_l, t_r, \sigma_0)}{\left(I_H(t_l, t_r, \sigma_0)\right)^\alpha}, \\
C &= \frac{H_{ini}}{I_H(t_l, t_r, \sigma_0)}.
\end{aligned} \tag{5.4}$$

Equation (5.4) is the incremental model form for any kind of repeated loading test. The parameters A , B and C must be found for each test condition. Note that A and C involve the terms H_{ini} and $\varepsilon_{vp,ini}^0$. Hence, these parameters account for the initial state of the material. Because the initial state tends to be affected by specimen-to-specimen variability, these parameters help to capture such information. Note that H_{ini} used here is not related to that found in the rate model, which serves mainly a numerical regularization purpose. The parameter B is dependent only on the loading conditions, t_l, t_r and σ_0 (although there will be a spread in B values over replicates due to variability); hence, parameter B is unique for each test condition. This parameter will be the most important parameter for the incremental model and represents the rate of permanent deformation growth.

5.3 Characterization of Incremental Model

The incremental model shown in (5.4) was characterized for the first loading groups in the composite loading tests. The parameters A , B and C were characterized for each test condition, e.g., 1.6 s-100 s SQ or 0.4 s-0.9 s HV. The incremental model involves a nonlinear equation in N . Attempts to fit the model directly to the test data using optimization as a black-box were not successful. Hence, a multi-step process was used to fit the data. First, if

the model is valid, plotting $y = \varepsilon_{vp} (C + N)^\alpha$ versus N should give straight lines, provided C and α are known. The value of α was fixed, and C was iteratively changed until the best straight line fit was obtained. The straight line was then fit to the equation, $y = A + BN$, to get the parameters A and B . In general, it is observed that B is not sensitive to variations in A and C . This property makes the model attractive and keeps the characterization procedure robust with respect to B , which is the most important property of the model. The fitted parameters are averaged over the replicates and listed in Table 5.2. It was found that an α value of 0.85 worked best in providing good fits for all the test conditions. Note that the parameter C tends to be of the same order as the number of load pulses in the test. This phenomenon occurs because of the way it enters the model form. Also note that the parameter B for an SQ test is always higher than the B for a HV test with the same load time and rest period. Although this type of behavior is not expected of B , owing to its ratio form, it is nevertheless an interesting observation. The above-mentioned behavior is, however, expected for both $I_H(t_l, t_r, \sigma_0)$ and $I_{vp}(t_l, t_r, \sigma_0)$. The incremental model fits for the SQ and HV tests are shown in Figure 5.1 and Figure 5.2, respectively. As seen from these figures, it is clear that the incremental model is able to fit each of the test conditions well.

Table 5.2 Averaged Incremental Model Parameters from Composite Loading Group #1

Test Type	Load Time (s)	Rest Period (s)	A	B	C
SQ	0.1	100	0.2096	4.579E-03	100.0
SQ	0.4	100	0.2454	5.781E-03	93.3
SQ	1.6	100	0.3368	6.686E-03	98.3
SQ	0.1	0.9	1.947	3.210E-03	550
SQ	0.4	0.9	0.3829	3.819E-03	100
SQ	1.6	0.9	0.2158	6.182E-03	71.7
HV	0.1	100	0.1268	2.963E-03	85.0
HV	0.4	100	0.2351	3.769E-03	115
HV	1.6	100	0.2112	4.293E-03	81.7
HV	0.1	0.9	0.7765	2.341E-03	300
HV	0.4	0.9	0.4416	3.050E-03	133
HV	1.6	0.9	0.0614	4.946E-03	26.7

Figure 5.1 Incremental model fits for SQ tests for a) 0.1 s – 100 s, b) 0.1 s – 0.9 s, c) 0.4 s – 100 s, d) 0.4 s – 0.9 s, e) 1.6 s – 100 s and f) 1.6 s – 0.9 s.

Figure 5.2 Incremental model fits for HV tests for a) 0.1 s – 100 s, b) 0.1 s – 0.9 s, c) 0.4 s – 100 s, d) 0.4 s – 0.9 s, e) 1.6 s – 100 s and f) 1.6 s – 0.9 s.

5.4 Trends in Model Parameter B

It would be instructive to look at the trends in parameter B as a function of test type and rest period to better understand the possible forms of the creep functions, $D_1(t)$ and $D_2(t)$. Figure 5.3 shows that parameter B for tests with 100 s rest periods (SQ and HV) seems to follow a curved shape whereas for 0.9 s rest periods it seems to follow a straight line type of behavior. The similar trends between the SQ and HV type tests are certainly worth noting.

Figure 5.3 Plots of incremental model parameter, B for a) SQ tests and b) HV tests.

5.5 Verification of the Incremental Model

The composite tests provide a means of verifying the incremental model and testing some of the assumptions made as part of the model. Although the best way to verify the incremental model is to predict the response of subsequent loading groups in a given composite loading test, in practical terms that is not possible because the incremental model works only after steady state conditions have been reached (as discussed in Chapter 4), and the start of each new loading group involves transient responses that are not captured by the incremental model. These transient responses are not defined entirely by the parameters A and C . Because these transient responses can be significant, it is not possible for an incremental model to predict the behavior of such composite loading tests. (A rate model, on the other hand, would be able to do so.) For this reason, a different approach is followed for verification. As long as steady state is reached within each of the composite loading groups, the incremental model should be valid. The parameter B measured in a given composite loading group should match the B obtained for the same loading condition in the first group (not necessarily related to the same specimen). For example, consider the second loading group in the CSQ (1) with a load condition of 0.1 s-100 s SQ. Parameter B for this group should be similar in value and preferably within the variability of the 0.1 s-100 s SQ tests. The fitting procedure described in the previous section is applied in characterizing B for the composite loading groups. The

fitted values are shown in Table 5.3. The values shown in Table 5.2 for 0.9 s rest period tests are averaged values for the replicates.

