The Conservatism of the Net-Section Stress Criterion for the Failure of Cracked Stainless Steel Piping

E. SMITH

Manchester University, Manchester, UK

ABSTRACT

The failure of cracked stainless steel piping can be predicted by assuming that failure conforms to a net-section stress criterion, using as input an appropriate value for the critical net-section stress together with a knowledge of the anticipated loadings. The stresses at the cracked section are usually calculated via a purely elastic analysis based on the piping being uncracked. However because the piping is built-in at its ends into a larger component, this limits the amount of elastic follow-up and, consequently, use of the net-section stress approach in this manner can lead to conservative failure predictions. This paper quantifies the extent of this conservatism, and shows that it can be quite marked. There is an additional measure of conservatism due to the fact that unstable failure need not necessarily be associated with the onset of crack extension. A key parameter with regard to both these conservatisms is L_{EFF} , a length parameter which is a measure of the degree of elastic follow-up in the system.

INTRODUCTION

During recent years considerable attention has been given to the question of the integrity of cracked piping systems that are fabricated from ductile materials. This interest has been motivated, in large part, by the technological problem of intergranular stress corrosion cracking (IGSCC) of Type 304 stainless steel piping in Boiling Water Nuclear Reactor piping systems. The IGSCC cracks form at the inner surface of a pipe, and are usually associated with the heat-affected zones of girth welds; the cracks are circumferential and are able to grow slowly in service by an environmentally assisted mechanism. From a safety perspective, it is important to know whether accident condition loadings will allow a part-through stress corrosion crack to propagate unstably across the pipe thickness by a non-environmentally assisted (plastic) fracture mechanism, and then the resulting through-wall crack propagate around the pipe circumference and thus cause complete pipe severance (guillotine failure).

One particular methodology that has been developed to address this problem is the net-section stress methodology (ASME 1983), which in a sense can be viewed as one of the two key elements of the CEGB R6 methodology (Milne et al. 1988). The net-section stress methodology uses as input a knowledge of the anticipated loadings and the critical net-section stress, which is usually taken to be the average of the yield and ultimate tensile stresses of the material under consideration. In applying the methodology, the usual practice is to calculate the stresses in the region of the cracked section via a purely elastic analysis based on the piping system being uncracked. However because the piping is built-in at its ends into a larger component, this limits the amount of elastic follow-up and, consequently

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use of the net-section stress approach in this manner gives conservative predictions for the onset of crack extension. There is also a measure of conservatism which stems from the fact that unstable failure need not necessarily be associated with the onset of crack extension, an issue that is usually addressed using tearing modulus procedures (Paris et al. 1979).

This paper focusses on the conservatism associated with the onset of crack extension predictions, and shows that it can be quite marked, particularly with small pipe radii and stiff piping systems.

GENERAL CONSIDERATIONS FOR A PIPING SYSTEM

In the context of the present paper's objectives, it is sufficient to consider the behaviour of a piping system containing a through-wall circumferential crack at an intermediate position in the system. Thus Figure 1 shows the cross-section of a circular cylindrical pipe, thickness t and radius $R \gg t$, which contains a through-wall crack with contained angle 2θ . Provided that plastic deformation is confined to the cracked section, the behaviour of this section can be decoupled from the remainder of the system, which is assumed to deform in accord with the laws of linear elasticity. Thus assuming that the deformation of the system occurs primarily by bending, the actual moment M and rotation ϕ at the cracked section are related to the moment M_u , calculated on the basis that the piping system is uncracked, by an expression of the form

$$M + \frac{EI\phi}{L_{EFF}} = M_u \quad (1)$$

where E is Young's modulus and $I = \pi R^3 t$ is the second moment of area of the piping at the cracked section. The moment M_u is the bending moment due to all sources that give non self-equilibrating static forces at the cracked section, i.e. pressure, deadweight, thermal expansion, applied loadings or displacements. Forces that are self equilibrating, i.e. arising for example from local thermal gradients or welding processes, are not considered in the present discussion. The effect of seismically induced (dynamic) forces has been considered separately (Smith 1991); it has been shown that their effects can be assessed in the same way as non self-equilibrating static forces.

