Application of PAULA-S Code to Shell Analysis

L. Lazzeri

Ansaldo Impianti S.p.A., Via G. D'Annunzio 113, I-16121 Genova, Italy

Abstract

The basic characteristics of PAULA-S code for the analysis of shells are given; in particular the following topics are discussed: generation of particular geometries, stress strain correlations, J integral computations, general organization of the program. Examples are given.

1. Introduction

The basic characteristics of a new shell finite element, have been described in detail by Lazzeri [1,2,3]; it is the aim of this paper to report about the further developments which have taken place recently, in order, to increase the capabilities of the shell code (PAULA-S) which makes use of such basic finite element.

The main finite element is a generalized shell with eight principal nodes, each with six dofs and eight auxiliary nodes for the definition of the geometry only; in fig.1 one may find both the actual and reference geometry, as the definition of it is made on a parametric basis.

Fig. 1

A modification (see later) is used for the membrane case. The element uses the following shape functions:

- 48 polynomial functions for the definition of the Geometry
- Hermitian functions for the definition of the radial displacement W (24 functions)
- mixed polynomial-Hermitian for the definition of the in plane displacements u, v (12 functions each).

The parametric approach is used in order to formulate the displacements and derivatives with

respect to the shell coordinates α, β up to the 2nd order as a function of the 48 basic parameters (nodal displacements and rotations).

2. Vlasov Shell Equations

The element makes use of the generalized shell equations formulated by Vlasov /4/, letting :

u,v,w, displacements α, β shell coordinates
A , B coefficients first quadratic form K1, K2 curvature radii
 ϵ elongations χ curvatures

the following table reports the dependance of ϵ, χ from the displacements and their derivatives :

Table 2.1						
Generalized shell equations (after Vlasov)						
	ϵ_α	ϵ_β	$\epsilon_{\alpha\beta}$	χ_α	χ_β	$\chi_{\alpha\beta}$
u		$\frac{1}{AB} \frac{\partial B}{\partial \alpha}$	$-\frac{1}{AB} \frac{\partial A}{\partial \beta}$	$\frac{1}{A} \frac{\partial K_1}{\partial \alpha}$	$\frac{1}{B} \frac{\partial K_2}{\partial \alpha}$	$\frac{(K_2 - K_1) \partial A}{AB \partial \beta}$
v	$\frac{1}{AB} \frac{\partial A}{\partial \beta}$		$-\frac{1}{AB} \frac{\partial B}{\partial \alpha}$	$\frac{1}{B} \frac{\partial K_1}{\partial \beta}$	$\frac{1}{A} \frac{\partial K_2}{\partial \beta}$	$\frac{K_1 - K_2}{AB} \frac{\partial B}{\partial \alpha}$
w	K_1	K_2		$-K_1^2$	$-K_2^2$	
$\partial u / \partial \alpha$	$1/A$					
$\partial v / \partial \alpha$			$1/A$			$-\frac{K_1 - K_2}{A}$
$\partial w / \partial \alpha$				$\frac{1}{A^2} \frac{\partial A}{\partial \alpha}$	$-\frac{1}{A^2 B} \frac{\partial B}{\partial \alpha}$	$\frac{2}{A^2 B} \frac{\partial A}{\partial \beta}$
$\partial u / \partial \beta$			$1/B$			$(K_1 - K_2) / B$
$\partial v / \partial \beta$		$1/B$				
$\partial w / \partial \beta$				$-\frac{1}{AB^2} \frac{\partial A}{\partial \beta}$	$\frac{1}{B^2} \frac{\partial B}{\partial \beta}$	$\frac{2}{AB^2} \frac{\partial B}{\partial \alpha}$
$\partial^2 w / \partial \alpha^2$				$-1/A^2$		
$\partial^2 w / \partial \alpha \partial \beta$						$-2/AB$
$\partial^2 w / \partial \beta^2$					$-1/B^2$	

The basic routine of the code computes the strains as a function of the basic 48 generalized displacements, however the values of the parameters in Table 2.1 have to be computed as a function of the particular geometry under condiseration. Basically assume that the shell geometry is uniquely defined by a set of parameters ξ, η (generally coincident with α, β) so that the Cartesian coordinates x, y, z are given as a function of ξ, η :

$$(1a,c) \quad x = x(\xi, \eta) \quad y = y(\xi, \eta) \quad z = z(\xi, \eta)$$

All the relevant parameters of the shell A,B, K1, K2 as well as the components of the orientation matrix are given as a function of the correlations 1a,c and their derivatives as shown in Table 2.2.