Table 5.3 Incremental Model Parameter B for Composite Loading History

Specimen ID	B for Composite Loading Groups				
	#1	#2	#3	#4	#5
CSQ (1)	2.439E-03 (0.1 s-0.9 s)	3.861E-03 (0.1 s-100 s)	2.502E-03 (0.1 s-0.9 s)	-	-
CSQ (2)	3.894E-03 (0.1 s-0.9 s)	5.976E-03 (0.1 s-100 s)	-	-	-
CSQ (3)	3.367E-03 (0.1 s-0.9 s)	6.813E-03 (1.6 s-100 s)	-	-	-
CSQ (4)	3.163E-03 (0.1 s-0.9 s)	5.763E-03 (1.6 s-100 s)	3.322E-03 (0.1 s-0.9 s)	-	-
CSQ (5)	4.039E-03 (0.4 s-0.9 s)	5.191E-03 (0.4 s-100 s)	-	6.004E-03 (0.4 s-100 s)	-
CSQ (6)	3.745E-03 (0.4 s-0.9 s)	5.395E-03 (0.4 s-100 s)	-	4.835E-03 (0.4 s-100 s)	-
CSQ (7)	3.676E-03 (0.4 s-0.9 s)	5.133E-03 (0.4 s-100 s)	-	5.297E-03 (0.4 s-100 s)	-
CSQ (8)	5.394E-03 (1.6 s-0.9 s)	7.466E-03 (1.6 s-100 s)	4.912E-03 (0.4 s-0.9 s)	-	-
CSQ (9)	6.229E-03 (1.6 s-0.9 s)	9.034E-03 (1.6 s-100 s)	5.741E-03 (0.4 s-0.9 s)	-	-
CSQ (10)	6.952E-03 (1.6 s-0.9 s)	9.310E-03 (1.6 s-100 s)	7.587E-03 (1.6 s-0.9 s)	-	-
CHV (1)	2.369E-03 (0.1 s-0.9 s)	3.536E-03 (0.1 s-100 s)	-	-	-
CHV (2)	2.240E-03 (0.1 s-0.9 s)	3.845E-03 (1.6 s-100 s)	2.360E-03 (0.1 s-0.9 s)	-	-
CHV (3)	2.394E-03 (0.1 s-0.9 s)	4.135E-03 (1.6 s-100 s)	2.558E-03 (0.1 s-0.9 s)	-	-
CHV (4)	3.407E-03 (0.4 s-0.9 s)	4.565E-03 (0.4 s-100 s)	-	4.781E-03 (0.4 s-100 s)	-
CHV (5)	2.566E-03 (0.4 s-0.9 s)	3.481E-03 (0.4 s-100 s)	-	3.542E-03 (0.4 s-100 s)	-
CHV (6)	3.173E-03 (0.4 s-0.9 s)	4.278E-03 (0.4 s-100 s)	3.333E-03 (0.4 s-0.9 s)	3.112E-03 (0.4 s-0.9 s)	-
CHV (7)	5.129E-03 (1.6 s-0.9 s)	6.060E-03 (1.6 s-100 s)	5.287E-03 (1.6 s-0.9 s)	6.089E-03 (0.4 s-100 s)	4.538E-03 (0.4 s-0.9 s)
CHV (8)	4.732E-03 (1.6 s-0.9 s)	5.386E-03 (0.4 s-100 s)	4.150E-03 (0.4 s-0.9 s)	5.734E-03 (1.6 s-100 s)	5.035E-03 (1.6 s-0.9 s)
CHV (9)	4.983E-03 (1.6 s-0.9 s)	-	-	-	-

By carefully studying the values in Table 5.2 and Table 5.3 the following observations can be made:

1. In most cases, the B values obtained for the first loading group show good repeatability when the same load condition repeats within a given specimen for both the SQ and HV

tests. The only exception is the 1.6 s - 0.9 s loading condition in group #3 of CSQ (10) (B for 1.6 s - 0.9 s is not matched within specimen-to-specimen variability when the load condition is repeated in this specimen). A possible reason for this occurrence could be the high level of strain that occurs in group #3 for CSQ (10) (around 2.2 % strain). It is possible that effects such as damage may be more important at this strain level making the B value appear elevated.

2. B values for loading groups #3 and #4 in CSQ (5) to (7) match B values for 0.4 s-100 s SQ (which is within the specimen-to-specimen variability).
3. B values for loading group #2 in CSQ (8) to (10) are consistently elevated compared to the B values for 1.6 s-100 s SQ. The reasons for this elevation are not clear.
4. In the case of the HV tests, the B values for the 0.4 s-100 s loading condition in CHV (4), (6) are slightly higher than the specimen-to-specimen variability observed for B for 0.4 s-100 s type loading.
5. As with the SQ tests, all the B values for loading groups present in the 1.6 s-0.9 s HV tests show a much higher value than their corresponding B values in the first loading groups (considering specimen-to-specimen variability).

Based on the above observations it appears that the incremental model certainly seems to work for many loading conditions and that it may be possible to avoid entirely repeating loading conditions in a given test following good observed repeatability of B values under repeated loading conditions on the same specimen. The incremental model is also very good at predicting behavior as long as only a single load condition is applied for a given specimen as seen from the fits to the material response (in fact this is the main purpose for using the incremental model). The reasons for the incremental model not working for any arbitrary loading condition in a composite test are not clear. It is possible that the steady state approximation has problems with composite loading histories. More importantly, it is possible that the material behavior is such that the creep functions used in the rate model may not necessarily be capable of reaching the steady state within a loading group. The apparent steady state observed might be a numerical consequence in that the incremental change in

$\langle D^* \sigma_b \rangle$ may not become constant in a mathematical sense, but when compared to the incremental value in the previous load pulse with a tolerance may be considered as having reached the steady state. The creep functions may also be such that the ‘apparent’ steady state takes a long time to reach and may be affected by composite load histories. That being said, it may still be possible to use some of the strengths of the incremental model by using composite tests in which loading times in the loading groups are increased. For example, one could use a characterization test in which the loading groups are 0.1 s-100 s, 0.1 s-0.9 s, 0.4 s-100 s, 0.4 s-0.9 s, 1.6 s-100 s and 1.6 s-0.9 s. This suggestion is based on the fact that B values match well for specimens for which the load time is increased in the groups (CSQ (3) and (4), CSQ (5) to (7), CHV (2) and (3), and CHV (5)). This observation will need to be verified with further testing, but, if true, would open the way for a viable characterization protocol for the incremental model.

5.6 Applications

An important application of the incremental model is to reduce the need for testing multiple loading conditions and to reduce the overall testing time. Given that the incremental model is able to fit the responses of cyclic tests with rest periods for both SQ and HV type tests, and given that the B values obtained for these tests are monotonic as a function of loading time, it is possible to run these tests for just a few loading conditions and then interpolate the B values for any other loading condition within the same test type (SQ or HV). Statistical information may be needed in using optimal values for the A and C parameters that reflect specimen-to-specimen variability and initial conditions. However, owing to the nature of the model, the values of A and C would not affect the prediction of permanent strains as significantly as B , provided enough pulses are applied.

