Extensive Modeling of High Temperature Deformation for Cyclically Softening Material

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ABSTRACT

A unified inelastic constitutive equation is suggested for inelastic deformation of 2 1/4Cr-1Mo steel at 550 °C. In this study, an evolution of an overstress is discussed, where the overstress is a difference between an applied stress and an internal stress. Another significant point suggested in this study is the distinction of the inelastic deformation depending on the evolution of the overstress. A mathematical model was developed based on these assumptions. Many numerical experiments were conducted to examine the validity of the present model. In comparison with the experimental results, the present model is shown to give good descriptions for a wide variety of loading conditions, including time-dependent plasticity, stress relaxation, creep and cyclic softening.

1. INTRODUCTION

Since a material subjected to high temperature environment reveals very complicated inelastic behavior, establishing an inelastic constitutive equation which precisely predicts strain-stress responses is one of the most important but difficult problems in high temperature structural analysis. A concept of unified inelastic constitutive equation is considered to be a useful alternative in the future high temperature structural analysis, where loading conditions will become more complicated and severe. Many unified models have been suggested since late 70's and some comprehensive studies are reported in [1,2].

A unified model does not distinguish a plastic deformation from that of creep, and a "unified" time-dependent inelastic strain represents the both. In many models, a resistance force (internal stress) against the deformation is introduced to represent rearrangements of micro-structure induced by the inelastic deformation, and the rest part of the applied stress is termed an overstress. In many unified models, the inelastic strain rate is given as a function of the overstress.

In this study, the authors try to introduce an evolution equation of the overstress into a unified inelastic constitutive model. Another significant point suggested in this study is to distinguish the inelastic deformation depending on the growth of the overstress. Based on these as-

sumptions, a mathematical model is developed for the inelastic deformation of 2 1/4Cr-1Mo steel at 550 °C. The simulative ability of the present model is examined through many numerical experiments.

2. FUNDAMENTAL CONCEPT

In this study, we consider only a uniaxial loading condition and all of the expressions are uniaxial ones. A total strain ϵ_t is a sum of an elastic strain ϵ_e and an inelastic strain ϵ_p ;

$$\epsilon_t = \epsilon_e + \epsilon_p. \quad (1)$$

Assuming a linear elasticity, we have

$$\epsilon_e = \frac{\sigma}{E}, \quad (2)$$

where σ is an applied stress and E is an elastic modulus.

The applied stress is divided into two components that are an internal stress α and an overstress q ;

$$\sigma = \alpha + q$$

$$\text{or } \dot{\sigma} = \dot{\alpha} + \dot{q}, \quad (3)$$

where $(\dot{})$ means a time-derivative. This equation is a consistency condition and is used to derive the inelastic strain rate.

In this study, it is assumed that the internal stress is intrinsically time-independent, then we have the following evolution equation of the internal stress;

$$\dot{\alpha} = H_a(\alpha)\dot{\epsilon}_p, \quad (4)$$

where H_a is a function of α (other arguments are added in the final form, but they take no effect in the following discussion in this section).

As is mentioned before, the evolution equation of the overstress is tried to be taken into account. In contrast to the internal stress, the overstress is assumed to be time-dependent and we have the following evolution equation of the Bailey-Orowan type.

$$\dot{q} = H_q(q, q_s)\dot{\epsilon}_p - Ef(q, q_s) \quad (5)$$

where q_s is a variable to normalize q . The first term of the right-hand side of eq.(5) represents a strain hardening of the overstress. The function $f(q, q_s)$ represents the time-dependency of the overstress in terms of a recovery.

Equations (3), (4) and (5) give the following description of the inelastic strain rate.

$$\dot{\epsilon}_p = \frac{\dot{\sigma} + Ef(q, q_s)}{H_a + H_q}. \quad (6)$$

Note that we can find a good analogy between the present model and the combined hardening plastic theory. As is well known, the internal stress represents the directional hardening. Therefore, the evolution equation of the overstress (eq.(5)) describes the isotropic hardening behavior but with a time-dependency in terms of the recovery function f . Indeed, if the function f is eliminated from eq.(5), the present model turns out to be a time(rate)-independent plastic constitutive model of the combined hardening type (but pure elastic region is out of consideration).

In the above model, the overstress dominates the time-dependency of the inelastic deformation. Therefore, in the case of stress relaxation for example, the inelastic deformation is driven by the relaxation (recovery) of the overstress. The strain hardening of the overstress should not be

caused by the recovery process of its own, thus the evolution equation of the overstress (eq.(5)) is not employed in such a process where $d|q|/dt \leq 0$. Instead, since the recovery of the overstress dominates the inelastic deformation in this case, we assume the following description of the inelastic strain rate.

$$\dot{\epsilon}_p = f(q, q_s), \quad \text{for } \frac{d|q|}{dt} \leq 0, \quad (7)$$

where the function $f(q, q_s)$ is that of eq.(5) which represents the recovery of the overstress. This equation (7) is applied for stress relaxation under a constant total strain or creep under a constant stress, and eq.(6) for tensile or compressive deformation with non-zero total strain rate where $d|q|/dt > 0$.

