Microplane Model for Concrete Subject to Plane Stresses

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Abstract

The Microplane Model, originally introduced for the description of the progressive fracture of concrete and rock, proves in this paper to be suitable also to the description of the multiaxial behavior of concrete, subjected either to tension or to compression, with or without shear. The microcracking, which is the most relevant cause of non-linearity, is assumed to be localized in the thin mortar layers (cement paste and fine aggregates) which separate the coarse aggregate particles; these layers, called "microplanes" or "weak planes", are assumed to be present both at the microscopic and at the macroscopic level. Each microplane is characterized by a uniaxial constitutive relationship between the normal stress σ_n and the normal strain ϵ_n (the shear stiffness is disregarded). The function σ_n is formulated in such a way that -by superimposing the responses of the randomly oriented microplanes- the behavior of the concrete as a whole is adequately described (loading in compression, strain-softening excluded; loading in tension, strain-softening included; unloading either from previous tension or from previous compression). Attention is focused on plane stress situations. Comparisons are made with many test data on the biaxial behavior of concrete.

1. Introduction and Nature of the Problem

The non-linear behavior of concretes and geomaterials is characterized by strain-softening, both in tension and in compression, but tensile strain-softening is particularly important because the concrete subjected to tension can easily reach a situation characterized by a decrease of stress at increasing strain, due to the limited value of the strain corresponding to the peak stress in tension (of course, a strain-softening situation is stable on condition that the strains in the concrete are efficiently controlled by the boundary restraints or by the reinforcement).

Any analytical model for the description of concrete behavior should in principle introduce tensile strain-softening, but the classical approach (based on the theory of plasticity, with scalar loading functions in order to satisfy the invariance conditions, and with a "global" view of the material properties) becomes rather complex.

Alternatively, an approach based on a "local" view of the material properties can be adopted: the material can be considered as a system of randomly oriented planes (called "microplanes" or "weak planes" in /5-10/ Fig. 1), all characterized by a very limited number of constitutive relationships between the stresses and the strains acting on the planes themselves. This approach has several advantages: (a) a unique stress-strain relationship between the normal stress σ_n and the normal strain ϵ_n is sufficient in most cases; (b) the tensorial invariance restrictions can be satisfied by simply considering a suitable distribution of orientations for the microplanes (for instance, isotropy requires that each orientation must be equally frequent); (c) concrete path-dependency can be adequately described even if a simple total stress-total strain relationship is adopted for each microplane.

The microplane approach seems to agree with the very nature of concrete, which is basically a two-phase material, with its non-linear behavior mostly concentrated in the thin

mortar layers separating the coarse aggregate particles (Fig. 1). These layers, which act as contact planes, bond planes and microcracked planes, are present only at the microscopic level, and make the inelastic phenomena highly oriented; should all possible orientations be considered, the microplanes could be assumed to be present also at the macroscopic level (Fig. 2).

Once the inelastic behavior of each microplane is known, the inelastic behavior of the material as a whole can be evaluated by superimposition. This idea, which is in principle contained in Coulomb's concept of rupture planes, first appeared in Taylor's work /1/ on the plasticity of polycrystalline metals; subsequently, Batdorf and Budiansky /2/ introduced the concept of "slip planes" in the analysis of the plastic behavior of metals, while Calladine /3/ used a similar approach in the analysis of the behavior of saturated clay. As observed in /3/, the displacements along the different microplanes should not be independent; nevertheless, kinematic independence is a reasonable assumption for materials having a large number of voids, such as saturated clay and concrete. Recently, the same approach was used by Zienkiewicz et al. /4/, in the so-called "multilaminate model" for rocks.

With reference to concrete, the "microplane model" was first developed by Bažant and Oh /5,6,7/ for the description of progressive tensile fracturing of brittle aggregate materials. The microplane model was then shown to be capable of modeling crack shear within the crack band theory (Bažant and Gambarova /8/), and path-dependency of concrete subjected to tension and shear (Bažant and Gambarova /9/). Of course, the microplane model must also describe the material's elastic behavior, for small values of the stresses and of the strains (see /5,7/, and also Gambarova and Floris /10/).

In the present paper, the microplane model is shown to be capable of modeling biaxial concrete behavior in compression-compression, compression-tension, tension-tension, as well as in compression and shear.