5.7 Summary

Making steady state assumptions, an incremental model has been derived as an approximation of the rate model that is shown to be capable of explaining permanent deformation in cyclic tests with rest periods (SQ and HV tests). Once the model has been

characterized for a particular test type using a few loading conditions, it can be used in predicting the response of any other loading condition within the same test type. A novel testing protocol using composite loading histories is presented. The use of the incremental model for arbitrary composite loading histories was found to be limited. With further testing, however, it may be possible to propose a characterization test for the incremental model.

Perhaps a more relevant application of the incremental model is to serve as an aid in understanding the rate model behavior and to help in the characterization of the rate model. Using the parameter, B , one can get an idea about the creep functions, $D_1(t)$ and $D_2(t)$, that can be used in characterizing the rate model. This is the approach used in the next chapter that deals with the rate model characterization.

Chapter 6 Characterization and Validation of Rate Model

Armed with an understanding of the rate model from Chapter 4 and the experience with the simplified incremental model in Chapter 5, this chapter focuses on the characterization of the rate model, and its verification with independent experimental data. The initial goal was to fit the model for varying load levels, shapes and times, as well as varying rest periods. However, as explained in the next section, it was decided to focus the model development for long rest periods only. The rest of the chapter contains the details of characterization and validation of rate model for different load levels, shapes and times, with focus on long rest periods.

6.1 Reasons for Modeling Long Rest Period Tests

The initial goal for the model was to be able to explain the behavior of both long and short (0.9 s) rest period tests together using a single rate model. The motivation for using the very short rest period of 0.9 s stems from early pioneering testing done in the 1970's by Monismith and co-workers at the University of California, Berkeley. A haversine loading test with 0.1 s load time and a 0.9 s rest period became the standard for determining permanent deformation performance for asphalt concrete in compression. When developing the test protocol for use with the model, it was decided to have the 0.9 s rest period as the short rest period condition and 100 s rest period as the long rest period condition.

Subsequent development of the model however involved significant difficulties when trying to predict both 100 s and 0.9 s rest period tests together. Predicting the 0.9 s rest period tests alone was also a challenge. The problem was exacerbated by the fact that the 0.9 s rest period was comparable in magnitude to the load times of 0.1 s, 0.4 s, and 1.6 s while the long rest period of 100 s was completely different. For the model, the effect of rest period is manifested through the back stress function, which is related to the creep function based on viscoelasticity concepts. The requirements for predicting a 0.9 s rest period test typically involves a relaxation function that relaxes slowly during the first 0.9 s. Owing to the interrelationship between the creep function and the relaxation function that comes from the

model definition, the constraint on the shape of the relaxation function would lead to a creep function that would not be suitable for modeling the differences between the load times, 0.1 s, 0.4 s, and 1.6 s. In other words, the model may be able to explain the behavior of a single 0.9 s rest period test but not multiple tests with different load times together. To explain further, consider the requirement of having a gradual relaxation in the first 0.9 s after unloading. This would typically be accomplished by having a Generalized Maxwell model with relaxation times greater than 0.9 s so as to allow gradual relaxation in that range. Given that the rest period of 0.9 s is comparable to the load times involved, this would also mean that during loading, the creep function would look more or less linear in the first 0.9 s of loading. Eventually the function may be nonlinear in shape but the time at which this occurs may be much greater than 0.9 s. The trends in the tests as a function of load times may however call for a pronounced nonlinear shape for creep loading times of less than 1.6 s and this requirement would be violated by the need to have gradual relaxation as also stipulated by the model for explaining the tests. Thus it would appear that the model form as conceived may not be suitable for very short rest periods like 0.9 s that are comparable in magnitude to the load times being investigated. Another issue with the 0.9 s rest period tests is that at the end of a block of loading, a long rest period needs to be provided to measure the permanent strain owing to the fact that 0.9 s is not sufficient for the recoverable deformation to completely recover. This exposes the material to the effect of both long and short rest periods in the same test and it is possible that this changes the deformation mechanisms.

The final and yet most compelling reason for choosing to model only long rest period tests was the fact that pavements in the field (interstates) are almost never subjected to short rest periods like 0.9 s on a regular basis. Arguments from two different approaches are used to support this. The first approach uses traffic design estimates to show that rest periods are much greater than 0.9 s. To explain this, it is necessary to introduce a few terminologies used in traffic engineering. A typical interstate is subject to the passing of a wide variety of vehicles with different axle loads and frequencies. The Equivalent Single Axle Load (ESAL) is a construct used to get the equivalent effect of the diverse traffic loading in terms of the number of wheel passes of an 18000 lb single axle wheel. Table 6.1 shows the average rest

period between ESAL passes for different values of total design ESAL passes over the life of the pavement. A design life of 20 years is assumed. Total design ESAL of 30,000,000 is considered a high volume design and one can see that even for this condition, the average rest period is 21 s with lower design ESAL values giving even longer effective rest periods.

The second approach uses field measurements of truck arrival frequency data collected by the North Carolina Department of Transportation (NCDOT) to conclude that rest periods on the pavements are greater than 0.9 s (Stone et al, 2010). The data was collected by NCDOT using Weigh-In-Motion (WIM) technology at 44 sites across the state of North Carolina from 1997 to 2007. The maximum Average Annual Daily Truck Traffic (AADTT) from the 44 sites was 11583; and was found to be at site 528, on Interstate 85, which is a four-lane freeway. Assuming that 90 % of the truck traffic is on the right lane, the average minimum rest period for each direction was found to be 16.6 s. While this number is lower than the estimate using ESAL concept, it is still much higher than 0.9 s. In other words, it may be more pertinent to model long rest period tests for asphalt concrete in compression and hence the focus was shifted to long rest periods of 100 s. The goal of modeling both 100 s and 0.9 s rest periods, while ideal, was considered too ambitious given the complexities involved.