The parameter L_{EFF} in relation (1) is a length parameter, which is a measure of the degree of elastic follow-up or flexibility of a piping system, with a high L_{EFF} value implying significant elastic follow-up. It is a function of the geometrical parameters of the system and can be obtained via a very simple procedure (Cotter, Chang and Zahoor 1988), which involves a separation of the complete piping system into two elastic parts at the cracked section with the system ends remaining fixed, the application of equal and opposite moments M_A to the cut faces and the correlation of M_A with the rotational discontinuity ϕ_A generated at this section, i.e.

$$L_{EFF} = \frac{EI\phi_A}{M_A} \quad (2)$$

For a straight pipe segment, built-in at its ends, and containing a cracked section at the mid-length position, $L_{EFF} = L$, the pipe length (Tada, Paris and Gamble 1980).

Now the moment (M) -rotation (ϕ) relation for a section of the piping containing a non-growing circumferential through-wall crack of prescribed length will be of the form shown in Figure 2, the initial region corresponding to small-scale yielding behaviour while at the other extreme there is an approach to limit-load conditions; the general level of the curve will be lower with a larger crack size. This curve is applicable until crack extension occurs, when there is a cut-off. Assuming, for a given crack size, that the onset of crack extension occurs at a given value of ϕ , i.e. ϕ_c , this corresponding to a critical value (M_c)

of M , then crack extension occurs when the line given by equation (1) intersects the curve shown in Figure 1. It is immediately seen that there is a reserve margin, corresponding to a moment of magnitude $EI\phi_c/L_{EFF}$, if the onset of crack extension is predicted using as input the moment calculated on the basis of an uncracked piping analysis. This reserve margin is unaffected by the magnitudes of the applied loadings or displacements, but does depend on their nature, e.g. where and how they are applied, and the system's geometrical characteristics; these factors govern the magnitude of the elastic follow-up parameter L_{EFF} . The reserve margin also depends on the magnitude of ϕ_c , the value of ϕ at the onset of crack extension; the margin will increase with the ductility of the material, i.e. the greater is the value of ϕ_c .

An estimate of the difference between M_c and the enhanced value $M_c + (EI\phi_c/L_{EFF})$ can be obtained by assuming: (a) that the material does not work-harden, (b) that the rotation at the cracked cross-section is all plastic, (c) that general yield precedes the onset of crack extension. The cracked section is therefore subject to the plastic limit moment, i.e. $M_c = M_p$, and following Tada, Paris and Gamble (1980), the stress distribution across the section can be described by a tensile stress σ_o acting within the region above the neutral axis (see Figure 1), while a compressive stress of similar magnitude is operative below the neutral axis. Since it is assumed that pipe deformation proceeds primarily by bending so that the effect of any resultant tensile force acting perpendicular to the cracked cross-section is ignored, the location of the neutral axis, defined by the angle α , is readily shown by balancing forces, to be given by $\alpha = \theta/2$. Furthermore, moment equilibrium provides the relation

$$\frac{M_p}{4\sigma_o R^2 t} = \cos\alpha - \frac{1}{2} \sin\theta = \cos \frac{\theta}{2} - \frac{1}{2} \sin\theta \quad (3)$$

Since the plastic rotation about the neutral axis at the onset of crack extension is ϕ_c , this is related to the crack tip opening displacement δ_I associated with the onset of crack extension by the expression

$$\phi_c = \frac{\delta_I}{R(\sin\alpha + \cos\theta)} = \frac{\delta_I}{R\left[\sin \frac{\theta}{2} + \cos\theta\right]} \quad (4)$$

