INVERSE AND DIRECT TRANSFER FUNCTIONS FOR THE FATIGUE FOLLOW-UP OF PIPING SYSTEMS SUBMITTED TO STRATIFICATION

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1. Introduction

Thermal stratification has been observed in several PWR systems since a couple of years. The locations where stratification is known to occur are the surge line and the main feedwater lines but other lines like the pressurizer spray line may also exhibit the phenomenon. The large number of thermal cycles of great amplitude produced by the stratification has raised some concern about the damage induced by fatigue in these lines.

Temperature and displacement sensors have been installed on these lines and records have been stored during measurement campaigns. This paper deals with the various interpretation problems which were encountered and the way followed to solve them.

2. Main problems to be solved

The physical parameters which influence the fatigue of a highly loaded point are usually very limited in number. In most cases, the knowledge of the local pressure and of a few values of fluid temperature (usually less than 3) is sufficient to allow the evaluation of the local stresses by use of direct transfer functions. The problem of investigating the thermal stratification in a pipe is notably more complex since, due to the non linearity of the temperature distribution over a cross section, even in an unrestrained pipe, stratification induces local stresses and a pipe curvature. The stresses at a given location in a restrained pipe submitted to stratified flow are therefore function not only of the local temperature distribution but also of the curvatures induced by the stratification at all other locations. Usually the stratification pattern is variable along the length of the line.

The local temperature distribution as well as the distribution of the curvature along the line (or the linear thermal gradient which is proportional to the curvature) must therefore be gained from the monitoring sensors. To reach a sufficient accuracy, a minimum of 5 measuring points per section are required and a minimum of 4 or 5 measuring sections along the length of the pipe has to be contemplated.

Ideally, the measurements should give the fluid temperature what is difficult, the measurements of the pipe outer wall temperature being by far more easy to perform. This leads either to assume that the external wall temperatures are a fair representation of the water temperatures (what appears to be inaccurate and unconservative particularly for transients) or to develop an algorithm allowing the calculation of the fluid temperature from the external wall values.

Once the local curvatures are known, they must be introduced in a global model of the line which will be used to determine the forces and moments at all locations of interest. The problem is

complicated by the fact that the gaps at the pipe whip restraints are usually insufficient to accommodate the large displacements caused by the stratification. Contact forces with these restraints take place and completely disturb the distribution of moments.

The main problems to be solved in the interpretation are therefore :

- a practical way of coping with the complex temperature and stress patterns both in a cross section and axially;
- the calculation of the fluid temperature distribution from the measured outside wall temperature;
- the global behaviour of the line (influenced by the local conditions in each cross section);
- the gap/contact situation on some pipe whip restraints (not dealt with in this paper).

The way by which these problems have been solved is described in detail in the next paragraphs.

3. Complexity of the temperature and stress fields

The temperature field in the pipe presents important variations in azimuthal and radial directions while the variations are smoother in the axial direction. In addition, the temperature field is strongly variable in time. In each cross section, the temperature presents a mirror symmetry with respect to the vertical plane. Based on these observations, it has been decided to express the temperature and local stress fields in the pipe as a Fourier's series of the azimuthal angle around the pipe. The temperature will thus be expressed as :

$$T(r, \theta, t) = \sum_{k=0}^n T_k(r, t) \cos k \theta \quad (1)$$

In order to keep the equations linear, the thermal as well as the mechanical properties of the pipe material as the thermal conductivity, the thermal capacity, the Young's modulus and the thermal expansion coefficient, will be considered independent of the temperature.

The thermal conduction equations in transient regime will be solved for each harmonic of the Fourier's series. A technique of eigenvalues and eigenfunctions similar to the one of the eigenmodes and eigenfrequencies in structural dynamics (see ref. 1 and 2) will be used for each harmonic. The temperature at a given time t for the point of coordinates r and θ in the pipe will be :

$$T_k(r, t) = \left[M_{0k}(r) - \sum_{l=0}^m M_{lk}(r) \right] T_{fk}(t) + \sum_{l=1}^m \kappa \phi_k^2 M_{lk}(r) \int_0^t e^{-\kappa \phi_k^2 M(t-\tau)} T_{fk}(\tau) d\tau \quad (2)$$

with

$M_{lk}(r)$ = eigenform of index l for the harmonic k

ϕ_k = eigenvalue of index l for the harmonic k

κ = thermal diffusivity of the pipe material

$T_{fk}(t)$ = component of harmonic k of the fluid temperature

$M_{0k}(r)$ = steady state temperature distribution of harmonic k
for a fluid temperature equal to unity in the same harmonic.