Table 6.1 Average Rest Period as a Function of Total Design ESAL Passes for a 20 Year Design Life

Total ESAL Passes	Average Rest Period (s)
1,000,000	630.7
3,000,000	210.2
10,000,000	63.1
30,000,000	21.0

6.2 Characterization of Rate Model for Long Rest Period Tests

The rate model was characterized using repeated creep and recovery haversine loading tests with long rest periods as discussed in Chapter 3. A total of nine different test conditions were used in the characterization: three load times: 0.1 s, 0.4 s, and 1.6 s with a 100 s rest period after each load pulse, and three peak deviatoric stresses: 620 kPa (90 psi), 827 kPa (120 psi),

and 1034 kPa (150 psi). The three load times were needed to define the creep functions, $D_1(t)$ and $D_2(t)$ reasonably well. The three different stress levels were used to help define the stress dependence characteristics of asphalt concrete in compression.

Owing to the complex nature of the rate model (an integral within another with no direct way of characterizing the inner integral separately), an optimization approach was chosen for characterization of the model. While one would wish that such a black-box approach would hopefully yield all the model parameters, the reality is that such approaches rarely work right out of the box. One needs to gain an idea of the range of values that each of the parameters tends to take, which can then be used to define the bounds for the parameters and their initial values in a nonlinear optimization scheme. The incremental model was used as a step in gaining a better understanding of the possible shapes of the $D_1(t)$ and $D_2(t)$ creep functions. Next, extensive effort was spent in obtaining parameters for the rate model manually by trial and error such that the haversine tests for the nine different loading conditions could be fit by the model.

After finding success in fitting the tests manually, a parallel optimization program was developed in Matlab (The Mathworks, Inc., 2006). The message passing interface was handled using bcMPI (The Ohio State University Research Foundation, 2006). The program was run on the High Performance Computing (HPC) clusters at North Carolina State University. For the parallel optimization code, a very fine time step mesh was used to ensure that the computations were as accurate as possible. Once the parallel optimization program was successfully able to converge to the solution, a study was performed to make the time steps as coarse as possible while still having the error in the permanent strain solution not exceed 1 % of that obtained from the converged fine-time-step solution. With such coarse time steps, it was found that the optimizations could be performed serially on the HPC clusters within a day in most cases.

For the optimizations, the function FMINCON provided as part of the Optimization Toolbox in Matlab was used. FMINCON is a gradient-based optimization tool that uses the sequential quadratic programming (SQP) method. The optimization approach for

characterization was done in two different ways. The first used a two-step optimization approach in which the parameters for the hardening function (q, H_1 , and relaxation function parameters E_2 and τ_2) were provided as inputs to the optimization, which then solves for the rest of the parameters: α, E_1, p_1, p_2 , and G_1 . The second approach used a more or less blackbox-like approach where the parameters, $\alpha, E_1, E_2, q, p_1, p_2$, and G_1 , were found using optimization. In both approaches the relaxation times, τ_1 and τ_2 , for permanent strain and hardening evolution respectively were provided. The optimization usually had significant difficulty when optimizing for the relaxation times and it was found more efficient to fix the relaxation times. The relaxation times can typically be supplied in a decade-like format such as [0.1, 1] s for example or in any other format such that the time scales of interest involved in the modeling are covered. For all of the optimizations, using two Prony terms for the relaxation functions was found to be sufficient. For both approaches, the parameters, H_1 and G_2 , can be set arbitrarily as a result of the model having a ratio form. The rest of the parameters would adjust themselves around the values provided for H_1 and G_2 . For these characterizations, $H_1 = 215.8$ and $G_1 = 0.0022$. Flowcharts for the two-step optimization approach and the blackbox-like optimization approach are given Figure 6.1 and Figure 6.2 respectively. A listing of the interface code used in performing the two-step optimization in parallel is provided in Appendix A.

Figure 6.1 Flowchart for two-step optimization approach.

Figure 6.2 Flowchart for blackbox-like optimization approach.

Both optimization approaches eventually give a set of parameters that lead to very similar model fits and predictions. Given that using the second blackbox-like approach without having an idea about the model parameters is unlikely to be successful initially, the two-step approach for optimization is given more importance in this research and all fits and validations are shown using parameters obtained from the two-step approach. Notwithstanding this, model parameters for the blackbox-like approach will also be listed.

In both optimization approaches, the following definition was used to calculate the error involved in optimization:

$$\text{error} = \left(\sum_{i=1}^n \left(\frac{|\epsilon_{vp}^{data}(N_i) - \epsilon_{vp}^{pred}(N_i)|}{(\epsilon_{vp}^{data}(N_i))} \right) \right) / n, \quad (6.1)$$

where n is the number of load cycles in the test, and ϵ_{vp}^{data} and ϵ_{vp}^{pred} are the permanent strains measured and predicted respectively at the end of the i^{th} load cycle. When multiple test conditions are involved, the average of the error in all test conditions was computed. The error definition shown is the average relative error and it was found to work well for the optimization of rate model parameters.

6.2.1 Parameters from Two-Step Optimization Approach

The first step in the two-step optimization approach is in getting a good estimate of the hardening function involving the stress-dependence parameter, q , and the relaxation function parameters, E_2 and τ_2 . Parameters, E_2 and τ_2 , were obtained using the incremental model discussed in Chapter 5. Recall that the incremental model parameter, B , tracks permanent strain evolution and has certain trends for haversine and square shaped loads as a function of

load time. The parameter is given as $B = \frac{I_{vp}(t_l, t_r, \sigma_0)}{(I_H(t_l, t_r, \sigma_0))^\alpha}$, where t_l is the load time, t_r is the

rest period, σ_0 is the peak deviatoric stress in the test, I_H is the steady state increment in the hardening function $\langle D_2 * \sigma_b \rangle$, and I_{vp} is the steady state increment in the $\langle D_1 * \sigma_b \rangle$ function.

