Toughening Analysis for Warm Prestressing

FAN Xuejun

Taiyuan Univ. of Tech. Taiyuan, China

YU Shouwen

Tsinghua Univ., Beijing, China

ABSTRACT

In this paper, Chell's theory for warm prestressing is re-examined. It is found that the J_e -integral defined by Chell should be reformulated. As a new control parameter, the integral $\bar{J}_w$ is proposed to explain WPS phenomenon. By comparison with published WPS data, $\bar{J}_w$ -integral is more available.

1 INTRODUCTION

Warm Prestressing (WPS) of a cracked structure can result in an apparent elevation of its fracture toughness at lower temperature. These beneficial effects have been found to the pressure vessel of a PWR during a loss of coolant accident (Cheverton et al. 1983; Bryan et al. 1986). Chell and his coworkers (1980, 1981) have produced a series of papers which outline a theory of warm prestressing based on J_e -integral parameter, elevated only from elastic strains and distortions.

In present paper Chell's theory is re-examined. It is found that J_e -integral defined by Chell should be reformulated. As a new control parameter, the integral $\bar{J}_w$ is proposed to explain the WPS phenomenon. The predicted results based on $\bar{J}_w$ -integral is compared with the published experimental data.

2 GENERAL ANALYSIS

Referred to Fig.1, the dashed line Γ_1 represents the residual plastic boundary at the temperature T_1 , i.e., step 1 of the warm prestressing process. Γ_2 represents the boundary of the compressive plastic region which occurs in unloading at T_2 during the step 2 of the loading sequence. The cracked structure is then cooled to temperature T_3 . On

Fig.1 Schematic representation of plastic and residual zones during WPS

re-loading at T_3 during step 3, the new plastic zone, which is bounded by the path of Γ_3 , is developed. According to the relative sizes of the plastic zones formed during each loading step, there are three cases to be considered (Chell 1980). Assuming that the small scale yielding condition holds in the present analysis, three cases thus can be defined by

$$\text{Case 1} \quad \frac{K_1}{\bar{\sigma}_1} > \frac{K_1 - K_2}{\bar{\sigma}_1 + \bar{\sigma}_2} > \frac{K_3 - K_2}{\bar{\sigma}_2 + \bar{\sigma}_3} \quad (1)$$

$$\text{Case 2} \quad \frac{K_1}{\bar{\sigma}_1} > \frac{K_3 - K_1}{\bar{\sigma}_3 - \bar{\sigma}_1} > \frac{K_1 - K_2}{\bar{\sigma}_1 + \bar{\sigma}_2} \quad (2)$$

$$\text{Case 3} \quad \frac{K_3 - K_1}{\bar{\sigma}_3 - \bar{\sigma}_1} > \frac{K_1}{\bar{\sigma}_1} > \frac{K_1 - K_2}{\bar{\sigma}_1 + \bar{\sigma}_2} \quad (3)$$

where K_i is the stress intensity factor during the i th loading step, and $\bar{\sigma}_i$ the flow stress at T_i ($i=1,2,3$). For case 1 the development of new active plastic zone is controlled by the magnitude of stress intensity factor ($K_3 - K_2$), and for case 2, by ($K_3 - K_1$). Case 3 corresponds to all WPS effects wiped out by step 3.

3 FAILURE CRITERION

In order to describe failure in the presence of the plastic and residual zones following warm prestressing, Chell et al postulated as a failure criterion the attainment of a critical force on all mobile dislocations at low temperature. This force is given by the counter integral, termed J_e , which elevated only from elastic strains and distortions, i.e.,

$$J_e = \int_{\Gamma_3} W^e dy - \sigma_{ij} n_j \beta_{ix}^e ds \quad (4)$$

where

$$W^e = \int_0^{\epsilon_{ij}^e} \sigma_{ij} d\epsilon_{ij}^e \quad (5a)$$

$$\beta_{ij}^e = \beta_{ij} - \beta_{ij}^p, \quad \beta_{ij} = \frac{\partial u_i}{\partial x_j} \quad (5b)$$

It is assumed that the low temperature failure will be by cleavage and this will occur when $J_e = J_c$, the critical value of J_e . If small scale

plasticity is considered, the failure criterion at temperature T_3 would be

$$J_e = \frac{K_c^2}{E'} \quad (6)$$

where K_c is the fracture toughness at T_3 and $E' = E$, Young's modulus for plane stress and $E/(1-\nu^2)$ where ν is Poisson's ratio, in plane strain.