2. Fundamental Assumptions - Material Stiffness Matrix

The microplane model is based on three fundamental assumptions, in its actual formulation; a fourth assumption refers here to the special case of a material subjected to a load process with either plane stresses or plane strains:

- a) the local strains (i.e. the strains on each microplane, also called microstrains) are the resolved components of the applied strains (macroscopic strain tensor):

$$\epsilon_n = n_i n_j \epsilon_{ij} \quad (1)$$

in which n_i, n_j = direction cosines of the normal to the microplane, and the repeated latin lower case subscripts indicate a summation over 1,2,3. The use of resolved strains reflects closely the microstructure of a brittle aggregate material, where the deformations are mostly concentrated in the matrix (cement paste) and tend to coincide with the relative displacements of the hard inclusions (coarse aggregate particles); these displacements correspond to the macroscopic strains.

- b) the shear stiffness of the microplanes is neglected; this assumption is correct for concrete with many thin cracks (the crack opening tends to prevail over the crack slip, and the shear stiffness due to aggregate interlock tends to be zero) and for concrete with open cracks and large slip values /11/; this assumption is also remarkable for its intrinsic simplicity, but can be dropped /7/ if a more general approach is needed.
- c) only the normal stiffness of the microplanes is introduced, and the possible coupling between the normal microstress σ_n and the shear strains is disregarded (this coupling tends to become negligible in the same situations discussed in point b). Both the elastic and inelastic behavior of the concrete is described by assuming that the normal microstress σ_n acting on the microplane of any orientation is a function of the normal microstrain ϵ_n on the same plane:

$$\sigma_n = F(\epsilon_n) \epsilon_n \quad (2)$$

- d) for the description of plane stress or plane strain situations (as in this paper), only the microplanes at right angles to the reference plane are considered (see Fig. 2): as a result, the model may be too stiff in certain situations characterised by concrete deterioration along the slanting microplanes (with respect to the reference plane), but

in the end this limitation is more than balanced by the computational simplifications which are a consequence of the assumption itself.

In order to work out the material tangential stiffness matrix, it is necessary to write the equilibrium conditions between the macrostresses σ_{ij} and the microstresses σ_n ; these conditions may be written by means of the virtual work principle:

$$\delta w = \pi \sigma_{ij} \delta \epsilon_{ij} = 2 \int_0^\pi \sigma_n \delta \epsilon_n d\phi \quad (3)$$

where the virtual work δw refers to a unit-radius cylinder, having unit thickness, while $\delta \epsilon_{ij}$ and $\delta \epsilon_n$ represent a system of virtual strains ($\delta \epsilon_n = n_i n_j \delta \epsilon_{ij}$).

From Eq. 3:

$$\sigma_{ij} = \frac{2}{\pi} \int_0^\pi \sigma_n n_i n_j \delta \epsilon_{ij} d\phi \quad (4)$$

Furthermore, according to Eq. 1, $dF(\epsilon_n) = (dF/d\epsilon_n) d\epsilon_n = (dF/d\epsilon_n) n_k n_m d\epsilon_{km}$, and thus differentiation of Eq. 4 finally yields:

$$d\sigma_{ij} = D_{ijkm}^* d\epsilon_{km} \quad (5)$$

in which $D_{ijkm}^* = \frac{2}{\pi} \int_0^\pi a_{ijkm} (dF/d\epsilon_n) d\phi$, with $a_{ijkm} = n_i n_j n_k n_m$

(the suffix * stands for "microplane system")

The coefficients D_{ijkm}^* are the tangent stiffnesses of the material considered as a microplane system.

It can be easily demonstrated /10/ that the microplane model can describe the linear elastic behavior of any material, that has Young's modulus E and Poisson ratio $\nu^* = 0.33$ (plane stresses) or $\nu^{**} = 0.25$ (plane strains and triaxial behavior), on condition that $F(\epsilon_n)$ is made equal to E_n , with $E_n = 1.5 E$ or $2 E$ respectively. In order to achieve any Poisson ratio ν , additional elastic (volumetric) strains or (hydrostatic) stresses must be added. As for plane stresses, the bulk modulus of the additional volumetric strains must have the following value: $k^* = E^*(1+\nu)/9(\nu^*-\nu)$; under these circumstances, the Young's modulus of the material is achieved if E^* is kept equal to $E(1+\nu^*)/(1+\nu)$, and E_n is kept equal to $1.5 E^*$.