Parameters, E_1, τ_1 , and E_2, τ_2 associated with the creep functions, $D_1(t)$ and $D_2(t)$ respectively were obtained using optimization such that the B values from such parameters fit the B values obtained from the 100 s rest period tests with square shaped loading (SQ). For this process, α , was taken as 0.85. E_2 and τ_2 obtained from this optimization were then used for the two-step optimization while E_1 and τ_1 obtained in this step were ignored. Based on this strategy, for the final step of the two-step optimization, the parameter values were set at: $E_2 = [1\text{E-}03, 1.037\text{E-}4]$, and $\tau_2 = [0.6948, 2.146]$. Instead of the SQ tests one could very well also use the haversine (HV) tests. The stress-dependence parameter, q , was obtained by performing multiple optimizations for different values of q in the range of 0.1 and 0.9 and choosing the parameter that yields the lowest error. Because q controls load shape effects, and because one of the goals for the model is to be able to predict both square and haversine loading tests, for the first step of the two-step optimization used to obtain q , both SQ tests (3 load conditions; 0.1 s-100 s SQ 827 kPa, 0.4 s-100 s SQ 827 kPa, and 1.6 s-100 s SQ 827 kPa) and HV tests (9 load conditions listed initially) were utilized. Table 6.2 lists the errors in optimization as a function of parameter, q . The lowest error is obtained for $q = 0.2$, however, qualitatively there is very little difference in fits for $q = 0.2$ to $q = 0.5$. Recall from

Chapter 4 that as the parameter, q , goes closer to zero, the hardening function in a SQ and a HV test with the same load time and rest period become more similar until they become identical at $q = 0$. With the parameters, q , E_2 and τ_2 , now available, the second step of the two-step optimization was carried out for the HV tests to determine the parameters, α , E_1 , p_1 , p_2 , and G_1 .

Table 6.2 Error (Objective Function) in Optimization as a Function of Parameter q when Fit to SQ and HV Tests

q	Error
0.1	0.1218*
0.2	0.0688
0.3	0.0706
0.4	0.0701
0.5	0.0706
0.6	0.0735
0.7	0.0722
0.8	0.0700
0.9	0.0757

* - Optimization not converged.

Table 6.3 summarizes the parameters provided as an input to the two-step optimization process. The rest of the parameters obtained by optimization of HV data are summarized in Table 6.4. Note that the parameter $H(t_{ini} = 0)$ is the initial value of the hardening function at the beginning of the test and is set as 50000. It serves only a regularization role and it is usually less than 50 % of the increment in the hardening produced in the first load pulse in the test. Note that the parameter, p_2 , is related to the stress dependence function for permanent strain evolution and is the power on the applied load. With a value of $p_2 = 9.275$, the power is quite high and shows a very highly nonlinear stress dependence. This power is justified given the highly nonlinear behavior of asphalt concrete in compression as a function of stress level as shown in Chapter 3. Figure 6.3 shows a plot of the stress dependence function obtained from the optimization as a function of stress level. This shows that the

stress amplification is linear until around 800 kPa and then steeply increases beyond that. For example, the stress dependence function has a value of 3038 kPa for an applied stress of $\sigma = 827$ kPa and for $\sigma = 1034$ kPa the value is 5461 kPa. That is, for a 25 % increase in applied stress, there is an 80 % increase in the stress dependence function when going from $\sigma = 827$ kPa to $\sigma = 1034$ kPa. This agrees well with the observations made on the effect of stress on overall strain evolution in Chapter 3. Chapter 3

Table 6.3 Parameters Provided as Input to Two-Step Optimization of HV Tests

Parameter	Value
E_2	[1E-3, 1.037E-4]
τ_1	[1,0.1]
τ_2	[0.6948, 2.146]
q	0.2
H_1	215.8
G_2	0.0022
$H(t_{ini} = 0)$	50000

Table 6.4 Parameters Obtained by Two-Step Optimization of HV Tests

Parameter	Converged Value	Initial Value	Lower Bound	Upper Bound
E_1	[6.738,15.15]	[2,3]	[1E-3,1E-3]	[20,20]
α	0.8982	0.8	0.6	0.99
p_1	0.9222	1	0.7	1.5
p_2	9.275	9	8	13
G_1	6.564	4	0.01	100

Figure 6.4 shows creep and relaxation functions for permanent strain and hardening based on the parameters obtained from the optimization. Firstly, one can see that the creep functions are mostly linear except for some initial curvature. In the case of the relaxation functions, most of the relaxation occurs within the first 15 s in the case of $E_1(t)$ for permanent strain and most of the relaxation in $E_2(t)$ for hardening occurs within the first 10 s.

Figure 6.3 Stress dependence function for permanent strain evolution from two-step optimization for HV tests.

Figure 6.5 to Figure 6.7 show the fits to haversine tests using the parameters from the two-step optimization. The quality of fits is good with the fits falling within the variability seen in the replicates (solid lines) for the test conditions, although the following conditions have a slight underprediction or overprediction: 0.1 s-100 s HV 620 kPa, 1.6 s-100 s HV 620 kPa, and 1.6 s-100 s HV 827 kPa. It is important to note the level of variability seen between the replicates for the different test conditions. The difference between some replicates can be as high as 15 % from the average. This kind of variability is typically seen in asphalt concrete in compression as a result of the aggregate structure playing a significant role in the deformation. Since the aggregate structure has a high degree of variability from specimen to specimen, replicate variability is high in compression testing. A very notable observation that can be made in these fits is that the fits are good right from the beginning of the test with no need for having a separate model for predicting the initial part of the tests, which is often the case in traditional models for asphalt concrete.

Figure 6.4 Creep and relaxation functions for permanent strain and hardening: a) creep function for permanent strain, b) relaxation function for permanent strain, c) creep function for hardening, and d) relaxation function for hardening.

Figure 6.5 Two-step optimization fits for 620 kPa haversine loading tests: a) 0.1 s-100 s HV 620 kPa, a) 0.4 s-100 s HV 620 kPa, and c) 1.6 s-100 s HV 620 kPa.

Figure 6.6 Two-step optimization fits for 827 kPa haversine loading tests: a) 0.1 s-100 s HV 827 kPa, a) 0.4 s-100 s HV 827 kPa, and c) 1.6 s-100 s HV 827 kPa.

Figure 6.7 Two-step optimization fits for 1034 kPa haversine loading tests: a) 0.1 s-100 s HV 1034 kPa, a) 0.4 s-100 s HV 1034 kPa, and c) 1.6 s-100 s HV 1034 kPa.

6.2.2 Parameters from Blackbox-Like Optimization Approach

The blackbox-like approach attempts to get most of the parameters from the optimization process with very few parameters given as input. This approach is likely to be preferable to the previous approach, however, it needs an experienced model user in order to be successful. For this approach, τ_1, τ_2, H_1 , and G_1 , were provided as an input and the rest of the parameters were obtained from the optimization. In this approach however, if both square and haversine tests need to be predicted by the model, then the optimization should be done for both SQ and HV tests, unlike the previous approach where the final optimization step could be performed for just HV tests. This is because there appears to be other solution sets that may explain the HV tests well but not the SQ tests. The optimization may converge to these solutions, which is not desirable. Hence the optimization needs to be done using both SQ and HV tests. In the two-step optimization approach some amount of information on the SQ tests comes from the parameter, q , that helps ensure that both HV and SQ tests can be predicted even when fitting only HV tests in the optimization.