Now with the net-section stress methodological approach, as it is usually employed, crack extension is presumed to occur when the moment M_{UNC} at the cracked section, calculated on the basis of the piping system being uncracked, equates with the plastic limit moment M_p . However, the onset of crack extension actually occurs when the moment M_{IN} at the cracked section, calculated on the same basis equals $M_p + (EI\phi_c/L_{EFF})$. Thus a measure of the factor of conservatism is the parameter $f = M_{IN}/M_{UNC}$, the physical meaning of f being that the cracked section can withstand a moment f times the moment predicted via the net-section stress approach without fracture initiating. Relations (3) and (4) show that f is given by the expression

$$f = \frac{M_{IN}}{M_{UNC}} = 1 + \frac{\pi E \delta_I}{4\sigma_o L_{EFF} \left[\sin \frac{\theta}{2} + \cos\theta\right] \left[\cos \frac{\theta}{2} - \frac{1}{2} \sin\theta\right]} \quad (5)$$

Having, as input, typical values for Type 304 stainless steel: $E = 30 \times 10^6$ psi (210×10^3 MPa), $\sigma_o = 50 \times 10^3$ psi (350 MPa), $\delta_I = 0.1$ in (0.25 cm) and $\theta = 60^\circ$, Table I gives the parameter f for a range of R and L_{EFF}/R values, remembering that $L_{EFF} = L$ for a straight pipe segment of length L , built-in at its ends, and with a cracked section at the mid-length position. Inspection of the results in Table I shows that there is a fair measure of conservatism in using the net-section stress approach (based on an uncracked piping analysis) to predict the onset of crack extension, when the pipe radius is small, and

with a stiff piping system (i.e. small L_{EFF}/R ratio). For example the factor of conservatism f exceeds two with $R = 5$ cm. if $L_{EFF}/R \gtrsim 50$, and with $R = 10$ cm if $L_{EFF}/R \gtrsim 15$.

INSTABILITY OF CRACK GROWTH

The paper's considerations so far have been with regard to the criterion for the onset of crack extension, and in particular with regard to the margin of conservatism associated with the application of the net-section stress criterion. There is also a measure of conservatism stemming from the fact that the onset of crack extension need not necessarily be associated with unstable failure. In order to address this issue, it is sufficient to base the discussion on the idealised model representation used in the preceding section, where it was assumed that (a) the material is non-work-hardening, (b) the rotation at the cracked cross-section is all plastic (i.e. the elastic rotation is assumed to be negligible) and (c) the onset of crack extension occurs subsequent to the attainment of the general yield state at the plastic limit moment M_p . If the crack extension regime is incorporated, the $M-\phi$ relation for a section containing a through-wall circumferential crack of prescribed initial size, i.e. the contained angle is initially $2\theta_0$, will be of the schematic form shown in Figure 3. The dotted lines a and b represent relation (1) at the onset of crack extension for two different L_{EFF} values. In case (a), crack growth is stable at the onset of crack extension and the input loadings M_u have to be increased for instability, which occurs when M_u is sufficiently high that the slope of the input $M-\phi$ line is the same as the $M-\phi$ relation for the cross-section, i.e. the dotted line a is translated to the position a¹. In case (b), which refers to a higher L_{EFF} value, instability is associated with the onset of crack extension.

To quantify this issue, it is sufficient to focus on the criterion for crack growth to be unstable at the onset of crack extension, with the applied loadings fixed. Thus with M_u fixed and M being equal to the limit moment M_p , relation (1) shows that the condition for crack growth to be unstable at the onset of crack extension is

$$\frac{EI}{L_{EFF}} + \left[\frac{dM}{d\phi} \right]_I < 0 \quad (6)$$

Following Tada, Paris and Gamble (1980), relation (6) can be expressed in the tearing modulus format (Paris et al 1979). Thus the magnitude of the J integral for a given crack area $A = 2Rt\theta$ is

$$J = - \frac{\partial M_p \phi}{\partial A} = \sigma_0 R \phi \left[\sin \frac{\theta}{2} + \cos \theta \right] \quad (7)$$

using relation (3), whereupon relations (3), (6) and (7) give the instability condition as

$$L_{EFF} \cdot \frac{2}{\pi} \left[\sin \frac{\theta}{2} + \cos \theta \right]^2 > \frac{E}{\sigma_0^2} \frac{dJ}{d\theta} - \frac{EJ}{\sigma_0^2 R} \left[\frac{1}{2} \cos \left[\frac{\theta}{2} \right] - \sin \theta \right] \quad (8)$$

