Modelling the Dynamical Behaviour of Nuclear Reactor Fuel Assemblies

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Abstract

The computation of the motion of fuel rods of nuclear reactor cores is generally difficult for the number of elements is large. This paper is devoted with an homogenization process likening the fluid-tube-spacer system as an equivalent elastic medium for which the constitutive laws are derived. Numerical schemes are briefly described.

1. Introduction

The mechanical behaviour of the fuel clads of nuclear reactors is of great importance with regards to safety. Indeed, the fuel elements can be subjected to flow-induced vibrations, seismic impact, pressure drop due to LOCA accident, etc.. ; it is then necessary to know a priori the displacement of the entire core, taking into account presence of the cooling fluid. Unfortunately, direct numerical calculations are not possible to carry out, because of the large number of fuel rods, about 45,000 for PWR cores. The sole remedy consists by replacing the fluid-fuel assembly coupled system by an equivalent homogeneous material. This method is general and is known as homogenization in the literature (see [1], [2]).

In the following section, we outline this theory in case of fluid-tube bundle interaction.

2. Basic equations

In order to simplify the presentation, we consider two-dimension problems, and we suppose that the fluid is perfect and incompressible. The rods are rigid, elastically mounted, linked by springs of stiffness k . In this situation, the motion equation of each rod is (see figures 1 and 2) :

$$m \frac{d^2 \vec{s}_o}{dt^2} = \int_{\gamma_o} p(x, t) \vec{n}_x d\gamma_o - k \sum_{\ell=1}^4 (\vec{s}_\ell(t) - \vec{s}_o(t)) - k_o \vec{s}_o(t) \quad (1)$$

where p is the pressure, $\vec{s}_o$ and $\vec{s}_\ell$ are respectively the displacements of the tube γ_o and its neighbouring ones ; k_o is the stiffness of each cylinder ; $\vec{n}_x$ is the normal unit on γ_o , oriented inside γ_o (γ_o denotes also the wall tube). The rod mass m , k and k_o are considered per length unit. p is related to the speed potential of fluid ϕ by

$$p = -\rho_o \frac{\partial \phi}{\partial t}, \quad \rho_o: \text{fluid specific density.} \quad (2)$$

Incompressibility implies :

$$\Delta\phi = 0 \text{ in } \Omega_0, \text{ domain occupied by the fluid.} \quad (3)$$

The boundary conditions satisfied by ϕ are :

$$\frac{\partial\phi}{\partial n}(x) = \frac{d\vec{s}_0}{dt} \cdot \vec{n}_x \text{ on each } \gamma_0, \quad \frac{\partial\phi}{\partial n} = 0 \text{ on } \Gamma, \quad (4)$$

Γ being the rigid wall of the cavity containing the bundle.

Introducing the added mass matrix ρH_0 calculated by the influence functions (Chen [3], Paidoussis [4], see also [5]), and the stiffness matrix S of the spring-tube system, we obtain, eliminating ϕ :

$$(m + \rho_0 H) \frac{d^2 \vec{s}}{dt^2} + S \vec{s} = 0,$$

where $\vec{s}$ is the vector of order $2N$ made up with the displacement-vectors $\vec{s}_0$, N being the total number of tubes.

3. Homogenization of the equations

For this operation, it is necessary to assume that the rods are identical (same m , k , k_0 and same radius) and the bundle is spatially periodic with a tube pitch ϵ small compared to the dimensions of the core. Multiplying eq. (3) by an arbitrary regular test-function $v(x)$, integrating by parts on Ω_0 and using the boundary conditions (4), we obtain :

$$\int_{\Omega_0} \text{grad } \phi(x, t) \cdot \text{grad } v(x) dx = \sum_{\ell=1}^K \frac{d\vec{s}_\ell}{dt} \cdot \vec{N}_\ell \vec{v}, \quad (5)$$

where $\vec{N}_\ell \vec{v} = \int_{\gamma_\ell} v(x) \vec{n}_x d\gamma_\ell$, ℓ refers to the ℓ^{th} tube in the entire bundle.

Because ϵ is small, an asymptotic analysis (see [6], [7]) leads to replace the first term of eq. (5) by $\beta \int_{\Omega} \text{grad } \phi \cdot \text{grad } v dx$.

Here Ω denotes the region occupied by both the fluid and the bundle. β is calculated as follows : let us introduce the functions $\chi_i(x)$ on the elementary cell C (see figure 3) satisfying :

$$\begin{cases} \Delta\chi_i(x) = 0 \text{ in } C^*, & \frac{\partial\chi_i}{\partial n}(x) = -\cos(\vec{n}, x_i) \text{ on } \gamma, \\ \chi_i(x_1 + \epsilon, x_2) = \chi_i(x_1, x_2 + \epsilon) = \chi_i(x_1, x_2), & \forall x. \end{cases}$$

Then we define

$$\beta_{ij} = \frac{1}{\epsilon^2 - \pi r^2} \int_{C^*} \left(\frac{\partial\chi_i}{\partial x_j}(x) + \delta_{ij} \right) dx, \quad i, j = 1 \text{ and } 2 \text{ (then } \beta_{ij} = \beta_{ji}).$$

Geometric symmetries imply that $\beta_{12} = 0$ and $\beta_{11} = \beta_{22} = \beta$ (say).