Table 2.2

Basic shell equations

Coefficients of the first quadratic form

$$A^2 = (\partial x / \partial \xi)^2 + (\partial y / \partial \xi)^2 + (\partial z / \partial \xi)^2$$

$$B^2 = (\partial x / \partial \eta)^2 + (\partial y / \partial \eta)^2 + (\partial z / \partial \eta)^2$$

Curvature radii

$$K1 = m_x^w \partial^2 x / \partial \xi_1^2 + m_y^w \partial^2 y / \partial \xi_1^2 + m_z^w \partial^2 z / \partial \xi_1^2$$

$$K2 = m_x^w \partial^2 x / \partial \xi_2^2 + m_y^w \partial^2 y / \partial \xi_2^2 + m_z^w \partial^2 z / \partial \xi_2^2$$

ds1 = distance between $\alpha, \beta = \text{const}$ and $\alpha + d\alpha, \beta + d\beta$

$$ds1 = A d\xi \quad ds2 = B d\eta$$

orientation matrix

$$m_x^u = 1/A \partial x / \partial \xi; \quad m_x^v = -1/B \partial x / \partial \eta; \quad m_x^w = 1/AB (\partial y / \partial \xi \partial z / \partial \eta - \partial z / \partial \xi \partial y / \partial \eta)$$

$$m_y^u = 1/A \partial y / \partial \xi; \quad m_y^v = -1/B \partial y / \partial \eta; \quad m_y^w = 1/AB (\partial z / \partial \xi \partial x / \partial \eta - \partial x / \partial \xi \partial z / \partial \eta)$$

$$m_z^u = 1/A \partial z / \partial \xi; \quad m_z^v = -1/B \partial z / \partial \eta; \quad m_z^w = 1/AB (\partial x / \partial \xi \partial y / \partial \eta - \partial y / \partial \xi \partial x / \partial \eta)$$

Note that the basic parameters are intercorrelated by Codazzi Gauss equations

$$\partial / \partial \alpha (1/A \partial B / \partial \alpha) + \partial / \partial \beta (1/B \partial A / \partial \beta) - K1 K2 A B$$

$$\partial / \partial \beta (A K1) = K2 \partial A / \partial \beta$$

$$\partial / \partial \alpha (B K2) = K1 \partial B / \partial \alpha$$

The code handles directly the basic geometries (cylinder, sphere) but any geometry can be analyzed if the following parameters calculation procedures are added in the routines

USER 1 α, β values at the Gauss points in the shell
 γ derivatives matrix ($\partial x / \partial \xi, \partial y / \partial \xi$ etc.)

USER 2 quadrature coefficients SURF

basic twelve parameters at the Gauss points

$$A, B, K1, K2, \partial A / \partial \alpha, \partial A / \partial \beta, \partial B / \partial \alpha, \partial B / \partial \beta, \partial K1 / \partial \alpha, \partial K1 / \partial \beta, \partial K2 / \partial \alpha, \partial K2 / \partial \beta$$

3. Membrane Elements

In some cases it is convenient to use membrane elements, whenever bending in the thickness is not relevant. In this case a modified element is available, as given by 16 nodes with 3 dofs each (rather than 8 nodes with 6 dofs) the shape functions formulation is in this case fully isoparametric. Theoretically Table 2.1 can be used even if the fourth, sixth column are not applicable as elongations only are of interest. However if one considers terms only, instability takes place as already shown by Novozhilov /5/ due to the presence of zero strain modes. The problem is solved very simply by adding the bending terms (hence the full Table 2.1 is implemented), however as the nodes have displacements only (no rotations) the element cannot deal with bending modes.

4. Stress Strain Correlations

The element can deal with three types of strain-stress correlation as mentioned herein:

a) Generalized Prandtl-Reuss correlation, in which the incremental theory of plasticity is used, however the yield locus is described in terms of stress resultants as suggested by Iliouchine /6/.

b) Generalized Henky Nadai correlation, in which the total theory of plasticity is used, again the theory of Iliouchine in terms of stress resultants is followed.

c) Generalized elastic non linear correlation, i.e. the elastic modulus is function of the stress status.

