Statistics of Cumulative Damage Based on Hysteretic Energy Dissipated

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ABSTRACT

The statistics of hysteretic energy dissipated by a nonlinear-hysteretic SDOF oscillator under random dynamic loading are discussed. It is observed that the first two moments (mean and variance) of the hysteretic energy dissipated by hysteresis behave in the same manner as the corresponding statistics of cumulative damage parameters based on linear damage accumulation. The paper also discusses how the hysteretic energy dissipated may be used as a measure of cumulative damage for both deteriorating and non-deteriorating structural components.

1.0 INTRODUCTION

It is recognized that the seismic performance of many structural systems depend on more than just one peak response[1-4]. Since well-designed earthquake resistant structures are supposed to tolerate deformations well beyond yield and several excursions into the nonlinear and inelastic range are expected during a strong earthquake load, it is apparent that the effect of the number and size of such loading cycles needs to be considered to evaluate the performance of a given design [1-4]. Liquefaction of loose sandy soils under earthquake loads is one other example where the cumulative effect of several loading cycles into the inelastic range needs to be accounted for[5-6]. The cumulative effect of several loading cycles into the nonlinear range also appears to be an important factor in the safety evaluation of surface panels of some flight structures under severe flow and acoustic pressures[7].

Here, the mean and coefficient of variation (c.o.v.) of the hysteretic energy dissipated by a nonlinear-hysteretic SDOF oscillator under random dynamic loading are discussed. It is observed that both the mean and c.o.v. of the hysteretic energy dissipated behave in the same manner as the mean and c.o.v. of cumulative damage parameters based on linear damage accumulation models. Next, the paper discusses how the hysteretic energy dissipated, ϵ_T , or the time-integral of the time-rate of the hysteretic energy dissipated raised to some power, $[\dot{\epsilon}_T]^m$, may be used as an indicator of cumulative damage for non-deteriorating systems and for deteriorating systems provided that the deterioration of the system per loading cycle can be established.

2.0 MEAN AND C.O.V. OF HYSTERETIC ENERGY DISSIPATED

A smooth hysteretic model together with an equivalent linearization technique [8,9] are used to investigate the behavior of the mean and, particularly, the coefficient of variation of the hysteretic energy dissipated. The basic features of the smooth hysteretic model are repeated here to discuss

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some of its parameters and introduce the notation used throughout the paper. Consider the SDOF system shown in Fig. 1. The hysteretic component of the restoring force in the spring is given by

$$\frac{\partial z}{\partial u} = A - \beta \frac{|\dot{u}|}{\dot{u}} |z|^{r-1} z - \delta |z|^r \quad (1)$$

where u = displacement of the oscillator relative to its support; and A , β , δ and r = parameters of the model that describe the shape of the hysteresis loops. A wide variety of hysteresis shapes may be represented[9] although only the hysteresis loops shown in Fig. 2 will be considered here. For these loops, $A > 0$, $(\beta + \delta) > 0$, and $(\delta - \beta) \leq 0$. The parameter r can be any number greater than or equal to zero. For $r = 0$, the system behaves as a linear elastic system and as $r \rightarrow \infty$ the system behaves like a elastic-perfectly plastic bilinear system.

The total restoring force in the spring is given by

$$F = \alpha K u + (1 - \alpha) K z \quad (2)$$

where K = elastic stiffness; and αK = postyield stiffness. The maximum restoring force for $\alpha = 0.0$ is given by

$$F_{max} = K \left(\frac{A}{(\beta + \delta)} \right)^{1/r} \quad (3)$$

Under a horizontal support motion with acceleration $a(t)$, the equation of motion for the oscillator is

$$m\ddot{u} + c\dot{u} + \alpha K u + (1 - \alpha) K z = -ma(t) \quad (4)$$

with the hysteretic component of the restoring force given by

$$\dot{z} = A\dot{u} - \beta |\dot{u}| |\dot{z}|^{r-1} z - \delta |z|^r \quad (5)$$

which is obtained from Eq. 1 by multiplying both sides of the equation by $\dot{u} = du/dt$. The rate of hysteretic energy dissipated at time t is

