Shape Optimization of Ceramics/Metal Joint Based on Reliability

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1. INTRODUCTION

New-Ceramics have a great potential in various engineering applications. However, most ceramics are brittle and poor in machinability. These drawbacks are overcome by introducing composite structure consisting of ceramics and metal. In most cases, ceramics and metal are joined at elevated temperature by methods such as brazing or diffusion bonding. Due to the large difference in thermal expansion coefficients of the two materials, significant magnitude of residual stress is produced during the cooling process after joining (Seo et al. 1989). Such residual stress created at the bonding region may cause cracking or reduce the bonding strength. Thus, it is desirable to minimize the size of the residual stress and various techniques are developed for this purpose. One such technique is to introduce an interlayer (Kobayashi et al. 1989). Another possible technique is to reduce stress by controlling the shape of the bonding zone (Koguchi et al. 1989). The authors treated the problem of reducing the residual stress through shape control as an optimum design problem and developed a numerical method to automatically determine the optimum shape based on the reliability.

2 SHAPE OPTIMIZATION METHOD

2.1 Representation of shape and its modification

A two dimensional shape optimization problem, shown in Fig.1, is considered. Both ceramic and metal parts are assumed to be cylindrical and joined coaxially, so that the problem can be treated as axisymmetric thermal-elastic problems. The modification of the shape can be achieved by moving the nodes. If the part of the boundary to be modified and the directions in which the nodes can move are given a priori, the modified shape can be uniquely defined by the distance of move a_i of the i -th node.

Fig.1 Description of shape and its modification.

2.2 Failure probability and constraint condition

(1) failure probability

As one of the methods to evaluate the strength of brittle materials such as ceramics, methods based on the weakest link hypothesis and the statistics have been proposed (Murata et al. 1986). According to this approach, the failure probability P_f can be estimated as,

$$P_f = 1 - \exp \left\{ - \int (\sigma / \sigma_{01})^m Y(\sigma, 0) dV \right\} \\ \approx 1 - \exp \left\{ - \sum_j V_j (\sigma_j / \sigma_{01})^m Y(\sigma_j, 0) \right\} \quad (1)$$

where, σ_j is the largest principal stress computed for each element and V_j is the volume of the element. The function $Y(\sigma_j, 0)$ in Eq.(1) is the heaviside step function.

(2) constraining condition

In case of practical engineering problems, there are certain restrictions so that the product can be machined or produced and fulfill the required function. As one of the geometrical constraints, the arc length is assumed to be constant and the optimum shape is sought under this condition. In other words, when the original arc length is L_0 , the arc length of the optimum shape is kept αL_0 , such that

$$L = \alpha L_0 \quad (2)$$

where α is a given constant and referred to as arc length factor.

2.3 Optimization problem and objective function

(1) optimization problem

The optimization problem can be stated that, to determine the shape for which the objective function W , which is the failure probability P_f in this case, is minimized under the given constraint condition, i.e.

$$W = P_f \Rightarrow \min \\ \text{under } L = \alpha L_0 \quad (3)$$

(2) optimization problem without subsidiary condition

By introducing the Lagrange multiplier λ , the constraint condition can be embedded in the objective function and an objective function W^* in the following form is derived.

$$W^*(a_i, \lambda) = 1 - \exp \left[- \sum_j V_j \left\{ \sigma_j(a_i) / \sigma_{01} \right\}^m \right] \\ - \lambda \{ L(a_i) - \alpha L_0 \} \quad (4)$$

2.4 Iterative procedure

Since the objective function W^* given by Eq.(4) is a highly nonlinear function in terms of a_i and λ , its stationarity conditions, i.e.

$$\partial W^* / \partial a_i = 0, \quad \partial W^* / \partial \lambda = 0 \quad (5)$$

also become nonlinear, it is impossible to solve these equations in a single step. Thus, the optimum solution is sought through an iterative procedure. If the approximate values obtained after the (n)th iteration are denoted by a_i^n and λ^n , these are corrected by Δa_i and $\Delta \lambda$ in the (n+1)th iteration to get new approximations a_i^{n+1} and λ^{n+1} , such that,

$$a_i^{n+1} = a_i^n + \Delta a_i, \quad \lambda^{n+1} = \lambda^n + \Delta \lambda \quad (6)$$

Considering the objective function W^* as a function of Δa_i and $\Delta \lambda$, it can be expanded in Taylor series. If the terms higher than the second

order terms in Δa_i and $\Delta \lambda$ are neglected, the objective function W^* is reduced to,

$$W^*(\Delta a_i, \Delta \lambda) = W_0^*(a_i^n, \lambda^n) + W_1^*(\Delta a_i, \Delta \lambda) + W_2^*(\Delta a_i, \Delta \lambda) \quad (7)$$

where, W_0^* , W_1^* and W_2^* represent the constant, the first order and the second order terms in Δa_i and $\Delta \lambda$, respectively. The above objective function, can be rewritten in the following matrix form.

$$W^*(\Delta a_i, \Delta \lambda) = W_0^*(a_i^n, \lambda^n) - \left\{ \begin{array}{c} \Delta a_i \\ \Delta \lambda \end{array} \right\}^T \left\{ \begin{array}{c} F_a \\ F_b \end{array} \right\} + \frac{1}{2} \left\{ \begin{array}{c} \Delta a_i \\ \Delta \lambda \end{array} \right\}^T \left| \begin{array}{cc} K_{aa} & K_{ab} \\ K_{ba} & K_{bb} \end{array} \right| \left\{ \begin{array}{c} \Delta a_i \\ \Delta \lambda \end{array} \right\} \quad (8)$$