Table 6.5 provides the parameter values input to the blackbox-like optimization while Table 6.6 shows the converged values from the optimization. The parameter, q , is slightly different at 0.337 compared to 0.2 for the previous approach. However, according to Table 6.2 there is not much difference in fit quality for $q = 0.2$ to $q = 0.5$. The overall quality of fits of HV and SQ tests from the blackbox-like optimization is practically identical to that obtained from the two-step optimization and so no plots of the fits are shown.

Table 6.5 Parameters Provided as Input to Blackbox-Like Optimization of HV and SQ Tests

Parameter	Value
τ_1	[1,0.1]
τ_2	[0.5, 2]
H_1	215.8
G_2	0.0022
$H(t_{ini} = 0)$	50000

Table 6.6 Parameters Obtained by Blackbox-Like Optimization of HV and SQ Tests

Parameter	Converged Value	Initial Value	Lower Bound	Upper Bound
E_1	[1.450,3.437]	[2,3]	[1E-3,1E-3]	[20,20]
E_2	[8.452E-04, 1.412E-04]	[5E-04, 2E-04]	[1E-3,1E-3]	[20,20]
α	0.8872	0.8	0.6	0.99
q	0.3370	0.7	0.1	0.8
p_1	0.8796	1	0.7	1.5
p_2	8.564	9	8	13
G_1	4.0958	4	0.01	100

6.3 Verification of Two-Step Optimization Model Using Square Loading Data

The predictive capability of the model is shown for square loading (SQ) creep and recovery tests. While the SQ tests were used in obtaining the parameters, q , E_2 , and τ_2 , the final stage in the optimization used only the haversine (HV) tests. Predictions for SQ tests with 827 kPa deviatoric stress are shown in Figure 6.8. With the exception of an underprediction for 0.1 s-100 s SQ 827 kPa, the rest of the test conditions are predicted very well. For the SQ tests as well the model has good prediction starting from the beginning of the test, eliminating the need for a separate model for the initial part of the tests.

Figure 6.8 Two-step optimization predictions for 827 kPa square loading tests: a) 0.1 s-100 s SQ 827 kPa, a) 0.4 s-100 s SQ 827 kPa, and c) 1.6 s-100 s SQ 827 kPa.

6.4 Validation Using Random Haversine Loading Test

The rate model was validated using a random loading test that involves three different haversine load pulses 0.1 s, 0.4 s, and 1.6 s, and three different peak deviatoric stresses 620 kPa, 827 kPa, and 1034 kPa for a total of nine unique loading pulses. The nine loading conditions were applied in a completely random order on the specimen. The uniform random distribution was used in generating the load history with each load condition having the same probability of being chosen for a particular cycle in the test. A total of 1350 load pulses was applied on each specimen, with two specimens being used for the testing as replicates. All the tests were performed by fellow graduate student, Yeong-Tae Choi at North Carolina State University.

Owing to the large number of load pulses in the test, only the frequency of application of each load condition is listed in Figure 6.9. In the figure, the following notation is used: 0.1 - 620 means 0.1 s haversine loading with 620 kPa peak deviatoric stress. The frequency of application of the load conditions is mostly uniform with the exception of the 0.4 s loading with 1034 kPa deviatoric stress. Figure 6.10 shows the prediction of the random haversine load test using the two-step optimization parameters. The model underpredicts the test data and the result appears surprising given that the model fit the haversine tests well.

To test the significance of the 1034 kPa deviatoric stress loading on the overall test, a simulation was performed for the random load history but with one modification: only the 1034 kPa loading pulses were permitted to induce deformation. This simulation result is included in Figure 6.10. Surprisingly 33 % of the load pulses in the random load test seem to contribute to 95 % of the overall permanent strain in the test. The significant effect of the 1034 kPa pulses on the overall test was not realized during the random load test protocol development and future random load test protocols would need to take this into account. The random load test inadvertently ends up demonstrating the significant stress level related nonlinearity of asphalt concrete in compression.

Figure 6.9 Frequency distribution of load conditions used in random haversine loading test.

Figure 6.10 Prediction of random haversine load test using two-step optimization model parameters.

As the test is largely dominated by the 1034 kPa load pulses, any issues with the 1034 kPa HV tests used in model characterization can manifest themselves with the kind of underprediction seen. Only two replicates per load condition (0.1 s-100 s HV 1034 kPa for example) were used for the 1034 kPa deviatoric stress condition in model characterization. If the deformation in one of the replicates was lower than the population average, the model would be fit for a level of deformation lower than the average and this could be a possible reason for the underprediction. Another possible reason for the underprediction could be the presence of mechanisms that become active under random application of load pulses, and that are not present in the regular application of a particular load condition as seen in the tests used for characterization.

6.5 Sensitivity Analysis of Two-Step Optimization Parameters

Given the large number of parameters involved in the model and the fact that they are obtained using optimization, a sensitivity analysis would give much needed insight into the landscape of the error function used in the optimization. Such an analysis can reveal the level of sensitivity involved with the various parameters and the information can be used to better understand the nuances in making the optimization work. A sensitivity analysis was performed on the following parameters: q, p_1, p_2 and G_1 . Owing to the computational expense involved in the model predictions and given the large number of parameters involved, the sensitivity analysis was performed by perturbing each parameter about the converged value while keeping the rest of the parameters equal to their converged values. That is, the parameters were assumed to be independent of each other. Such an analysis may not give the complete picture for the sensitivity of the variables but it is still a useful step in understanding the model better. Sensitivity analysis was not performed for the relaxation function parameters, E_1, E_2, τ_1 and τ_2 , as each parameter in turn has two terms and results obtained by varying one term independently of the other would not be very useful. The results of a sensitivity analysis for parameter α are also not very useful because changing α independently of the other parameters would appear to show very high sensitivity that is not indicative of the true sensitivity of α .

The error function listed in (6.1) was calculated for the range of parameter values within the bounds specified in Table 6.6 for each parameter considered for the analysis. Recall that the error function gives the average relative error over all test conditions when using the parameters $\beta=1$ and $\gamma=1$. For instance, a value of 0.05 indicates that the average relative error considering all tests is 5 %. Given the perturbation-like analysis for the converged model parameters, it was no surprise that only a single minimum was found for the error function in the entire range for each of the parameters, with the minimum occurring at the converged value. Next a finer analysis was performed with the range set more tightly around the converged value.

