A Stochastic Approach to the Problem for Stability of a Spherical Shell with Initial Imperfections

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1 INTRODUCTION

Let us consider a spherical shell subjected to uniformly distributed external pressure. It is observed in experiments that after the loading exceeds a certain value p^* (the upper critical load), the shell loses stability and a jump transition in a new stability form takes place. This transition is accompanied either by forming one buckle or - a group of buckles, joining afterwards in a deep buckle. When considering the problem for stability loss of such a shell usually the forming of one buckle is considered.

It is well known that spherical shells are very sensitive even to small initial imperfections, and the real critical value of the load p^* practically depends on these imperfections (Galletly 1976). The problem for stability loss of a shell with initial imperfections could be regarded from a deterministic point of view as well as from a probabilistic one (Volmir 1963; Bolotin 1965). When considering the problem deterministically it is necessary to solve the basic equations from the nonlinear shell theory for obtaining the critical loads and the corresponding stability forms. When considering the problem from a stochastic point of view, the probability for realization of one stability form or another is accepted to be a measure for the possibility for realization of this form. In this work we propose a stochastic method developed in (Vorovich 1959) for considering the stability problem for a spherical shell with initial imperfections at uniformly distributed external pressure. We shall consider briefly this method in the following paragraph.

2 SOLUTION METHOD

The random factors influencing the work of the shell are divided in the next three groups:

1. deviations in dimensions and elastic properties;

2. deviations in fastening parameters;

3. dispersion of the external loading

The first two groups of factors are expressed by a finite number of parameters a_i , $i=1,2,\dots,m$ with probability density function $\phi(a_1, a_2, \dots, a_m)$. The problem is stated the next way: Given the density function of the disturbing parameters $\phi(a_1, a_2, \dots, a_m)$. Find the density function of the deformational parameters of the shell $f(q_1, q_2, \dots, q_n)$. The probability density function $f(q_1, q_2, \dots, q_n, t)$ is determined following Smoluchovsky equation (Feller 1970; Feynman 1972) and then adding the conditions each density function is supposed to satisfy. In the certain case, of a static problem and no continuous random process, the function $f(q_i)$ is given by Gibbs distribution density:

$$(1) f(q_1, q_2, \dots, q_n) = 1/J \exp \left[(-U + \sum_{k=1}^n q_k F_k^{(1)}) \frac{2\gamma}{\delta \rho^2} \right] \cdot J = \int_{-\infty}^{+\infty} \int \exp \left[(-U + \sum_{k=1}^n q_k F_k^{(1)}) \frac{2\gamma}{\delta \rho^2} \right] dq_1 dq_2 \dots dq_n$$

where U is the potential energy of the shell, $\{F_k^{(1)}\} = F_k^{(1)} = E(F)$, F gives the force acting on the shell and E denotes the mathematical expectation, δ characterizes the working conditions of the shell and the bigger δ is the "calmer" are the working conditions and γ gives the energy dispersion.

Then the unconditional density $f^0(q_1, q_2, \dots, q_n)$ will be given by:

$$(2) f^0(q_1, q_2, \dots, q_n) = \int_{-\infty}^{\infty} \dots \int f(q_1, q_2, \dots, q_n, a_1, \dots, a_n) \phi(a_k) da_{k1}$$

We shall give below some of the main properties of the stochastic method considered.

1. It does not require the solution of the stability problem for the shell, at one or other loadings, but only to know the total energy of the shell as a function of the deformational parameters.

2. To obtain the distribution function of the parameters of deformation one does not need the deterministic relation between these parameter and the disturbance parameters.

3. The distribution law (9) includes all possible factors that could cause the random character of the shell bending, at static and at dynamic loadings as well.

3 ENERGY OF THE SPHERICAL SHELL

Let us now turn to the case of the spherical shell subjected to uniformly distributed external pressure. The

disturbing parameters are the deviations in the geometry of the initial mean surface from the ideal one. The process of forming one buckle is considered. It is assumed that at the first stage of the buckle development, up to achieving the low critical load, the behaviour of the buckle could be described using the theory of shallow shells. The problem is regarded as axisymmetric. To evaluate the full energy of the shell we have applied a solution variant following Ritz method (Volmir 1963). It is assumed that at the buckle boundary the conditions for clamping hold:

$$(3) w=0, \frac{dw}{dr}=0 \text{ at } r=c$$

where c is the buckle radius. Then we assume no radial displacements at the buckle edge, i.e.

$$(4) u=0 \text{ at } r=c$$

which leads to

$$(5) \frac{d^2 \Phi}{dr^2} - \frac{\mu}{r} \frac{d\Phi}{dr} = 0 \quad \text{at } r=c$$

