Simplified Creep Analysis of Perforated Plates Under Steady Creep Condition

T. Igari, K. Setoguchi

Nagasaki Technical Institute

S. Nomura

Nagasaki Shipyard & Engine Works

Mitsubishi Heavy Industries, Ltd., 1-1, Akunoura-Machi, Nagasaki 850-91, Japan

Abstract

A study was performed on the creep analysis method of perforated plates based on the equivalent solid plate concept.

A series of creep analysis by FEM for various materials, ligament efficiencies and loading conditions was performed on the perforated plate with triangular penetration pattern of circular holes. From the results of the analysis, prediction methods for both the equivalent solid plate property and the stress-strain concentration around the hole were proposed. The prediction method for the equivalent solid plate property had the compatibility with the elastic property shown in ASME B & PV Code Sec. III A-8000. The prediction method for the stress-strain concentration was basically based on the Neuber's rule together with the Hoff's analogy.

1. Introduction

In the design of high temperature equipments containing perforated plates, such as the intermediate heat exchanger for the high temperature gas cooled reactor, creep analysis of perforated plates is required. The simplified method based on the equivalent solid plate concept [1] is usually used for the stress analysis of perforated plates, because of the complexity of 3-dimensional analysis. In such cases, results of the analysis are controlled by the followings;

- A. Prediction of the equivalent solid plate property
- B. Prediction of the stress-strain concentration around the hole

For the elastic analysis, the above two items are presented in ASME B & PV Code A-8000 [2]. For the creep analysis, on the other hand, the proposal for A and B by Uragami et al. [3] and that for B by O'Donnell et al. [4] have been made. However, more widely applicable methods for various ligament efficiencies and materials are required.

This paper presents the more widely applicable prediction methods for both the equivalent solid plate property and the stress-strain concentration, based on the results of a series of elastic-creep analysis by FEM.

2. Creep Analysis of Perforated Plates by FEM

Perforated plates subjected to three kinds of constant in-plane load shown in Fig. 1 were considered. Three kinds of ligament efficiency η ($=h/p$), 0.3, 0.5 and 0.7, were chosen. Plane stress condition was assumed and Norton's creep law in the following equation was used

$$\dot{\epsilon}_c = A\sigma^n \quad (1)$$

where $\dot{\epsilon}_c$, σ , A and n are a creep strain rate, a stress and material constants respectively. Four kinds of material shown in Table 1 were used for the analysis.

An example of finite element model considering a regularity of configuration of the perforated plate is presented in Fig. 2. The analysis was made by MARC, element being a 2-D isoparametric one with 8 nodes. The boundary conditions were the same as those by Uragami et al. [3].

The creep analysis was performed to obtain the redistribution from the initial elastic state to the steady-state creep condition. From the results at steady-state condition, creep behavior of the equivalent solid plate and stress-strain concentration around the hole were obtained.

Examples of analytical results for the change of $\dot{\epsilon}_c^*$ and $\sigma\phi_{\max}$ with time are shown in Fig. 3, where $[*]$ and $[\phi_{\max}]$ denote the equivalent solid plate property and the maximum circumferential component respectively. Fig. 4 shows an example of creep behavior of the equivalent solid plate for various ligament efficiencies and loading conditions. The relation between the equivalent stress $\bar{\sigma}^*$ and the equivalent creep strain rate $\dot{\epsilon}_c^*$ is expressed as the group of parallel linear lines in log-log diagram, and the slope of the lines is equal to n which is the creep constant of the base material. Fig. 5 shows an example of the stress-strain distribution around the hole at the initial and the steady-state condition.

3. Prediction of the Equivalent Solid Plate Property

According to the method by Uragami et al. [3], the creep strain rate of the equivalent solid plate was assumed to be the following equation

$$\dot{\epsilon}_c^* = A^* \sigma^{*n^*} \quad (2)$$

where A^* and n^* were given by next equations

$$A^* = A^*(\eta, A, n) = f(n)\eta^{-mn}A \quad (3)$$

$$n^* = n \quad (4)$$

In eq.(3), Uragami et al. assumed that $f(n)$ was determined as a linear function of n for $n=5.6 \sim 9.0$, m being a constant. However, applicability of eq.(3) for various materials and temperatures strongly depends on the determination of $f(n)$.

