Study on Buckling of Square Tubes Under Bending Loads

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ABSTRACT

Buckling tests of square tubes under bending loads are conducted and the propagation behavior of local deformations after buckling and ultimate bending moments are investigated experimentally.

1 INTRODUCTION

Square tubes are characterized by their ability to support loads exceeding buckling loads without rapidly decreasing their strengths against such overload. Thus, square tubes can be used under loads greater than buckling loads if local deformations due to buckling are negligible. However, it is necessary to recognize their behavior after buckling. Through 4-point bending tests of square tubes and result analysis, the propagation behavior of local deformations after buckling and ultimate loads were obtained experimentally.

2 BUCKLING TESTS

2.1 Geometry and material properties of tested tubes

Square tubes of three different thicknesses (2.0 mm, 2.5 mm and 3.0 mm) and the same inside width were used. Material properties of the tested tubes are shown in Table 1.

Table 1 material properties

Temperature θ °C	Modulus of Elasticity E GPa	Yield stress σ MPa	Poisson's ratio ν
20	93.76	372.7	0.35
300	77.86	147.1	0.42

2.2 Buckling test apparatus

The test apparatus consisted of a loading machine, measuring instruments and heaters. The loading machine applied 4-point bending loads to the test tubes. To prevent local deformations of the tubes, blocks were inserted into the square tube at the center and the points to which loads were applied directly.

2.3 Test conditions and measured items

Loading directions are called loading modes 1 and 2 as shown in Fig.1. The measured items are deflections, loads, and deformations of cross sections due to buckling waves.

2.4 Results of buckling tests

2.4.1 Deformations after buckling

In loading mode 1, the tubes can be assumed to consist of two flanges and two webs. If a bending moment is applied as shown in Fig 2, a uniform tensile or compressive load acts on the flanges, and an in-plane bending load acts on the webs. Consider a rectangular plate under a compressive load of $-N_x$ per unit of length. Assume that in this plate there exists an initial deflection in the form of a sinusoidal wave having an amplitude α relatively small for the plate thickness, and an additional deflection is caused under a compressive load of $-N_x$. The maximum deflection w_{max} at the center of the plate is given by (Timoshenko et al. 1959)

$$w_{max} = \frac{\alpha}{1 - \beta N_x} \quad \text{and} \quad \beta = \text{constant} \quad (1)$$

Between local deformation u of the wall surface center generated in the tubes under the bending load and bending moment M , assume, as an analogy from Eq.(1), the following relation:

$$u = \frac{A}{1 - BM} \quad \text{or} \quad A + B u M = u \quad \text{and} \quad A, B = \text{constant} \quad (2)$$

From Eq.(2), it is evident that deformation u of the tube's wall surface center is proportional to the product uM .

Figure 3 shows applications of the relation in Eq.(2) to known measured or analyzed values. Evidently, the measured and analyzed values satisfy the relation in Eq.(2).

(1) Coefficient A

Figure 4 shows the relation between coefficient A and wall thickness t . Coefficient A represents the deformation of the wall surface with bending moment zero. This value, however, is too large to be taken as an initial deformation. Coefficient A can be considered as representing the wall surface deformation directly after buckling. As shown in Fig.4, the absolute value of coefficient A decreases as the wall thickness increases.

(2) Coefficient B

We assume that coefficient B can be represented as

$$B = K_m \cdot f(\theta) \cdot g(t) \quad (3)$$

Fig.1 Sketch of loading modes

Fig.2 Sketch of a square tube under bending load

where coefficient K_m is loading mode. Function $F(\theta)$ is related to temperature and includes the material properties; function $g(t)$ is related to thickness. If equation (3) holds true, the following relations can be obtained:

$$r_1 = \frac{B(\text{mode } 2)}{B(\text{mode } 1)} = \frac{K_{m2}}{K_{m1}} = \text{constant} \quad \text{and} \quad r_2 = \frac{B(\text{room temp.})}{B(300^\circ\text{C})} = \text{constant} \quad (4)$$

(a) Loading mode 1 at 300°C

(b) Loading mode 2 at room temperature

Fig.3 Relationship between deformation u and product uM

Fig.4 Relationship between coefficient A and wall thickness t

Fig.5 Relationship between coefficient B and wall thickness t

Table 2 shows values of r_1 and r_2

Table 2 coefficient r_1 and r_2

Thickness t mm	coefficient r_1	coefficient r_2
2.0	1.21	0.414
2.5	1.28	0.429

As the wall thickness increases, the values of coefficient B increase slightly, but these changes are equivalent to the amount of dispersion in test values. Thus, coefficient B can be represented by Eq.(3).

Coefficient B , like coefficient A , decreases as the wall thickness increases.

Figure 5 shows $\log B - \log t$ relations of the test results for loading mode 1 at room temperature. It gives

$$g(t) = t^{-1.53} \quad \text{and} \quad t = \text{wall thickness} \quad (5)$$

The form of function $f(\theta)$ is not determined only by the data at room temperature and 300°C.