Summing up, for an elastic material having a Young's modulus E and a Poisson ratio ν , subjected to plane stresses, the tangent stiffness matrix $\underline{D}$ has the following expression:

$$\underline{D} = \left[\underline{D}^{*-1} + \underline{C}^a \right]^{-1}$$

where $\underline{C}^a$ = additional compliance matrix = $\frac{\nu^* - \nu}{E(1+\nu^*)} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$\text{Eq. 5 becomes: } d\sigma_{ij} = D_{ijkm} d\epsilon_{km} \quad (5')$$

In order to obtain a concrete response σ_{ij} for a given strain history ϵ_{km} , a computer program has been set up. The integrals appearing in the coefficients D_{ijkm}^* are evaluated numerically: they are approximated by a finite sum, regarding 24 microplanes for a unit semicircle /10/.

The theoretical predictions of Sect. 3 were obtained with the above-mentioned program: the experimental strain histories were introduced as input data.

3. Modeling of Concrete Behavior

For modeling concrete behavior under increasing loads, it suffices to specify the stress-strain relation $\sigma_n(\epsilon_n)$ for loading in tension, loading in compression, unloading from tension and from compression, reloading in tension and in compression. In Fig. 3 the various branches of the stress-strain curve here adopted are shown.

Should the stress-strain curve $\sigma_n(\epsilon_n)$ be of the elastic-plastic type with strain hardening, the microplane model would be similar to the elastic-plastic models with many-faceted

failure surfaces (actually in the microplane model there are as many failure surfaces as there are orientations of the microplanes). The introduction of strain-softening (here limited to tension, as in /8,9,10/) excludes the microplane model from the classical theory of plasticity.

For modeling elastic and inelastic behavior in tension, strain softening included, it is necessary to adopt (for each microplane) a stress-strain curve suitable to the description of cracking up to complete fracture, at which σ_n reduces to zero: it follows that σ_n (as a function of ϵ_n) must first rise, then reach a maximum and then gradually decline to zero, or to a finite asymptote close to zero, as shown in the very recent tests by Reinhardt /12/. The following law has been adopted in this paper:

$$\sigma_n = E_n \epsilon_n \exp \left[- (a_1 \epsilon_n + b_1 \epsilon_n^2) \right] \quad (6)$$

which reduces to a linear-elastic relation for small strain values ($\epsilon \rightarrow 0$).

Eq. 6 is plotted in Fig. 4a for three different concrete strengths; the resultant stress-strain curves for the concrete as a whole are shown in Fig. 4b (uniaxial tension). Using Eq. 6, the fitting of the test data by Evans and Marathe /13/ (Fig. 5) in uniaxial tension, and by Kupfer et al. /14/ (Fig. 6) in biaxial tension is quite remarkable.

For modeling concrete behavior in compression, a simple curve with a horizontal plateau was adopted for the microplanes, as in /8,9,10/: strain-softening in compression was disregarded.

The equation of the $\sigma_n(\epsilon_n)$ law in compression coincides with Eq. 7, on condition that σ^* and ϵ^* are made equal to zero.

The asymptote σ_∞ (Fig. 3a) is related to concrete compressive cylindrical strength f'_c by the relation shown in the caption of Fig. 8.

The fitting of Kupfer et al. test data is very good for σ_1 , less good (and fairly sensitive to Poisson ratio ν) for σ_2 , both for biaxial compression (Fig. 7c), and for tension-compression (Fig. 7b). The agreement is equally good for low strength and high strength concretes ($f'_c = 18-60$ MPa). One has to observe that under monotonically increasing compression ($\sigma_2 < 0$) and tension ($\sigma_1 > 0$), some microplanes are always in compression and some others are always in tension, so that no unloading or reloading occurs, and no stress-strain laws for unloading and reloading are necessary.

For modeling concrete behavior when shear is applied, suitable laws for unloading after previous tension and after previous compression must be introduced: as a matter of fact, due to shear, some microplanes will unload after tension and will possibly become loaded in compression, some others will unload after compression and will possibly become loaded in tension.