In this paper, an improved form of $f(n)$ was proposed to extend the applicability of eq.(3), considering the Hoff's analogy [5] between a linear viscoelastic and a linear elastic materials for the case of $n=1$. Fig. 6 shows the relation between $f(n)$ and $1/n$, where $f(n)$ for $n=1$ was determined from the following relation obtained from eqs. (1) and (2) by setting $n=n^*=1$ and neglecting the elastic strain.

$$\frac{A^*}{A} = \frac{E}{E^*} \quad (5)$$

The constant m in eq.(3) was determined as [1.18] using the results for $n=4.45$, and E/E^* was determined from the data in ASME B & PV Code Sec. III A-8000.

Linearly approximated relation between $f(n)$ and $1/n$ as shown in eq.(6) was obtained by tying two points for $n=1$ and $n=4.45$, using average values of $\eta=0.3, 0.5, 0.7$ for $n=1$ and $\eta=0.5, 0.7$ of x and y load for $n=4.45$ respectively.

$$f(n) = \frac{0.591}{n} + 0.316 \quad (6)$$

Therefore, eq.(3) is expressed in the following form finally.

$$\frac{A^*}{A} = \left(\frac{0.591}{n} + 0.316 \right) \eta^{-1.18n} \quad (7)$$

Predicted results by the present and Uragami's methods are shown in Fig. 7, comparing them with FEM results. Predicted values by the present method coincide well with those by Uragami's method and results by FEM. The present method is considered to be a practical one which extends Uragami's method to wider range of the material constant n and has the compatibility with the elastic case for $n=1$.

4. Prediction of the Stress-Strain Concentration

Uragami et al. [3] proposed a prediction method for the stress-strain concentration in creep region, based on the elastic strain concentration, but applicable range for materials and temperatures is restricted. On the other hand, O'Donnell et al. [4] proposed a more general method. But this method requires elastic ($n=1$) and rigid plastic ($n=\infty$) solutions.

In this paper, a new approach fundamentally based on the Neuber's rule [6] was proposed. The perforated plate and the equivalent solid plate subjected to the same load were considered as shown in Fig. 8. Stress-strain relations for these two plates can be presented by the following equations, neglecting the elastic strain.

$$\dot{\epsilon} = A\sigma^n \quad (\text{Base material}) \quad (8)$$

$$\dot{\epsilon}^* = A^*\sigma^{*n} \quad (\text{Equivalent solid plate}) \quad (9)$$

The Neuber's rule presented by eq.(11) was assumed for the perforated plate made of the material of eq.(10) where $[\cdot]$ is neglected from eq.(8).

$$\epsilon = A\sigma^n \quad (10)$$

$$\sigma_{\phi\max} \epsilon_{\phi\max} = k_t^2 \sigma_{nom} \epsilon_{nom} \quad (11)$$

In eq.(11), ϵ_{nom} is an average strain of the perforated plate, and σ_{nom} is a stress related with ϵ_{nom} by eq.(10). $\sigma_{\phi\max}$ and $\epsilon_{\phi\max}$ also satisfy eq.(10). Elastic stress concentration factor k_t is expressed*1 as the following equation

$$k_t = \left(\frac{\sigma_{\phi\max}}{\sigma_{nom}} \right)_{n=1} = \frac{E^* y_{\max}}{E \eta} \quad (12)$$

1 Eq.(12) can be derived based on the following relations: $\epsilon_{nom} = \epsilon^$,
 $\sigma_{nom} = E\epsilon_{nom}$ (Perforated plate), $\sigma^* = E^*\epsilon^*$ (Equivalent solid plate),
 $\sigma_{\phi\max} = y_{\max} \sigma^*/\eta$ (Definition in A-8000)

where E^*/E and $Y_{\max}$ can be obtained from the data in A-8000.