Based on the strip yield model, Chell et al used this integral parameter to predict the magnitude of the resultant toughness elevation, which have demonstrated the agreement between predicted and measured failure values after warm prestressing. In authors' opinion, however, there are a few points in Chell's theory, which, may be deliberated.

i). On the non-determinacy of the J_e -integral. In Chell's theory J_e -integral is defined to include only elastic strains and distortions. In fact, the elastic deformation gradient, β_{ix}^e , given in eq.(5), is arbitrary, because a rigid body motion can be introduced. On elevating J_e based on the strip yield model, Chell actually replaced β_{ix}^e by β_{ix} , the total deformation gradient. In other words, the following integral was used

$$\bar{J}_e = \int_{\Gamma_3} W^e dy - \sigma_{ij} n_j \beta_{ix} ds \quad (7)$$

which, gives the agreement with the observed data during WPS process. Naturally, people now, may pose one question: would is it available to use $\bar{J}_e$ -integral as the control parameter for WPS analysis? The answer is negative.

ii). On the non-universality of $\bar{J}_e$ -integral. To illustrate this point, Mode III crack in perfectly-plastic materials is considered here. The exact mathematical solutions of stress and strain fields in plastic region near mode III crack tip can be derived (Rice 1968), and used to compute the $\bar{J}_e$ -integral. For case 2 as analyzed in last section, $\bar{J}_e$ is

$$\bar{J}_e = \frac{\bar{\tau}_3}{2\mu} \left[-\frac{1}{\bar{\tau}_1} + \frac{(K_3 - K_1)^2}{\bar{\tau}_3 - \bar{\tau}_1} \right] \quad (8)$$

The results given in (8), obviously, is wrong. It is therefore concluded that $\bar{J}_e$ -integral is applicable only to the case in which the strip yield model is used.

In view of the above mentioned facts, it may be necessary to find a new parameter to explain the WPS phenomenon. The paper presents a new control parameter, termed $\bar{J}_w$, i.e.,

$$\bar{J}_w = \int_{\Gamma_3} \Delta W dy - \sigma_{ij} n_j \Delta u_{i,x} ds \quad (9)$$

for WPS analysis, where

$$\Delta W = \int_{\epsilon_{ij}}^{\epsilon_{ij}} \sigma_{ij} d\epsilon_{ij} - \Delta \epsilon_{ij} \quad (10)$$

and Δu_i and $\Delta \epsilon_{ij}$ are the associated displacements and strain fields under

reloading step at T_3 . The failure criterion thus can be written as

$$\bar{J}_w \geq J_c = \frac{K_c^2}{E'} \quad \text{with} \quad E' = \begin{cases} E & \text{in plane stress} \\ 1/(1-\nu^2) & \text{in plane strain} \\ 2\mu & \text{in Mode III} \end{cases} \quad (11)$$

Based on the strip model for Mode I and on the exact analytical elasto-plastic solutions for Mode III, $\bar{J}_w$ -integral can be obtained as

Mode I		Mode III	
case 1	$\bar{J}_w = \frac{1-\nu^2}{E} \frac{(K_3-K_2)^2 \bar{\sigma}_3}{\bar{\sigma}_2 + \bar{\sigma}_3}$	case 1	$\bar{J}_w = \frac{(K_3-K_2)^2 \bar{\tau}_3}{2\mu(\bar{\tau}_3 + \bar{\tau}_2)}$
case 2	$\bar{J}_w = \frac{1-\nu^2}{E} \frac{(K_3-K_1)^2 \bar{\sigma}_3}{\bar{\sigma}_3 - \bar{\sigma}_1}$	case 2	$\bar{J}_w = \frac{(K_3-K_1)^2 \bar{\tau}_3}{2\mu(\bar{\tau}_3 - \bar{\tau}_1)}$
case 3	$\bar{J}_w = \frac{1-\nu^2}{E} K_3^2$	case 3	$\bar{J}_w = K_3^2 / 2\mu$

(12-13)

It can be seen that the identical solutions are obtained for Mode I and Mode III by use of the $\bar{J}_w$ -integral.

4 RESULTS AND DISCUSSIONS

For the propose of assessing structural integrity the parameter of main interest is K_f , the value of K_3 at low temperature on re-loading step when failure occurs. Since the value of this is not known beforehand, it may be necessary to perform several calculations to identify which case, as illustrated in section 2, is applicable. For instance, let $K_2/K_1=1$,

Fig.2 Resultant toughness elevation during warm prestressing

calculations show that $K_f = K_c$ when $K_1/K_c < \bar{\sigma}_1/\bar{\sigma}_3$. For this case the WPS effects are wiped out. When $K_1/K_c > \bar{\sigma}_1/\sqrt{\bar{\sigma}_3(\bar{\sigma}_3 - \bar{\sigma}_1)}$, $K_f/K_c = K_1/K_c + \sqrt{(\bar{\sigma}_3 - \bar{\sigma}_1)/\bar{\sigma}_3}$. And with the values of K_1/K_c between $\bar{\sigma}_1/\bar{\sigma}_3$ and $\bar{\sigma}_1/\sqrt{\bar{\sigma}_3(\bar{\sigma}_3 - \bar{\sigma}_1)}$, the condition $K_f/K_1 = \bar{\sigma}_3/\bar{\sigma}_1$ must be made, which means that the failure will occur once the development of the plastic zone coincides with the residual zone in warm prestressing process. Fig. 2 gives the results of K_f/K_c as a function of K_1/K_c for various ratios of K_2/K_1 and $\bar{\sigma}_3/\bar{\sigma}_1$. It should be noted that load step 1 and 2 are assumed to be at same temperature, i.e., $\bar{\sigma}_1 = \bar{\sigma}_2$. The case of $K_2/K_1 = 1$ corresponds to the case in which the warm prestressing load level is maintained unchanged during cooling, while $K_2/K_1 = 0$ corresponds to complete unloading after warm prestressing. The latter is relevant to the overpressure tests in a high temperature in engineering. Usually before a pressure is put into service, it needs to be tested by overpressure in a high temperature. This process at same time improves the fracture resistance of the vessel. From the Fig.2 it can be seen that the greatest elevation of the toughness is given by $K_f/K_c = \sqrt{1 + \bar{\sigma}_2/\bar{\sigma}_3}$ provided that $K_2/K_1 > (\bar{\sigma}_1 + \bar{\sigma}_2)/\sqrt{\bar{\sigma}_3(\bar{\sigma}_3 + \bar{\sigma}_2)}$.