The eigenforms $M_{lk}(r)$ are normalized in such a way that

$$M_{0k}(r) = \sum_{l=1}^{\infty} M_{lk}(r) \quad (3)$$

The stresses may be obtained by substitution of the expression 2 of $T_k(r, t)$ in the thermoelastic equations of harmonic k .

For each $M_{lk}(r)$ function ($l > 0$), the thermoelastic theory will give a stress function $\sigma_{lk}(t)$ which, after combination, gives the stress response of harmonic k :

$$\begin{aligned} \sigma_k(r, t) = & \left[\sigma_{0k}(r) - \sum_{l=0}^m \sigma_{lk}(r) \right] T_{fk}(t) \\ & + \sum_{l=1}^m k \phi_l^2 \sigma_{lk}(r) \int_0^t e^{-\kappa \phi_l^2 k(t-\tau)} T_{fk}(\tau) d\tau \end{aligned} \quad (4)$$

with

$\sigma_{lk}(r)$ = modal stress caused by the application of temperature distribution $M_{lk}(r)$

$\sigma_{0k}(r)$ = steady state stress caused by the $M_{0k}(r)$ temperature field

The last equation is valid for any one of the stress components. The same formulation may also be used for the displacements, strains, etc.

The stresses are calculated under the assumption of plane strain conditions. Under these conditions the harmonic 0 usually causes an axial force. To get the pipe free expansion, an axial force of opposite sign is superimposed.

The same reasoning may be done concerning the bending moment caused by the harmonic 1. The superposition allows to determine the free curvature of the pipe.

The hereabove formulation allows to calculate, for any section where the fluid temperature is known, the local stresses, the free axial expansion and the free curvature.

4. Calculation of the azimuthal fluid temperature distribution from the outer wall one

The hereabove equations may only be used if the fluid temperature is known. In fact as explained hereabove only the outside wall temperatures are available. A formulation has therefore to be developed to obtain the fluid temperature from the outside wall temperatures. In such a process, the transfer functions used may be called "inverse" since they allow to obtain the cause of the phenomenon from the knowledge of its effects in opposition with normal or "direct" transfer functions which permit to calculate the effects of a phenomenon from its causes. The equation (2), written for the external radius r_e may be put under the form :

$$T_k(r_e, t) = C_k T_{fk}(t) + \int_0^t G_k(r_e, t - \tau) T_{fk}(\tau) d\tau \quad (5)$$

with :

$$C_k = M_{0k}(r_e) - \sum_{l=0}^m M_{lk}(r_e)$$

$$G_k(r_e, t) = \sum_{l=1}^m \kappa \phi_l^2 M_{lk}(r_e) e^{-\kappa \phi_l^2 k t}$$

r_e being a constant, C_k is a constant and G_k is a function of t only.

The Equation (5) is the basis to obtain the inverse transfer functions. In the case where $T_k(r_e, t)$ is known and $T_{fk}(t)$ unknown, it may be considered as an integral equation in T_{fk} the solution of which may be written as :

$$T_{fk}(t) = D_k(r_e) T_k(r_e, t) + \int_{-\infty}^{+\infty} F_k(r_e, t - \tau) T_k(r_e, \tau) d\tau \quad (6)$$

The mathematical developments leading to the values of the constant D_k and to the expression of F_k are beyond the scope of this presentation. The F_k function is the inverse transfer function. As this function decreases sharply as t tends to $+\infty$ or $-\infty$, the final expression of $T_{fk}(t)$ may be put under the form :

$$T_{fk}(t) \equiv D_k(r_e) T_k(r_e, t) + \int_{t-\Delta^-}^{t+\Delta^+} F_k(r_e, t-\tau) T_k(r_e, \tau) d\tau \quad (7)$$

where Δ^- and Δ^+ are chosen in such a way as to give a good engineering accuracy to the expression of T_{fk} .

An important advantage of the use of inverse transfer function is that the accurate knowledge of the film heat transfer coefficient is not needed. By using the same value for the film coefficient in the development of the inverse and direct transfer functions a compensation of the error occurs leading to correct values for the temperatures, stresses and displacements.