The Neuber's rule for the material of eq.(8) can be written in the next form considering the Hoff's analogy

$$\sigma_{\phi_{\max}} \dot{\epsilon}_{\phi_{\max}} = k_t^2 \sigma_{\text{nom}} \dot{\epsilon}_{\text{nom}} \quad (13)$$

where the pairs of $\sigma_{\phi_{\max}} - \dot{\epsilon}_{\phi_{\max}}$ and $\sigma_{\text{nom}} - \dot{\epsilon}_{\text{nom}}$ satisfy eq.(8) respectively.

By the definition of the equivalent solid plate, next equation can be derived,

$$\dot{\epsilon}_{\text{nom}} = \dot{\epsilon}^* \quad (14)$$

On the other hand, σ_{nom} is related with σ^* in the next equation by eqs.(8), (9) and (14)

$$\sigma_{\text{nom}} = \left(\frac{A^*}{A} \right)^{\frac{1}{n}} \sigma^* \quad (15)$$

Stress concentration around the hole in creep region can be obtained as follows by eqs.(8), (9), (13), (14) and (15)

$$\frac{\sigma_{\phi_{\max}}}{\sigma^*/\eta} = \eta k_t^{\frac{2}{n+1}} \left(\frac{A^*}{A} \right)^{\frac{1}{n}} \quad (16)$$

where A^*/A can be estimated by eq.(7). Eq.(16) coincides with the definition of $Y_{\max}$ in A-8000 considering eq.(5) in the case of $n=1$.

Creep strain concentration around the hole can be expressed as follows using eqs.(8), (9) and (16)

$$\frac{\dot{\epsilon}_{\phi_{\max}}}{\dot{\epsilon}^*} = k_t^{\frac{2n}{n+1}} \quad (17)$$

The maximum circumferential creep strain $\epsilon_{\phi_{\max}}$ can be obtained by integration of eq.(17). Predicted values by eqs.(16) and (17) were found to coincide pretty well with FEM results, showing the reducing tendency of the stress concentration and the increasing tendency of the strain concentration respectively for the increase of n . However, some discrepancy was found especially for the strain concentration, resulting in a lower prediction for $\eta=0.7$ and a higher prediction for $\eta=0.3$.

Therefore, modification factor C was introduced to eq.(11) as shown in the following equation in order to improve the prediction method.

$$\sigma_{\phi_{\max}} \dot{\epsilon}_{\phi_{\max}} = (Ck_t)^2 \sigma_{\text{nom}} \dot{\epsilon}_{\text{nom}} \quad (18)$$

where C was assumed to be function of η and n as follows.

$$C = C(\eta, n) \quad (19)$$

Then eqs.(16) and (17) become as followings.

$$\frac{\sigma_{\phi_{\max}}}{\sigma^*/\eta} = \eta (Ck_t)^{\frac{2}{n+1}} \left(\frac{A^*}{A} \right)^{\frac{1}{n}} \quad (20)$$

$$\frac{\dot{\epsilon}_{\phi_{\max}}}{\dot{\epsilon}^*} = (Ck_t)^{\frac{2n}{n+1}} \quad (21)$$

Values of C were determined by the next equation, making the predicted strain concentration equal to the FEM results

$$C = \frac{\left[\left(\frac{\epsilon_{\phi_{\max}}}{\epsilon^*} \right)_{\text{FEM}} \right]^{\frac{n+1}{2n}}}{k_t} \quad (22)$$

where $(\epsilon_{\phi\text{cmax}}/\epsilon^*)_{\text{FEM}}$ is the value at the steady state obtained by FEM, and it was regarded as equivalent to $\dot{\epsilon}_{\phi\text{max}}/\dot{\epsilon}^*$ in eq.(21). Obtained results for C by eq.(22) is shown in Fig. 9, and a predictive equation (23) for C was determined from the figure.

$$C = (-1.359 \log_{10} \eta - 0.551) \left(\frac{1}{n} - 1 \right) + 1 \quad (23)$$