Using the stationarity condition, such that,

$$\partial W^* / \partial \Delta a_i = 0, \quad \partial W^* / \partial \Delta \lambda = 0 \quad (9)$$

a set of linear algebraic equations in terms of $\Delta a_i, \Delta \lambda$ are derived.

$$\left| \begin{array}{cc} K_{aa} & K_{ab} \\ K_{ba} & K_{bb} \end{array} \right| \left\{ \begin{array}{c} \Delta a_i \\ \Delta \lambda \end{array} \right\} = \left\{ \begin{array}{c} F_a \\ F_b \end{array} \right\} \quad (10)$$

By solving the above equation, $\Delta a_i, \Delta \lambda$ are determined and the new approximate solution a_i^{n+1}, λ^{n+1} can be obtained from Eq.(6). Further, the optimum solution for the given arc length αL_0 can be obtained by repeating the same procedure until the convergence is reached.

3 EXAMPLE OF SHAPE OPTIMIZATION

3.1 Example model

The model for the example problems is shown in Fig.2. It consists of ceramics (Al_2O_3) and metal (Cu) parts with the same size. It is assumed that the variations of material constants with temperature can be neglected and the values at room temperature shown in Table 1 are used. The parameters of the Weibull distribution which determines the failure probability of the ceramic part, are assumed to be,

$$\sigma_{01} = 50 \text{ MPa}, \quad m_1 = 15$$

Table 1 Mechanical properties of materials.

	Al_2O_3	Cu
Young's modulus (GPa)	370	130
Poisson's ratio	0.25	0.3
Thermal expansion coef. (1/K)	7.9×10^{-6}	17.7×10^{-6}

Fig.2 Example model and region to be modified.

3.2 Condition of optimization

The shapes are optimized under the conditions which are given as the combination of the following loading conditions and the constraint conditions on the part of the boundary to be modified.

loading condition

- (1) uniform tensile stress of 100 MPa is acting as an external load

(2) thermal stress due to temperature drop by 100 °C

(3) both (1) and (2) are acting simultaneously

boundary to be modified

(1) side surface of the metal part 2 mm from the interface (A-B in Fig.2)

(2) part of the interface 2 mm from the edge (A-C in Fig.2)

arc length : L

$$L = 1.005 L_0 \sim 1.10 L_0$$

3.3 Procedure of optimization

The procedure of optimization is shown using the problem under the uniform tensile force. The side surface of the metal is optimized in this case. The distribution of the largest principal stress for the original shape is shown in Fig.3. The maximum stress in the ceramics part occurs at the point P on the edge of the interface and its value σ_{max} is 165 MPa. The failure probability of the joint P_f is 3.07×10^{-7} . Noting that the stress field has a certain singularity at the edge of the interface (Hein et al. 1971), the value σ_{max} obtained by FEM should be considered as a relative measure.

As the first case, the shape is optimized for $\alpha = 1.005$. Then, this optimum solution is used as the initial value to obtain those for larger values of α .

The optimized shapes and the failure probability after optimization are shown for different values of arc length factor in Fig.4. In case of this example, concave shapes are obtained as optimum shape when positive value is chosen as the initial value of λ and convex shapes are obtained viceversa. When the optimum shape is convex and α is larger than 1.02, fully converged solution is not obtained. Such cases for which the convergence is not reached are shown by open circles and those for which converged solution is obtained are shown by solid circles. It can be seen from Fig.4 that the failure probability decreases with the increase of arc length factor when the shape is concave. The failure probability becomes 1.46×10^{-7} which is about the half of the failure probability before optimization. On the contrary, when the shape is convex, it rapidly increases with the arc length factor.

Fig.3 Distribution of the largest principal stress in the original model under uniform external load.

Fig.4 Optimum shapes and the failure probability under uniform external load.

Fig.5 Variation of the maximum stress in ceramics under external load with arc length factor.

Fig.6 Distribution of the largest principal stress in the optimum shape under uniform external load ($\alpha=1.10$).

For reference, the relation between the maximum tensile stress in ceramic part σ_{max} and α is shown in Fig.5. The maximum stress is roughly reduced by 25% when the shape is concave and $\alpha=1.10$. The distribution of the largest principal stress for the same case is shown in Fig.6.

3.4 Loading condition and optimum shape

To investigate the effect of the loading condition, shapes are optimized under two different loading conditions. For the case under the thermal load, the results of the optimization are shown in Fig.7. Unlike the case under external load, the failure probability reduces when the shape is convex. Similarly, it can be seen from Fig.8 that only concave shape is

Fig.7 Optimum shapes and the failure probability under thermal load.

Fig.8 Optimum shapes and the failure probability under combined load.

effective to reduce the failure probability when both thermal and external loads are acting.

3.5 Part of the optimization and its influence

To clarify the effect of the location to be modified, the interface between ceramic and metal parts is chosen as the part to be optimized. It is assumed that the thermal and the external loads are acting simultaneously in this case. The results of the optimization are shown in Fig.9. The failure probability is reduced when the shape of the ceramic part at the interface is convex. Comparing these results with those for the cases in which the side surface of the metal part is modified, it can be seen that it is more effective to control the shape of the metal part than to control that of the interface.

Fig.9 Optimum interface shapes and the failure probability under combined load.

4 CONCLUSION

A shape optimization technique based on reliability is proposed as one of the means to reduce the failure probability of ceramics/metal joints. Its effectiveness is demonstrated through simple numerical examples and it is shown that the shape optimization can be an useful tool in designing ceramics/metal joints.

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