Assuming the following expressions for the deflection and the initial deflection respectively

$$(6a) w=f \left(1 - \frac{r^2}{c^2}\right)^2$$

$$(6b) w_0=f_0 \left(1 - \frac{r^2}{c^2}\right)^2$$

and then applying the basic nonlinear equations of the theory of shallow shells, we obtain for the total energy of the shell:

$$\mathcal{G} = \frac{3}{2} \pi \frac{Eh^4}{R} \hat{\mathcal{G}}$$

$$(7) \hat{\mathcal{G}} = A_1 (\xi^4 + 4\xi^3 \xi_0 + 3\xi^2 \xi_0^2 + 4\xi \xi_0^3) - A_2 (\xi^3 + 3\xi^2 \xi_0 + 2\xi \xi_0^2) + A_3 (\xi^2 + 2\xi \xi_0) - A_4$$

$$\text{where } A_1 = \frac{23-\mu}{84(1-\nu)k}, A_2 = \frac{3-\mu}{6(1-\nu)}, A_3 = \frac{4}{3(1-\mu^2)k} + \frac{(7-2\mu)k}{30(1-\nu)} - \hat{\sigma}$$

$$A_4 = \frac{3(1-\nu)}{2} \hat{\sigma}^2 k$$

$$\xi = \frac{f}{h}; \xi_0 = \frac{f_0}{h}; \hat{\sigma} = \frac{qR^2}{2Eh^2}; k = \frac{c^2}{Rh}$$

4 SOME RESULTS AND COMMENTS

In this paragraph the probability density function of the deflections as well as some probabilities are obtained. Following (1) and substituting (7) the joint distribution law for ξ and ξ_0 looks like:

$$(8) f(\xi, \xi_0) = \frac{1}{J} \exp(-\mu \hat{\mathcal{G}}(\xi, \xi_0)) ; J = \int_{-\infty}^{\infty} \exp(-\mu \hat{\mathcal{G}}(\xi, \xi_0))$$

$$\mu = \frac{4}{3} \frac{\pi E h^4}{R} \frac{\gamma}{\rho^2 \delta}$$

The unconditional density function of ξ then is given by:

$$(9) f^0(\xi) = \int_{-\infty}^{\infty} f(\xi, \xi_0) \phi(\xi_0) d\xi_0$$

where $\phi(\xi_0)$ is the probability density of ξ_0 .

After assuming for $\phi(\xi_0)$ a symmetric triangle distribution, with parameter h , the function $f^0(\xi)$ is obtained in a closed analytic form. Now one can easily evaluate the probability

$P_b = \text{Prob}(-b \leq \xi \leq b) = \int_{-b}^b f^0(\xi) d\xi$. It is given in the next simple form, after truncating the exponents in series:

$$(10) P_1 = h \int_{-a}^a F(\xi_0) d\xi_0 + h^2 \int_{-a}^a \xi_0 F(\xi_0) d\xi_0 - h^2 \int_0^a \xi_0 F(\xi_0) d\xi_0$$

$$\text{where } F(\xi_0) = \frac{\sum_{n=0}^{\infty} (-1)^n \mu^n / n! \sum_{i=0}^{2n} D_{2i}^n / (2i+1)}{\sum_{n=0}^{\infty} (-1)^n \mu^n / n! \sum_{i=0}^{2n} D_{2i}^n A^{2i+1} / (2i+1)}$$

and D_i is given by the formula:

$$\exists^n = \left(\sum_{i=0}^4 B_i \xi_i \right)^n = \sum_{i=0}^{4n} D_i^n \xi_i$$

On fig 1. the probability (10) is given as a function of the variance $S_{\xi_0}^2$, for different values of the load.

Some comments are made explaining the results obtained and connecting them with experiments. Considering the relation on fig 1. one can easily observe that in the case $0 \leq \sigma_{\text{lor}}, \sigma_{\text{lor}} = 0.155$, the increase of the variance leads to decreasing the probability P_1 . This is explained by the fact that, in this case one only stability form exists, the initial one, and it is in the interval $|\xi| < 1$.

For pressures $\sigma > \sigma_{\text{lor}}$, for large positive ξ_0 , the only stable form is out of the interval $[-1, 1]$. For small $\xi_0 > 0$, $\hat{\sigma} > \hat{\sigma}_{\text{lor}}$, for the spherical shell there exists one only stable form in the interval $[-1, 1]$ and it is the initial one. This form possesses a higher level of potential energy than the other ones realized at $|\xi| > 1$. What concerns the case $\xi_0 < 0$, then the stability of the form in the interval $[-1, 1]$ increases when ξ_0 increases. Thus decreasing the variance of

the initial deflections, we decrease the possibility for reaching large enough negative initial deflections, and consequently we decrease the probability P_1 .

We shall now turn to the probability for a jump transition in another stability form completely different from the initial one. For $\sigma > \sigma_{lcr}$ a jump change of the stability form is realized after overtaking an energetic barrier $\Theta^*(\xi_*)$ (Volmir 1956). Moreover a "clap" is realized only if the initial deflections do not exceed a given value $\xi_{0*}(\hat{\sigma})$ (Bolotin 1965). If $\xi_0 > \xi_{0*}(\hat{\sigma})$ the deformations increase without a jump. So, for the probability P_* we obtain:

$$(11) P_*(\hat{\sigma}) = \int_{-\infty}^{\xi_{0*}(\hat{\sigma})} \int_{\xi_*}^{\infty} f(\xi, \xi_0) d\xi \phi(\xi_0) d\xi_0$$

Finally the probability P_* is obtained in a closed analytic form and its determination is driven to the calculation of some simple integrals. On fig 2 the probability for a jump transition P_* is drawn as a function of the loading σ , for different variances $S_{\xi_0}^2$.

The results obtained can be applied for determining the acceptable initial imperfections, given the value of the load, as well as the bearable loadings, given the initial imperfections (their probability density function). On this basis different kinds of safety problems can be solved for spherical shells with initial imperfections at uniform external pressure.

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Figure 2.

Figure 1.