On the other hand, it can be shown that :

$$\vec{N}_\ell \vec{v} = -\pi r^2 \text{grad } v(x_\ell) + O(\epsilon^3),$$

where x_ℓ is the center of the circle γ_ℓ , so that we can make the approximation :

$$\sum_{\ell=1}^K \frac{d\vec{s}_\ell}{dt} \cdot \vec{N}_\ell \vec{v} \simeq -\frac{\pi r^2}{\epsilon^2} \int_{\Omega} \frac{d\vec{s}}{dt} \cdot \text{grad } v(x) dx = \frac{\pi r^2}{\epsilon^2} \frac{d}{dt} \int_{\Omega} \text{div } \vec{s}(x, t) v(x) dx. \quad (6)$$

In this transformation, we have replaced the $\vec{s}_\ell$ by a continuous field $\vec{s}(x,t)$; it is also assumed, in a aim of simplification, that the tubes are fixed near the boundary Γ (see [6] for more realistic boundary conditions). Because v is arbitrary, relation (6) leads to

$$-\beta \Delta \phi(x,t) = \frac{\pi r^2}{\epsilon^2} \operatorname{div} \frac{\partial \vec{s}}{\partial t}(x,t). \quad (7)$$

Now, it remains to homogenize the rod motion equation. It is noted that $\sum_{\ell=1}^4 (\vec{s}_0 - \vec{s}_\ell)$ is a five-points approximation by finite difference of $-\epsilon^2 \Delta \vec{s}(x,t)$, so that eq.(1) becomes, after some handling :

$$(-k\epsilon^2 \Delta + k_0 + m \frac{\partial^2}{\partial t^2}) \vec{s}(x,t) = \pi r^2 \rho_0 \operatorname{grad} \frac{\partial \phi}{\partial t}(x,t). \quad (8)$$

The boundary conditions associated to (7), (8) are :

$$\vec{s} = 0, \quad \frac{\partial \phi}{\partial n} = 0 \text{ on } \Gamma. \quad (9)$$

Numerical scheme : a semi-explicit scheme can be used, for instance :

$$\left\{ \begin{array}{l} (-k\epsilon^2 \Delta + k_0) \vec{s}^n + m \frac{\vec{s}^{n+1} - 2\vec{s}^n + \vec{s}^{n-1}}{\Delta t^2} = \pi r^2 \rho_0 \operatorname{grad} \frac{\phi^n - \phi^{n-1}}{\Delta t}, \\ -\beta \Delta \frac{\phi^{n+1} + \phi^n}{2} = \frac{\pi r^2}{\epsilon^2} \operatorname{div} \frac{\vec{s}^{n+1} - \vec{s}^n}{\Delta t}, \end{array} \right. \quad (10)$$

where the index n means that the quantities must be considered at time $n \Delta t$. Of course, a stability condition holds. Fully implicit schemes can be also defined. The different differential operators with respect to x are approached by standard finite element method.

Remarks :

- 1 - Absence of mechanical links between adjacent assemblies can be considered : equations (7), (8) are only valid inside each assembly and it is necessary to add equations at each common interface (see [6]).
- 2 - Replacing $\partial/\partial t$ by $i\omega$, it can be shown that equations (6), (7) have an infinite discrete set of real eigenfrequencies ω_n which tend to infinity if $k > 0$ (presence of spacers) while these eigenvalues lie inside a finite interval when $k = 0$ (absence of spacer).

4. Extension to three-dimension case

Because the fuel rods are long, we have to take into account their bending, and moreover the presence of an axial flow with velocity u . ϕ is concerned only with the motion of fluid generated by the transverse displacement of the tubes, so that the homogenized equations are (see [6]), z being the axial coordinate :

$$\left\{ \begin{array}{l} (m \frac{\partial^2}{\partial t^2} - k\epsilon^2 \Delta_x + EI \frac{\partial^4}{\partial z^4}) \vec{s}(x,z,t) = \pi r^2 \rho_0 \operatorname{grad}_x (\frac{\partial}{\partial t} + u \frac{\partial}{\partial z}) \phi(x,z,t), \\ -\beta \Delta_x \phi(x,z,t) = \frac{\pi r^2}{\epsilon^2} \operatorname{div} \frac{\partial \vec{s}}{\partial t}(x,z,t). \end{array} \right. \quad (11)$$

Here, $\Delta_x = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}$, $\operatorname{grad}_x = \operatorname{col} (\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, 0)$, $\vec{s} = \operatorname{col} (s_1, s_2, 0)$; EI is the rod stiffness. The term in v results from the Lightfill's theory (see [8], [9] for instance).

Of course, we must add two boundary conditions at each tube end. Numerical schemes of the type (10) can be used for solving equation (11).

Remarks :

- 1 - The method is easily extended to fast breeder reactor cores (figures 4 and 5). It can be seen that the homogenized constitutive laws of fuel pins are similar to eq. (11), except that $\beta \Delta_x$ must be replaced by $\text{div} (\beta \text{grad.})$, where β is a matrix. In the same way, it is possible to obtain the equations governing the global displacements of the assemblies considered as beams, in which case the term $\Delta_x \vec{s}$ disappears because there is no spacer between the adjacent ones : a computer code based upon this method is presently in course of creation at C.E.A. (Centre d'Etudes Nucléaires, Saclay).
- 2 - Another homogenization process based on averaging the conservation laws, has been investigated by Benner and Schumann [10].

5. Conclusion

We have transformed, via homogenization, a complicated problem into a set of equations easy to solve by numerical method. Some assumptions to simplify the model are been done in order to present the process in few words ; nevertheless, the technique here described can be extended with no difficulty to more realistic situations taking into account nonlinear spacers, absence of mechanical link between adjacent assemblies, shocks, etc. ... The method may be also applied to heat exchangers and condensers.

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FIGURE 1 : Configuration of tubes and spacers.

FIGURE 2 : Example of PWR core.

FIGURE 3 : Elementary cell for PWR core.

FIGURE 4 : Fuel assembly of fast breeder reactor.

FIGURE 5 : Examples of elementary cells for fast breeder reactor (figures not to scale).