5. Global behaviour of the line

Figure 1 shows the layout of the surge line of the DOEL 2 power plant with the location of the provisional measuring devices (thermocouples and displacement sensors) used to monitor some heatups and cooldowns. This provisional measuring installation is typical (number and position of the captors) of what may be used for a permanent monitoring.

The use of the inverse transfer functions combined with the stress and strain analysis described hereabove allows to calculate the local stresses, the axial free expansion and free curvature of the pipe in each measuring section.

In order to assess the global behaviour of the line, it will be assumed that all these parameters vary linearly from measuring section to measuring section along the length of the line. The global behaviour of the line may then be determined as follows :

- influence functions are defined as shown in Figure 2 for both the free average axial strain and the free curvature of each measuring section. These functions have a value equal to 1 in the concerned section and decrease to zero in the next adjacent section;
- a finite element model of the line is built and load cases are calculated for each of the unit influence functions;
- in addition, load cases corresponding to unit end displacements in each space direction are also calculated;
- these unit load cases are multiplied by the local values of the end displacements, free axial strains and free curvatures and their summation provides the global displacements, forces and moments of the line.

The accuracy of this global model may easily be checked by performing a comparison between the response of the model and the measured values of the displacements. As shown in Figure 1, besides the 5 measuring sections equipped with thermocouples, the vertical displacement of the line is measured at three locations. These displacements are compared to the calculated ones in Figures 3 and 4 for a typical transient the recorded temperatures of which are given in Figure 5 for what concerns the Section 1 (temperatures for the other sections are very similar to Section 1 and are not shown). Examination of Figures 3 and 4 shows a very good agreement between the calculated and measured values with an excellent synchronization in time.

This validates the model of the line and thus the forces and moments which are obtained by the combination of the unit load cases.

The stresses at the most highly loaded point (which for Doel 2 is located in the elbow under the pressurizer) may then be calculated by summing the local stresses caused by the stratification with the stresses resulting from the forces and moments in the line. In the calculation of the stresses, use is made of stress intensification factors either directly extracted from the code or defined with the help of a detailed finite element model of the pipe particular discontinuity. In addition, the stresses are multiplied by an uncertainty factor of 1.2 to conservatively cover all the uncertainties and approximations introduced in the calculations. Figure 6 shows the time evolution at this highest loaded point for a cooldown (a little more than 24 hours).

6. Fatigue analysis

Once the local stresses and the force and moment distributions are calculated, the peak stresses at the points of interest may easily be evaluated leading to time history curves of the various stress components. The fatigue analysis based on the rainflow method is then straightforward; the following steps are performed in turn :

- identify first the extremes of the stress diagram
- identify the smallest cycles first, count them and eliminate them from the time history diagram and proceed further to finally take into account the largest cycles.

7. Conclusion

- Provisional measuring devices were installed on the surge line of the Doel 2 power plant to monitor the stratification in the line. From measurements obtained during the heatup and cooldown periods, it has been shown that a detailed fatigue monitoring of the actual transients occurring in the line was feasible. The main problems which were solved are :
 - the complexity of the temperature and stress fields as well in space as in time;
 - the determination of the temperature and stress fields from the knowledge of the outside wall temperatures;
 - the calculation of the global line behaviour and its addition to the local effects;
 - the effect of intermittent contacts with pipe whip restraints.
- The implementation of a permanent monitoring system may be contemplated as an alternative for the justification of the additional fatigue induced by the stratification on the surge lines.
- The usage factors obtained by this method are notably smaller than the ones calculated by classical design analysis. The period of observation, limited to a few heatups and cooldowns, is however too short to draw a final conclusion on this last point.

8. References

1. B.A. Boley, J.H. Weiner : "Theory of Thermal Stresses" - R.E. Krieger Publishing Company, Malabar, Florida, 1985.
2. H.S. Carslaw and J.C. Jaeger : "Conduction of Heat in Solids" - Clarendon Press Oxford, 1959.

Figure 1 Layout of the DOEL 2 Surge Line

Figure 2 Influence Functions of the Various Sections

Figure 3 Comparison of Measured and Calculated Displacements for Sensor 1 (DOEL 2)

Figure 4 Temperature Distribution in Measuring Section 5 (DOEL 2)

Figure 5 Temperature Distribution in Measuring Section 1 (DOEL 2)

Figure 6 Stresses in the DOEL 2 Surge Line during a Cutdown