$$\dot{\epsilon}_T = (1 - \alpha) K (\dot{u} z) \quad (6)$$

and the total hysteretic energy dissipated is

$$\epsilon_T = (1 - \alpha) K \int_0^\infty \dot{u}(t') z(t') dt' \quad (7)$$

The mean rate of hysteretic energy dissipated at any time t' is given by

$$\mu_{\dot{\epsilon}_T}(t') = (1 - \alpha) K E[\dot{u}(t') z(t')] \quad (8)$$

where $E[\dot{u} z]$ = is one element of the response covariance matrix that is particularly well predicted by the equivalent linearization[9]. The mean of the hysteretic energy dissipated is simply

$$\mu_{\epsilon_T}(t) = (1 - \alpha) K \int_0^t E[\dot{u}(t') z(t')] dt' \quad (9)$$

which is also accurately predicted by the equivalent linearization[9].

The mean-square energy dissipated $E[\epsilon_T^2(t)]$ is given by

$$E[\epsilon_T^2(t)] = (1 - \alpha)^2 K^2 \int_0^t \int_0^t E[\dot{u}(t') z(t') \dot{u}(t'') z(t'')] dt' dt'' \quad (10)$$

Assuming that the two-time joint distribution of $\dot{u}$ and z is normal the above equation for the mean-square hysteretic energy dissipated can be written

$$E[\epsilon_T^2(t)] = (1 - \alpha)^2 K^2 \int_0^t \int_0^t \{E[\dot{u}(t')z(t')]E[\dot{u}(t'')z(t'')]\} \quad (11)$$

$$+ E[\dot{u}(t')\dot{u}(t'')]E[z(t')z(t'')] + E[\dot{u}(t')z(t'')]E[\dot{u}(t'')z(t')]\} dt' dt'' \quad (12)$$

Note that all the quantities in the integrand of Eq. 16 are elements of the two-time (t' and t'') response covariance matrix which can be calculated as described in Ref. 10.

It was observed[10] that the mean and c.o.v. of the hysteretic energy dissipated computed from the above equations compare well with Monte Carlo simulation results for both the stationary and the non-stationary conditions. Figures 4a to 5c show the simulation data (for the stationary conditions) plotted on normal and lognormal probability paper for various durations of the load.

The properties of the SDOF freedom system used are shown in Fig. 3. The input motion was represented by a segment of a filtered Gaussian white noise. The power spectral density of the input motion is described by the Kanai-Tajimi spectrum

$$S(\omega) = S_o \frac{1 + 4\zeta_B^2(\omega/\omega_B)^2}{[1 - (\omega/\omega_B)^2]^2 + 4\zeta_B^2(\omega/\omega_B)^2} \quad (13)$$

with $\omega_B = 15.6$ rad/sec and $\zeta_B = 0.64$.

Also shown in Figs. 4a to 5c are straight lines that define the normal (Figs. 4a-4c) and lognormal (Figs. 5a-5c) probability distribution functions using the mean and c.o.v. of the hysteretic energy dissipated computed from the random vibration analysis. The results of the Kolmogorov-Smirnov goodness-of-fit test are shown in Table 1. To pass the test at the 5% and at the 10% significance levels, the values in Table 1 should be less than 14.1 and 12, respectively.

Table 1 Kolmogorov-Smirnov goodness-of-fit test results.

Distribution	Duration	$S_o (in^2/sec^3)$			Distribution	$S_o (in^2/sec^3)$		
NORMAL	2.00	29.4	15.4	12.0	LOGNORMAL	24.1	11.4	11.8
NORMAL	4.00	22.6	14.0	13.6	LOGNORMAL	21.5	10.8	8.2
NORMAL	8.00	4.2	19.4	8.6	LOGNORMAL	13.0	10.2	6.8
NORMAL	12.0	6.0	4.4	8.8	LOGNORMAL	4.8	7.4	6.8
NORMAL	16.0	13.4	6.8	6.2	LOGNORMAL	26.4	9.6	6.0

Note that the curves representing the normal distribution for $S_o = 2.0 in^2/sec^3$ and durations of 2.0 and 4.0 seconds, obtained using the mean and c.o.v. computed with the random vibration analysis, intersect each other at the left tail of the distributions(see Fig. 3a). This implies that the normal distribution is not appropriate for small load durations.