Figure 6.11 to Figure 6.14 show the variation of the error function as a function of a change in the parameters. The most sensitive parameter appears to be q while the least sensitive appears to be p_2 (surprisingly). In the case of the stress-dependence parameter, q , a 50 % change leads to about a 600 % change in the error. The quality of the fit degrades rapidly away from $q = 0.2$, which seems to contradict the observation from Table 6.2 that the quality of fits are identical for $q = 0.2$ to 0.5. In actuality, there is no contradiction as the results in Table 6.2 show the interdependence of the model parameters and it may be possible to change some of the parameters and still not see an appreciable change in the error function provided the rest of the parameters can adjust themselves around the changes. Hence from the point of view of making the optimization work using suitable choice of initial values and parameter bounds, the results from the sensitivity analysis may not be very useful as the parameters do not show as much sensitivity as observed in the analysis so long as the rest of the parameters can be adjusted suitably. In the case of the stress-dependence parameter, p_2 , the error remains within 15 % for a broad range of values. This low sensitivity may be because p_2 mostly affects the 1034 kPa deviatoric stress tests as seen from Figure 6.3 and these tests account for only 33 % of all the test conditions used in calculating the error function.

Figure 6.11 Error function for q with two-step optimization parameters.

Figure 6.12 Error function for p_1 with two-step optimization parameters.

Figure 6.13 Error function for p_2 with two-step optimization parameters.

Figure 6.14 Error function for G_1 with two-step optimization parameters.

Chapter 7 Conclusions and Future Work

7.1 Conclusions

Research on viscoplastic modeling of asphalt concrete in compression contributed the following: 1) use of experimental data to show that viscoplasticity models that involve functions of stress and viscoplastic strain at a given time alone may not be suitable for modeling creep and recovery behavior of asphalt concrete in compression; 2) development of a novel uniaxial viscoplasticity rate model for asphalt concrete in compression; and 3) development of an incremental model based on the rate model.

The proposed viscoplasticity rate model is applicable to compressive creep and recovery experiments at 54°C with 1) several hundreds of cycles of loading including the secondary creep region, 2) haversine loading shapes at three different peak deviatoric stress levels, 620 kPa, 827 kPa, and 1034 kPa, and square loading shapes at 827 kPa peak deviatoric stress, and 3) long rest periods that allow complete viscoelastic recovery. A notable advantage of the model is the ability to predict the primary creep region behavior. Compared to many of the existing viscoplasticity models, the proposed uniaxial viscoplasticity model has a relatively simple model form with a physical mechanical analog that can be used to intuitively understand the model behavior.

7.2 Future Work for the Proposed Viscoplasticity Model

The ultimate goal of viscoplasticity modeling of asphalt concrete would be to develop a three-dimensional model that is applicable to a wide range of loading conditions and temperature conditions, and that can be implemented into a finite element program for carrying out rutting performance evaluation of pavements. The proposed uniaxial compression model is but a first step towards the realization of this goal and the following may be done to expand its capabilities.

1. Checking the applicability of the model to different asphalt mixes, and for a wide range of test parameters: temperature, confinement pressure, peak deviatoric stress, load shape, load time, and rest period.
2. Generalization of the uniaxial model to three-dimensions. Challenges lie in understanding the effect of a three-dimensional stress state on hardening evolution. Owing to the fact that hardening is not directly measured, but is rather used to explain the material behavior; this may require systematic development of testing protocol to experimentally identify the effect of stress state on the hardening function.
3. Once the material model is finalized, accurate and convergent integration algorithms should be developed before the model can be incorporated into a finite-element structural model. The resulting structural model can then be used to predict rutting under realistic traffic loading, thus achieving the ultimate objective of rutting prediction.

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APPENDIX

Appendix A. Optimization Code Listing

The interface file written in Matlab for two-step optimization is listed below. Matlab and bcmpi (an MPI extension for Matlab) were used to perform the optimization using the function FMINCON in the Optimization Toolbox from Matlab. The main purpose of the listing is to provide all options and inputs used in the optimization so as to allow duplication of the results shown in the dissertation. Sub functions that are called by the interface file are not listed.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Program that runs parallel optimization of CLT/Flow number data using
% rate form parameters.
% Model form:
% epsilonVP = <integral(D1(t-tau)(d(sigmaVP/HEff^alpha)/d(tau))d(Tau))>,
% the integral is never allowed to reduce.
% HEff = <integral(D2(t-tau) (d(sigmaH)/d(tau))d(Tau))>
% the quantity in < > is not allowed to reduce.
% sigmaVP = (G1*<sigma(t)-sigmaY1VP>)^p1+(G2*<sigma(t)-sigmaY2VP>)^p2
% Note : G1 and G2 are inside the bracket for numerical reasons (no high
% powers on sigma directly)
% sigmaH = H*(<sigma(t)-sigmaYH>)^q
% D1, D2 are creep functions , sigma is the load history.
% D1(t) = Generalized Maxwell, D2(t) = Generalized Maxwell
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               Input Parameters
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

load summaryCLTFlowAllTests.mat testConditionsList startingIndexList
    endingIndexList
MPI_Init(); % bcmpi
global MPI_COMM_WORLD MPI_ANY_SOURCE MPI_ANY_TAG
myComm = MPI_COMM_WORLD;
mySize = MPI_Comm_size(myComm);
myRank = MPI_Comm_rank(myComm);

if (myRank == 0)
    tic
end

% Fitting for 100 s rest period haversine tests
indexList = [7,8,9,13,14,15,19,20,21];
```

```

% index numbers that correspond with all haversine creep and recovery test
% data
startingIndexListUsed = startingIndexList(indexList);
endingIndexListUsed = 300*ones(12,1);
% using a reduced part of the tests for fitting for shortening
% optimization time
testConditionsListUsed = testConditionsList(indexList,:);

numJobs = length(startingIndexListUsed);

% coarse time step mesh
loadTimeStepParams = 0.01*ones(24,1);
numerIntParams = ones(24,1)*[0.01,5,10,0.5];