It is fairly obvious that the incremental theory of plasticity better agrees with the experimental data, however numerical difficulties quite often make such advantage completely useless. A predictor corrector scheme is used for a better solution (see ref./2/). A similar correction scheme can be used in the case of the total theory. In all the cases a "balance procedure" is used, i.e. the difference in loads between the applied and equilibrated loads is computed and the difference is added to the next step; the addition of this procedure has been found beneficial in reducing computing difficulties.

5. Structure-Fluid Interaction

The code can accept distributed masses as due to the dynamic fluid actions, in particular it is planned to introduce the procedure described in the companion paper in ref./7/ as a direct sullayer.

6. Fracture Mechanics Computations

The code can perform J integral computations according to a modified procedure described herein. From the general definition of the J integral the following correlation is derived :

$$J = - \oint_{\Gamma} U d\eta + \oint p w d\eta + \oint d\beta \left(\frac{\partial U}{\partial \xi} N_{\alpha} + \frac{\partial V}{\partial \xi} N_{\alpha\beta} + \frac{\partial \Theta v}{\partial \xi} M_{\alpha} + \frac{\partial \Theta v}{\partial \xi} M_{\alpha\beta} \right) - \\ - d\alpha \left(\frac{\partial V}{\partial \xi} N_{\beta} + \frac{\partial U}{\partial \xi} N_{\alpha\beta} + \frac{\partial \Theta v}{\partial \xi} M_{\beta} + \frac{\partial \Theta v}{\partial \xi} M_{\alpha\beta} \right)$$

where

U is strain energy density

p pressure

ξ, η are (in this case) a set of coordinates measured along and perpendicular to the crack

Θu represents rotation around u axis

such basic correlation is used, taking advantage of the correlations for the computations of the rotations.

The postprocessor JIASS evaluates the contour integral along Γ path on a numerical basis (i.e. the values on the Gauss points are recorded and the integral is evaluated by direct numerical integration).

7. General Organization

The program is organized as to deal with a substructure organization as follows :

start

Program SAS

- Assemble overall geometry
- Establish substructures geometry and nodes correlations

N = 1, Number of substructures

Program SAPEA

- Read substructure geometry as independent from others
- Read basic characteristics, assemble stiffness

For each step, For each substructure

Program SAPEA

- Compute displacements of substructure, stresses, strain
- Assemble stiffness matrix for substructure
- Store data for JIASS

Program SAS

- Assemble overall stiffness matrix
- Compute total displacements in the global structure

AT the end

Program JIASS

- Computation of J integral

8. Examples

8.1 Cracked pipe

An example of the use of PAULA-S code is given herein; the case of a cracked cylindrical shell is examined. The model considers half shell (for obvious symmetry considerations) in two rows of three elements each for a total of 64 nodes (23 active) and 138 dofs. The applied load is a thrust perpendicular to the circumferential crack; an increase of 10% of the nominal load (roughly corresponding to the first onset of non linearity) is applied at each step. The following parameters are considered :

- a) J integral (on two paths)
- b) equivalent stress (total)
- c) axial and bending stress

8.2 Mixed geometry

An example of the use non conventional geometry (a cylinder and a rotation hyperboloid) is given. Three strain hardening rules are used and compared.

References

- /1/ LAZZERI L. "On Shells Finite Elements Analysis" Finite Elements Models Conference, Shanghai, 1982.
- /2/ LAZZERI L. "PAULA 82: a code for Non Linear Analysis of Pipes and Shells" L6/7, 7 SMiRT

Conf, Chicago, 1983

- /3/ LAZZERI L. "Non Linear Analysis of Pipes and Shells" Res Mechanica, Vol.10/2, 1984
- /4/ VLASOV V.Z. "General Theory of Shells and its Applications in Engineering" Translated NASA, TN-64, 19833
- /5/ NOVOZHILOV "The Theory of thin Shells" Transl. from Russian
- /6/ ILIOUCHINE A. "Plasticité" Eyrolles, Paris
- /7/ LAZZERI L., CRECONI S., SCALA M. "Progresses in the Computation of Added Masses for Fluid Structure Interaction" B1/7, 8 SMIRT Conf., Brussels, 1985.

Fig.2 - Data for cracked pipe

Fig.3 - Data for mixed geometry