Extensive sensitivity analyses conducted using the random vibration analysis[10] show that the behavior of the c.o.v. of the hysteretic energy dissipated under random dynamic loading behaves as the c.o.v. of linear cumulative damage parameters[11], namely: (1) the c.o.v. is inversely proportional to the square root of time; (2) it decreases when the bandwidth of the response process (load-effect process) decreases; (3) it decreases when the system ductility response increases (i.e., when the damping increases); and (4) for the same system ductility it is inversely proportional to the square root of the system elastic frequency when the input process is a white-noise.

3.0 CUMULATIVE DAMAGE PARAMETER

With the classical low-cycle cumulative damage model for metallic materials it is postulated that under constant amplitude cyclic load, the number of cycles to failure, N_f , can be related to the plastic deformation range by an equation of the type[3]

$$N_f = C^{-1}(u_p)^{-c} \quad (14)$$

where c and C are structural performance parameters. Assuming linear damage accumulation and letting the damage per cycle of size u_{pi} be denoted $1/N_{fi}$, the cumulative damage after N cycles of size u_{pi} is

$$D = \sum_{i=1}^N \frac{1}{N_{fi}} = C \sum_{i=1}^N (u_{pi})^c \quad (15)$$

For constant amplitude loading, Eq. 14 yields

$$N_f(u_p)^c = C^{-1} = \text{constant} \quad (16)$$

If, instead of the plastic deformation range, the hysteretic energy dissipated per cycle is used as the damage parameter, Eq. 16 is replaced by

$$N_f(\epsilon_c(u_p))^m = C_1^{-1} = \text{constant} \quad (17)$$

where $\epsilon_c(u_p)$ is the hysteretic energy dissipated in a cycle of amplitude u_p and m is chosen such that a best fit to the data can be obtained. Then, the cumulative damage after N cycles of different amplitude u_{pi} is given by

$$D = C_1 \sum_{i=1}^N (\epsilon_c(u_{pi}))^m \quad (18)$$

which is equivalent to Eq. 15.

Under random loading the energy dissipated by hysteresis and the damage parameter D are random quantities. A generalization of Eq. 18 to random loading is

$$D(t) = C_1 \int_0^t (\dot{\epsilon}_T(t'))^m dt' \quad (19)$$

where all the symbols have been previously defined. The mean and variance of $D(t)$ coincide with the mean and variance of the hysteretic energy dissipated if $m = 1$. If m is an integer, those statistics can be calculated, in approximation, assuming a joint gaussian distribution function and expressing the higher order moments as a combination of products and sums of the second-order moments. The validity of this approximation is currently being tested by comparing the random vibration results with the results from Monte Carlo simulation.

For deteriorating structural components it is necessary to take into consideration the variation of hysteretic energy dissipated per loading cycle as the system deteriorates. Since the deterioration of the system stiffness and strength can be considered in the random vibration analysis[9], it is possible to modify the previous formulation to account for the deterioration of the system.

4.0 SUMMARY AND CONCLUSIONS

A well-known smooth hysteretic model together with an equivalent linearization technique predict well the mean and c.o.v. of the hysteretic energy dissipated by nonlinear-hysteretic SDOF oscillators under random dynamic loading. The first two moments of the hysteretic energy dissipated by the nonlinear-hysteretic SDOF oscillator behave in the same manner as the first two moments of linear cumulative damage parameters. The paper also discusses how the time integral of the time rate of the hysteretic energy dissipated raised to some power may be used as an indicator of cumulative damage.

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Figure 1. SDOF nonlinear-hysteretic system.

Figure 2. Hysteresis loops.

Figure 3. Properties of SDOF investigated.

a)

b)

c)

Figure 4. Distribution function of hysteretic energy dissipated — normal probability paper: a) $S_0 = 2.0$; b) $S_0 = 50.0$; and c) $S_0 = 100.0$ in (in^2/sec^3) .

a)

b)

c)

Figure 5. Distribution function of hysteretic energy dissipated — lognormal probability paper: a) $S_0 = 2.0$; b) $S_0 = 50.0$; and c) $S_0 = 100.0$ in (in^2/sec^3) .