With the tearing modulus format, the material tearing modulus T_{MAT} is related to the slope of the J-crack growth resistance curve:

$$T_{MAT} = \frac{E}{\sigma_0^2} \frac{dJ_{MAT}}{da} \quad (9)$$

where δa is an increment of crack length. Noting that $\delta a = R\delta\theta$, the condition (i.e. (8)) for crack growth to be unstable at the onset of crack extension, i.e. when $J = J_{IC}$ and when $\theta = \theta_0$ say, can be written in the form

$$T_{APP} = \frac{L_{EFF}}{R} \cdot f_1(\theta_o) > T_{MAT} - \frac{EJ_{IC}}{\sigma_o^2 R} f_2(\theta_o) \quad (10)$$

where T_{APP} is the applied tearing modulus and where $f_1(\theta_o)$ and $f_2(\theta_o)$ are given respectively by the expressions

$$f_1(\theta_o) = \frac{2}{\pi} \left[\sin \frac{\theta_o}{2} + \cos \theta_o \right]^2, \quad f_2(\theta_o) = \left[\frac{\frac{1}{2} \cos \left[\frac{\theta_o}{2} \right] - \sin \theta_o}{\sin \left[\frac{\theta_o}{2} \right] + \cos \theta_o} \right] \quad (11)$$

For a material, like Type 304 stainless steel, with a high crack growth resistance, it can be assumed (Tada, Paris and Gamble 1980) that $EJ_{IC}/\sigma_o^2 R \ll T_{MAT}$, whereupon the instability condition (10) can be expressed in the simplified form

$$T_{APP} = \frac{L_{EFF}}{R} \cdot f_1(\theta_o) > T_{MAT} \quad (12)$$

The parameter L_{EFF} therefore plays a crucial role both with regard to the degree of conservatism associated with the net-section criterion for the onset of crack extension, and the criterion for unstable crack growth.

DISCUSSION

The present paper's considerations have shown that there is a significant safety margin in using the net-section stress methodology to predict the condition for the extension of a circumferential crack, if the stresses at the cracked section are calculated on the basis that the piping is uncracked. There is also an additional measure of conservatism which stems from the fact that unstable failure need not necessarily be associated with the onset of crack extension. A key parameter with regard to both these margins is L_{EFF} , a length parameter which is a measure of the degree of elastic follow-up in the system. It depends on the geometry of the system, the location of the cracked section and the location of the applied loadings; it also depends on the position of restraints in the piping system.

The considerations in this paper are clearly relevant to the determination of the critical crack size under accident loading conditions. However there are situations, such as for example in deciding whether or not to remove pipe-whip restraints in a system, where it is desirable to demonstrate that a leak-before-break situation exists. In other words, it has to be demonstrated that there is a sufficient margin between the size of crack that will give rise to a detectable leakage under normal operating conditions and the size of crack that is unstable under accident conditions. Just as the piping system's flexibility, i.e. L_{EFF} , affects the unstable crack size, as has been demonstrated in this paper, it will also influence the size of crack that will give a detectable leakage. This aspect of the overall leak-before-break problem is the subject of continuing research.

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Table 1. The factor of conservatism $f = M_{IN}/M_{UNC}$ associated with the onset of crack extension for a crack with $\theta = 60^\circ$.

R cm	L_{EFF}/R			
	10	20	50	100
5	6.44	3.72	2.09	1.55
10	3.72	2.36	1.55	1.27
20	2.36	1.68	1.27	1.14
40	1.68	1.34	1.14	1.07

FIG. 1
The cracked section of a pipe.

FIG. 2
The moment-rotation relation (full curve) for a pipe

FIG. 3
The idealised moment-rotation relation (full curve).