2.4.2 Ultimate strength

To determine the ultimate strength of a flat plate with simply supported edges under a compressive force, we use Kármán's study (Kármán et al. 1932). In Fig.6, a plate having length l and width b is compressed. All forces are supported by two slender fragments of width w and loading is uniform. If stress is σ , total force P is given by

$$P = 2wt\sigma \quad (6)$$

where w represents effective width, which is given by

$$w = \frac{\pi}{\sqrt{12(1-\nu^2)}} \sqrt{\frac{E}{\sigma}} t \quad (7)$$

If the stress distribution in the flange of a square tube is represented by the dotted line in Fig.7, we have

$$2w\sigma = \sigma_1 b = (M/I) \cdot (b/2) \cdot b \quad (8)$$

If moment of inertia I of the square tube is represented by inside width b_i and wall thickness t , and if $t/b_i \ll 1$, $b_i = b$, $\sigma = \sigma_y$ (yield stress),

Fig.6 Effective width and stress distribution of a flat plate

Fig.7 Assumption of stress distribution in flanges and webs of a square tube

putting Eq.(7) into Eq.(8) yields

$$M_{1k} = \frac{2}{3} \frac{\pi}{\sqrt{3(1-\nu^2)}} \sqrt{E\sigma_y} b t^2 \quad (9)$$

$$M_{2k} = r_m M_{1k} = \frac{2}{3} \frac{\pi}{\sqrt{3(1-\nu^2)}} r_m \sqrt{E\sigma_y} b t^2 \quad (10)$$

where M_{1k} and M_{2k} are ultimate bending moments for loading modes 1 and 2, respectively.

(1) Effect of wall thickness

Figure 8 shows the $\log M_C - \log t$ relation obtained from the test results of loading mode 2. The plotted points are nearly in a line and we have

$$\bar{M}_{rc} = 3.96 t^{1.57} \quad (\text{room temp.}) \quad (11)$$

$$\bar{M}_{hc} = 2.56 t^{1.60} \quad (300^\circ\text{C}) \quad (12)$$

According to Eqs.(9) and (10), the ultimate bending moment is proportional to the square of the wall thickness. However, experimental results are represented by Eqs.(11) and (12) which show that the ultimate bending moment is proportional to the wall thickness to the power of 1.5-1.6.

(2) Effect of loading modes

Figure 9 shows the effect of loading modes on the ultimate bending moment. Evidently, the ratio between ultimate bending moments M_{1c} in loading mode 1 and M_{2c} in loading mode 2 does not depend on the wall thickness, temperature, etc.

Thus, from the gradient of the best-fit line passing the origin, we have

$$r_m = \frac{\text{Ultimate bending moment in loading mode 2}}{\text{Ultimate bending moment in loading mode 1}} = \frac{\bar{M}_{2c}}{\bar{M}_{1c}} = 0.91 \quad (13)$$

(3) Effect of temperature θ

By putting the values of the material constants in Table 1 into Eq.(9), a ratio r_θ can be obtained as

$$r_\theta = \frac{\bar{M}_{k1}(300^\circ\text{C})}{\bar{M}_{k1}(20^\circ\text{C})} = 0.59 \quad (14)$$

Figure 10 shows the effect of temperature on the ultimate bending moment, proving that the ratio between ultimate bending moments $\bar{M}_{hc}$ at 300°C and $\bar{M}_{rc}$ at room temperature does not solely depend on wall thickness, loading direction, etc. Thus, from the gradient of the best-fit line passing the origin, we have

$$r_\theta = \frac{\text{Ultimate bending moment at } 300^\circ\text{C}}{\text{Ultimate bending moment at room temp.}} = \frac{\bar{M}_{hc}}{\bar{M}_{rc}} = 0.62 \quad (15)$$

This value agrees well with that in Eq.(14).

(4) Validity of estimated formula from Kármán's theory

Figure 11 shows the relation between the estimated values M_k calculated from Eqs.(9) and (10) and test values M_C . The calculated values provide a conservative estimation. The best estimate is represented by a line passing the origin at gradient 1.42 and the ultimate bending moment M_C is given by

$$\bar{M}_c = 1.42 \bar{M}_k \quad (16)$$

Although the coefficient 1.42 is not definite, it may be the result of loading rates. The material constants currently in use are for a strain rate of about 0.05/min. In tests, a strain rate of about 0.25/min is likely generated at the outermost end. However, there have been no tests quantitatively sufficient to draw any conclusions.

Fig.8 Relationship between ultimate bending moment M_C and wall thickness t

Fig.9 Effects of loading modes on ultimate bending moment

Fig.10 Effect of temperature on ultimate bending moment

Fig.11 Relationship between estimated values M_k and test values M_C

3 CONCLUSION

The relation between buckling wave growth and bending moment was represented by an experimental formula obtained as an analogy of an equation derived from the results of bending theory for plates having an initial curvature. A formula was also obtained for the ultimate bending moment by applying Kármán's formula for critical loads in plates to square tubes. They were proven that the experimental formula represents the behavior of test and analysis values very well.

4 REFERENCES

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