% fine time step mesh
% loadTimeStepParams = 0.01*ones(24,1);
% numerIntParams = [
%     0.005,1,200,0.005;
%     0.01,1,100,0.01;
%     0.01,1,100,0.01;
%     0.005,1,200,0.005;
%     0.01,1,100,0.01;
%     0.01,1,100,0.01;
%     0.005,1,200,0.005;
%     0.01,1,100,0.01;
%     0.01,1,100,0.01;
%     0.005,1,200,0.005;
%     0.01,1,100,0.01;
%     0.01,1,100,0.01;
%     0.005,1,200,0.005;
%     0.01,1,100,0.01;
%     0.01,1,100,0.01;
%     0.005,1,200,0.005;
%     0.01,1,100,0.01;
%     0.01,1,100,0.01;
%     0.005,1,200,0.005;
%     0.01,1,100,0.01;
%     0.01,1,100,0.01;
%     0.005,1,200,0.005;
%     0.01,1,100,0.01;
%     0.01,1,100,0.01;
% ];
%
% loadTimeStepParams = [0.005;0.01;0.01;0.005;0.01;0.01;0.005;0.01;
%     0.01;0.005;0.01;0.01;0.005;0.01;0.01;0.005;0.01;0.01;0.005;0.01;
%     0.01;0.005;0.01;0.01];

% The time step generation creates constant time steps in the load period
% and graduated time steps in the rest period according to the rule:
% timeStep =
% factor*sqrt(t - (N*tp+(N-1)*tr)), where N is the Nth load pulse, and the
% time steps are created in the rest period following the Nth load pulse.

```

```

% The time steps are initially kept uniform until numUniformSteps
% after the start of the rest period and then generated in a graduated
% manner until the timestep reaches the maximum value at which point,
% timesteps are generated uniformly at the maximum timestep size allowed.
% column order of numerIntParams = [initialUniformTimeStep,maxTimeStep,
% numUniformTimeSteps,factor];

numerIntParamsUsed = numerIntParams(indexList,:);
loadTimeStepParamsUsed = loadTimeStepParams(indexList,:);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               Pre-calculate Hardening Function
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
EHArray = 1E-3*[1;0.10366];
tau2Array = [0.69475;2.146];
H1Ini = 50000;
q= 0.2;
sigmaYH = 0; % yield stress for hardening
H1 = 215.8539;

tag = 0;
endTag = 99; % tag that signals worker to exit loop.
numProcsUsed = min(numJobs,mySize-1);
if (myRank == 0)
    if ( (mySize-1) == numJobs)
        % assigns a particular process to an index and has no
        % communication of hardeningCellMatrix - hence no work for manager
    else
        hardeningCellMatrix
            =callGenMaxwellStateIntManager(numJobs,myComm,mySize);

        for dest = 1:numProcsUsed
            MPI_Send(dest,tag,myComm,hardeningCellMatrix);
        end
    end
else
    if ( (mySize-1) == numJobs)
        hardeningCellArray = callGenMaxwellStateIntWorkerEfficient(
            EHArray,tau2Array,q,sigmaYH,H1,H1Ini,
            testConditionsListUsed(myRank,:),
            startingIndexListUsed(myRank,:),
            endingIndexListUsed(myRank,:),numerIntParamsUsed(myRank,:),
            loadTimeStepParamsUsed(myRank,:),myRank,myComm,mySize);
    else
        callGenMaxwellStateIntWorker(EHArray,tau2Array,q,sigmaYH,H1,
            H1Ini,testConditionsListUsed,startingIndexListUsed,
            endingIndexListUsed,numerIntParams(indexList,:),
            loadTimeStepParams(indexList),myRank,myComm,mySize);
        hardeningCellMatrix = MPI_Recv(0,tag,myComm);
    end
end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```

%                               Input Parameters for Optimization
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
alphaIni = 0.8;
EVPSArrayIni =[2;3];
p1VPIni = 1;
p2VPIni = 9.0;
sigmaY1VPIni = 0;
sigmaY2VPIni = 0;
G1VPIni = 4.0;
G2VPIni = 0.0022;

x0 = [EVPSArrayIni;alphaIni;p1VPIni;p2VPIni;sigmaY1VPIni;sigmaY2VPIni;
      G1VPIni];
lb = [1E-3;1E-3;0.6;0.9;8;0;0;1E-2];
ub = [20;20;0.99;1.5;11;400;400;100];
tauVPArray =[1;0.1];
beta = 1;
gamma = 1;
weightFactors = ones(length(indexList),1);

options = optimset('LargeScale','off','Display',
    'iter','FunValCheck','on');

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               Optimization
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if (myRank == 0)
    objFun = @(x)RF6M5OptimFitnessCLTFlowNoDataManager(
        [x;G2VPIni],weightFactors,numJobs,myComm,mySize);
    [xSoln,fval,exitflag,output,lambda,grad,hessian] = fmincon(objFun,
        x0,[],[],[],[],lb,ub,[],options);
    for dest = 1:numProcsUsed
        MPI_Send(dest,tag,myComm,0,0,endTag);
    end
else
    if ( (mySize-1) == numJobs)
        RF6M5OptimFitnessCLTFlowNoDataWorkerEfficient(tauVPArray,
            testConditionsListUsed(myRank,:),startingIndexListUsed
            (myRank,:),endingIndexListUsed(myRank,:),beta,gamma,
            hardeningCellArray,myRank,myComm,mySize);
    else
        RF6M5OptimFitnessCLTFlowNoDataWorker(tauVPArray,
            testConditionsListUsed,startingIndexListUsed,
            endingIndexListUsed,beta,gamma,hardeningCellMatrix,
            myRank,myComm,mySize);
    end
end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               Wrapping Up
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

if (myRank == 0)

```

```

numPronyVP = length(tauVPArray);
EVPAArraySoln = xSoln(1:numPronyVP);
alphaSoln = xSoln(numPronyVP+1);
p1VPSoln = xSoln(numPronyVP+2);
p2VPSoln = xSoln(numPronyVP+3);
sigmaY1VPSoln = xSoln(numPronyVP+4);
sigmaY2VPSoln = xSoln(numPronyVP+5);
G1VPSoln = xSoln(numPronyVP+6);
G2VPSoln = G2VPIni;
clear objFun
if ((mySize-1) ~= numJobs)
    clear hardeningCellMatrix
end
save optimDB.mat
t = toc;
msg = sprintf('Completed in %g hrs.',t/3600);
disp(msg);
end

MPI_Finalize();

```