Stress-strain concentration predicted by eqs.(20) and (21) together with eq.(23) is shown in Fig. 10 and Fig. 11 respectively in comparison with predicted results by Uragami's method and FEM results. Predicted values by the present method coincide well with FEM results, showing a good agreement with prediction by Uragami's method for the case of $\eta = 0.3$ which they studied mainly. Results of comparison of prediction for the stress concentration by the present and O'Donnell's method are shown in Fig. 12 for the case of equibiaxial loading ($\sigma_x^* = \sigma_y^*$, $\eta = 0.5$). A good agreement is obtained between them. The present method has the feature to make a prediction based on the Neuber's rule together with the data in A-8000 for $n=1$ without additional FEM analyses.

5. Conclusion

Prediction methods for both the equivalent solid plate property and the stress-strain concentration around the hole were proposed in the creep analysis of perforated plates based on the equivalent solid plate concept.

For the equivalent solid plate property, an extension of Uragami's method was made and a predictive equation was proposed which had a compatibility with the elastic data in ASME B & PV Code Sec. III A-8000 for the special case of $n=1$.

For the stress-strain concentration, a prediction method basically based on the Neuber's rule was proposed, which had a compatibility with the elastic data in A-8000 for the special case of $n=1$.

References

- [1] O'Donnell, W. J. and Porowski, J. S., "Design Analysis of Perforated Plates and Shells", Pressure Vessels and Piping Design Technology -1982- A Decade of Progress, Ed. Zamrik, S. Y. and Dietrich, D., 359-373 (1982) ASME.
- [2] ASME B & PV Code Sec. III, Division 1, Appendix A (1980) ASME.
- [3] Uragami, K., Nakamura, K., Asada, K. and Kano, T., "Simplified Inelastic Analysis Method of Ligament Plates", J. High Pressure Institute (in Japanese), 19, 57-65 (1981).
- [4] Porowski, J. S., O'Donnell, W. J., Tanaka, T. and Badlani, M., "Stress and Strain Concentrations in Perforated Structures Under Steady Creep Conditions", J. Pressure Vessel Technology, 102, 419-429 (1980).
- [5] Odqvist, F. K. G. and Hult, J., "KRIECHFESTIGKEIT METALLISCHER WERKSTOFFE", (1962) Springer-Verlag.
- [6] Neuber, H., "Theory of Stress Concentration for shear-Strained Prismatical Bodies with Arbitrary Nonlinear Stress-Strain law", Trans. ASME Ser. E, 28, 544-550 (1961).

Table 1 Material Properties Used for Analysis

Material No.	Elastic		Creep		Note
	E (MPa)	ν	A $\left(\frac{1}{\text{hr}} \right)$ $(\text{MPa})^n$	n	
I	1.329×10^5	0.3	1.678×10^{-10}	4.45	Hastelloy X, 930°C, $\sigma \geq 9.8 \text{ MPa}$
II	1.329×10^5	0.3	1.539×10^{-8}	2.47	Hastelloy X, 930°C, $\sigma < 9.8 \text{ MPa}$
III	1.329×10^5	0.3	1.201×10^{-18}	6.13	Hastelloy X, 650°C (Only for creep)
IV	1.546×10^5	0.3	7.796×10^{-26}	9.00	SUS 304, 550°C

Fig. 1 Loading Conditions

Fig. 2 An Example of Finite Element Model

Fig. 3 An Example of Time History of Creep Strain Rate of the Equivalent Solid Plate and the Maximum Circumferential Stress around the Hole

Fig. 4 Creep Behavior of the Equivalent Solid Plate (Material I)

Fig. 5 An Example of the Stress-Strain Distribution around the Hole

Fig. 6 Relation between \$1/n\$ and \$f(n)\$

Fig. 7 Prediction of the Equivalent Solid Plate Property

Fig. 8 Definition of the Equivalent Solid Plate

Fig. 9 Modification Factor C for the Neuber's Rule

Fig. 10 Stress Concentration around the Hole

Fig. 11 Creep Strain Concentration around the Hole

Fig. 12 Comparison with O'Donnell's Results for Stress Concentration