Since direct observations of unloading after tensile strain-softening are very limited (see, for instance, /12/), the law for unloading after tension must be based mostly on speculation although a check is possible by comparison with the considerable quantity of test data regarding crack shear (aggregate interlock tests /8,10/).

With reference to a uniaxial test, at the onset of unloading after tensile strain-softening, concrete stiffness is low because the microcracks are open, then gradually increases because the microcracks tend to close-up, but further on the stiffness decreases again due to the formation of new microcracks parallel to the compressive stress. The limiting compressive stress is of the same order of magnitude as the standard cylindrical compressive strength. The same behavior may be assumed for each microplane. Based on these considerations, the stress-strain law qualitatively plotted in Fig. 3a,b was adopted /8,10/. The equation is as follows:

$$\sigma_n = -a_2 + \beta \arctan \left[(E'_n / \beta) \cdot (\epsilon_n - b_2) \right] \quad (7)$$

where: $a_2 = -(\sigma_\infty + \beta\pi/2)$, $b_2 = \epsilon^* - (\beta/E'_n) \tan \left[(\sigma^* + a_2) / \beta \right]$

σ^*, ϵ^* are the coordinates of the reversal point at which unloading begins

$E'_n, \beta, \sigma_\infty$ are three additional material constants /10/
(see the plots in Fig. 8a,b,c).

For unloading after previous compression, a simple asymptotic curve has so far been adopted (Fig. 3b):

$$\sigma_n = \sigma^{**} \exp \left[\frac{E''_n}{\sigma^{**}} (\epsilon_n - \epsilon^{**}) \right] \quad (8)$$

where $\sigma^{**}, \epsilon^{**}$ define the point at which unloading from previous compression, or reloading in tension begins.

Eq. 8 has not been checked against available test data.

The microplane model, implemented with the laws represented by Eqs. 6,7,8 was used for predicting concrete shear response in two cases experimentally studied by Fouré /15/(Fig.9): in the first test, loading is sequential (uniaxial compression, followed by shear at constant compressive stress, Case 1) ; in the second test, loading is proportional (compression and shear, at constant shear-to-compression ratio, Case 2) . Microplane model prediction is quite satisfactory. Since the strains are not given in /15/, the strain histories (used as input data in the microplane approach) were calibrated so as to keep the compressive stress (sequential loading) and the shear-to-compression ratio (proportional loading) as constant as possible.

Concluding Remarks

Concrete is modeled as a system of planes (microplanes) characterized by a uniaxial non-linear constitutive law, which relates the normal stress and the normal strain acting on each microplane: from the virtual work principle, an explicit formula for the coefficients of the material stiffness matrix can be obtained, in the form of an integral over a unit semicircle (plane stresses and plane strains).

Assuming the microstrain acting on each microplane to be equal to the macrostrain acting on the same plane, concrete non-linear behavior can be modeled in a stable manner, including strain softening and cracking.

The microplane model is globally path-dependent, even if the uniaxial stress-strain law of the microplanes is given a total stress-total strain, path-independent formulation, as here and in previous papers on the subject.

For monotonic loading, the number of parameters required by the uniaxial stress-strain curves (loading in tension and in compression, unloading, reloading) is very limited, yet the agreement between the theoretical predictions and the test data is good (and in many cases very good), with reference to a variety of biaxial stress paths, such as biaxial tension, biaxial compression, tension and compression, compression and shear, pure tension and pure compression.

Particularly important features of the model are: first that it applies to load histories in which the principal stress directions rotate, and second, that it is suitable to the description of concrete behavior even if the directions of the principal stresses and those of the principal strains no longer coincide.

Acknowledgements

Financial support from grants provided by the Italian Ministry for Higher Education (M.P.I.) in the years 1983-1984 is gratefully acknowledged. Thanks are due to Prof. Zdenek P. Bazant of Northwestern University for his valuable suggestions.

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Fig.1 - Sketch of the microplanes.

Fig.2 - Idealization of the material as a microplane system.

Fig.7 - Fits of Kupfer's test results /14/: (a) uniaxial compression; (b) compression-tension; (c), (d) biaxial compression.

Fig.8 - Assumed elementary material properties.

Fig.9 - Fits of Fouré's test results /15/: compression and shear; $f'_c = 27$ Mpa; $E_c = 31860$ Mpa.