Beneficial effects arising from warm prestressing have been considered to the pressure vessel of a FWR during a loss of coolant accident. According to the magnitude of unloading during thermal shock, the actual fracture resistance can be predicted by use of $\bar{J}_w$ -integral. It is concluded that the total stress intensity factor at failure increases with the magnitude of unloading decreasing. However, there exists a limit state, i.e., the elevation of the toughness can not exceed the value of $K_c(T_1)/K_c(T_3)$.

The predicted results by use of $\bar{J}_w$ -integral have been compared with the published experimental data (Chell 1980) in table 1, in which a good agreement is made.

Table 1. Comparison of the predicted values K_f based on $\bar{J}_w$ -integral with experimental data (Chell 1980)

Spec.	Load step 1 ^a		Load step 2			Load step 3			Predicted
	T_1	K_1	T_2	K_2	$\bar{\sigma}_2$	T_3	K_f	$\bar{\sigma}_3$	
No.	(C)	(MPa $\sqrt{m}$)	(C)	(MPa $\sqrt{m}$)	(MPa)	(C)	(MPa $\sqrt{m}$)	(MPa)	(MPa $\sqrt{m}$)
1	16	59.3	*	41.8	514	-73	70.8	548	67.7
2	18	62.1	*	42.1	514	-73	71.8	548	70.9
3	21	60.4	*	40.7	514	-73	69.9	548	69.0
4	24	67.6	24	0	480	-73	75.8	548	69.9
5	24	65.9	24	0	480	-76	78.5	548	69.9

6	20	91.5	*	44.9	514	-73	94.7	548	104.6
7	*	88.2	*	41.2	490	-75	99.3	548	100.7
8	21	64.8	*	41.1	530	-96	70.7	581	78.4
9	20	65.8	*	47.6	538	-101	72.0	595	81.5
10	20	63.8	20	0	480	-99	67.7	590	60.6
11	24	61.2	24	0	480	-101	74.6	595	60.6
12	24	84.7	*	41.2	540	-104	88.5	600	101.0
13	24	82.4	*	49.7	539	-103	86.4	598	100.0
14	24	59.2	*	42.4	600	-158	64.8	720	80.6
15	24	62.3	24	0	480	-154	52.4	710	47.0
16	27	85.8	*	37.6	600	-159	89.6	720	87.6
17	24	58.9	*	25.8	658	-196	58.9	835	72.6
18	21	70.4	*	24.1	658	-196	65.2	835	70.8
19	24	84.3	*	24.4	658	-196	72.9	835	71.2

*Temperature varied during load step.Unloading completed at temperature T_3 .

aCalculated using K_{IC} =51,45,37 and 35 MPa $\sqrt{m}$ at 73 C, -101 C,-157 C and -196 C respectively.

$\bar{\sigma}_1$ equal to 480 MPa in all cases.

Acknowledgment: Thanks to Prof. Hwang Keh-chih and Prof. Yang Wei for helpful discussions.

REFERENCES

- Cheverton,R.D., Iskander,S.K. and Whiteman,G.D. (1983). Integrity of PWR Pressure Vessels during Overcooling Transients, Nucl. Safety, 24(2), 196-204.
- Bryan,R.H., Bass,B.R., Merkle,J.G., Pugh,C.E., Robinson G.C. and Whiteman,G.D. (1986). The heavy-section steel technology pressurized-thermal shock experiment PTSE-1, Engng. Fracture Mech., 23(1), 81-97.
- Chell,G.G. (1980). Some fracture mechanics applications of warm prestressing to pressure vessels , 4th Int. Conf. on Press. Vess. Tech., Paper C22/80, pp.117-124.
- Chell,G.G., Haigh,J.R. and Vitek,V. (1981). A theory of warm prestressing: experimental validation and the implications for elastic plastic failure criteria, Int. J. Fracture, 17(1), 61-81.
- Rice,J.R. (1968). Mathematical analysis in the mechanics of fracture, Fracture II, Ed. H.Liebowitz, 192-314.