3. MATHEMATICAL MODEL AND NUMERICAL EXPERIMENTS

3.1 Mathematical model

A mathematical model was developed for a uniaxial inelastic deformation of 2 1/4Cr-1Mo steel at 550°C through experimental data analyses based on the fundamental concept. This material reveals a cyclic softening behavior at this temperature. Experimental data used in this study come from creep-fatigue tests conducted in our previous studies. All the tests were conducted in a very high vacuum environment of 0.1 μ Pa. It was already found that the stress-strain responses do not differ from those observed in the air environment. Details about the test program and the test results are shown in [3-5].

The functions H_a , H_q and f were determined as the following equations (8) to (12). Evolution equations (12) to (15) were also determined for α_L , q_{sI} and q_{sD} , where q_s is divided into two parts of q_{sI} and q_{sD} . α_L and q_{sI} describe the cyclic softening behavior which is measured by the accumulated inelastic strain. A function g in eq.(8) represents a rate-dependency of the internal stress which is effective when the inelastic strain rate is very small. When the inelastic strain rate is larger than 10^{-7} s^{-1} , the value of g becomes very close to 1, then H_a becomes almost rate-independent.

- Functions

$$H_a = h_1(\alpha_L - \alpha \text{sgn}(q))^n g(f, \sigma) \quad (8)$$

$$H_q = h_2 \left\{ \ln \left(\frac{q_{sI} + q_{sL}}{|q|} \right) \right\}^\beta \quad (9)$$

$$f = k \left\{ \ln \left| \frac{q_{sI} \text{sgn}(q) + q_{sD}}{q} \right| \right\}^{-1/\lambda} \text{sgn}(q) \quad (10)$$

$$g = \exp \left(- \left| \frac{f}{B_1} \right|^{-\mu} \right) \quad (11)$$

$$\mu = B_2 \exp(B_3 |\sigma|) \quad (12)$$

- Evolution equations

$$\dot{\alpha}_L = k_a(\alpha_{st} - \alpha_L) |\dot{\epsilon}_p|, \text{ initial value } \alpha_{L0} \quad (13)$$

$$\dot{q}_{sI} = k_q(q_{st} - q_{sI}) |\dot{\epsilon}_p|, \text{ initial value } q_{s0} \quad (14)$$

$$\dot{q}_{sD} = h_3(q_{sL} \dot{\epsilon}_p - q_{sD} |\dot{\epsilon}_p|) \quad (15)$$

where E , h_1 , h_2 , h_3 , n , β , λ , k , k_a , k_q , α_{L0} , α_{st} , Q_{s0} , Q_{st} , Q_{sL} , B_1 , B_2 and B_3 are material constants. Their values are given in table 1.

Together with eqs.(1) to (4) and (6), these equations make a complete set of constitutive model.

3.2 Numerical experiments

Numerical experiments were conducted to examine the validity of the present model. The program included the following loading conditions.

(1) Monotonic tension tests under a total strain rate ranging from 10^{-6} to 10^{-3} s^{-1} . The calculated results and the corresponding experimental results are shown in Fig.1.

(2) Total strain-controlled cyclic push-pull tests which include symmetric continuous, unsymmetric continuous and strain-hold waves. The calculations were conducted for 18 different loading conditions. Some of the results are shown in Figs. 2 and 3. In Fig.3, total strain was held constant for $t_h(=600\text{s})$ at the tension peak of each cycle.

(3) Creep tests under constant nominal stress. The results are shown in Fig.4 with the corresponding experimental results.

In comparison with the experimental results, the agreements are found to be very good in all the cases.

4. CONCLUSION

A unified inelastic constitutive equation is suggested and a mathematical model is developed for the inelastic deformation of 2 1/4Cr-1Mo steel at 550 °C.

Many numerical experiments were conducted to examine the simulative ability of the model. The calculated results were compared to the experimental ones and it was shown that the present model give good descriptions of the inelastic deformation under a wide variety of loading conditions, including rate-dependent plasticity, stress relaxation, creep and cyclic softening.

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Table 1 Material Constants

Constants	2.25Cr-1Mo, 550°C
E	1.88×10^5 MPa
λ	0.29
k	1.60×10^{-6} s ⁻¹
n	3.0
h_1	0.346 MPa ⁻²
k_a	0.835
α_{L0}	160.0 MPa
α_{st}	132.5 MPa
β	1.42
h_2	9.34×10^5 MPa
q_{s0}	199.1 MPa
q_{st}	106.4 MPa
k_q	2.12
q_{sL}	100.8 MPa
h_3	221.1
B_1	6.56×10^{-8} s ⁻¹
B_2	0.358
B_3	5.37×10^{-3} MPa ⁻¹

Fig.1 Calculated and experimental results of monotonic tension

a) Stress-strain responses at the 1st cycle

b) Stress-strain responses at the 50th (40th) cycle

Fig.2 Calculated and experimental results of cyclic loading ($\dot{\epsilon} = 10^{-6}/10^{-3}$ s⁻¹)

a) Stress-strain responses at the 1st cycle

b) Stress-strain responses at the 50th cycle

c) Stress relaxation at the 1st and the 50th cycle ($\epsilon_{tr}=0.02$)

d) Stress relaxation at the 1st and the 50th cycle ($\epsilon_{tr}=0.01$)

Fig.3 Calculated and experimental results of cyclic loading ($\dot{\epsilon}_t=10^{-3}/10^{-3} \text{ s}^{-1}$, $t_h=600 \text{ s}$)

Fig.4 Calculated and experimental results of creep deformation