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On the consideration of phenomenological uncertainty

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Abstract: An approach to evaluate the influence of phenomenological and human error uncertainties in reliability assessments is proposed. A simple scheme is developed to update initial estimates of the proneness to failure as the time during which the system operates increases. Conclusions are drawn with respect to the significance of very high reliability estimates.

1 Introduction

The reliability assessment of complex systems, structures or materials requires, as a basic step, the identification of the relevant failure modes. Thus, the final result is generally a statement about the proneness to failure, which is *conditional* on the assumption that all significant failure modes have been duly considered. Nevertheless, there is always a nonzero likelihood that physical phenomena leading to pertinent failure modes be unknown at the time the system enters into operation. This is usually designated as *phenomenological uncertainty* and becomes particularly relevant in connection with new or developing technologies, or when established or tested technologies are applied in a different scale or environment. Similarly, there is also a nonzero likelihood that pertinent failure modes be neglected due to lack of experience, competence or insight of the analyst, or simply as a consequence of human error, even when sufficient knowledge is available at the time. This likelihood, denoted as *human error uncertainty*, also ought to be considered in the reliability assessment.

It is clear then that in any attempt to produce an assessment of the total proneness to failure of a system it will be necessary to deal, in a quantitative manner, with both phenomenological and human error uncertainties, in order to produce results convincing to the general public or useful in a consistent decision making process. In this work, an approach to assess both likelihoods in order to have priors for the evaluation of the total proneness to failure is discussed. Next, based on the assumption that the *time-to-first-failure* (tff) in all failure modes is Weibull distributed with common shape parameters, which is introduced only for the sake of analytical convenience within a parametric study, a simple scheme is developed to update the initial estimates of the proneness to failure as the time during which the system operates increases. A number of conclusions are drawn, such as (1) the lack of significance in typical applications of computational efforts to evaluate very high reliabilities, i.e. very low proneness to failure and (2) the importance of service records to update initial estimates in connection with new technologies.

2 Consideration of an unknown failure mode

Let us admit that the system under consideration is subjected to a reliability analysis, which leads to the identification of N relevant modes of failure. It will be further assumed that the times-to-first-failure, denoted by T_i , in all modes $i = 1, 2, \dots, N$, are Weibull distributed random variables, with scale and shape parameters α_i and β_i , respectively. Hence, in mode i :

$$F_i(t) = 1 - \exp \left[- (t/\alpha_i)^{\beta_i} \right], \quad t > 0 \quad (1)$$

The time-to-first-failure (tff) of the system is of course:

$$T = \min (T_1, T_2, \dots, T_N) \quad (2)$$

In order to obtain simple expressions that may serve as guidelines in more general situations, it will be further assumed that the shape parameter is common in all failure modes, i.e.:

$$\beta_i = \beta, \quad i = 1, 2, \dots, N \quad (3)$$

In such case, the total proneness to failure ¹ of the system is:

$$F(t) = 1 - \exp \left[- (t/\alpha)^\beta \right], \quad t > 0 \quad (4)$$

in which:

$$(1/\alpha)^\beta = \sum_{i=1}^N (1/\alpha_i)^\beta \quad (5)$$

just as it follows from the properties of the Weibull distribution. It will be admitted now that an unknown failure mode exists, say mode $N+1$. In this case, the correct distribution of the tff of the system, given that also $\beta_{N+1} = \beta$, would then be given by:

$$F^*(t) = 1 - \exp \left[- (t/\alpha^*)^\beta \right], \quad t > 0 \quad (6)$$

in which:

$$(1/\alpha^*)^\beta = (1/\alpha)^\beta + (1/\alpha_{N+1})^\beta \quad (7)$$

In general, however detailed the reliability assessment of any complex system, unknown failure modes will exist. Of these, only failure modes with scale parameters α_i smaller than, say, 3α will be relevant, i.e. will affect the tff predictions of the system. Thus, the proneness to failure if an unknown failure mode exists would be $F^*(t)$; otherwise, it would be just $F(t)$. The proposal here is to *complete the reliability analysis* by first assessing the likelihood that an unknown failure mode $N+1$ exists, updating later the reliability estimate using operating experience and Bayes' Theorem.

The consideration of unknown failure modes is possible if expert opinion on the risk associated with the development and application of new technologies is available. Based on the degree of innovation or on the period during which the innovative aspects have been tested, experts may turn in estimates of the following two quantities:

¹In this paper, the uncertainty about an event that has already taken place is designated as *likelihood*, while uncertainty about a future event is denoted as *proneness*. *Relative frequencies* apply always to past events, while *probability* is reserved to a function associated to events in the sample space, as defined in the axiomatic Theory of Probability. By convention, likelihood and proneness take values between zero and one and, as measures of uncertainty, are equivalent. The relative frequency must obey the laws of the theory of probability. On the other hand, likelihood and proneness may be chosen to, but not necessarily conform to the rules of the Theory of Probability.

1. The likelihood $\mathcal{L}_{N+1}$ that an unknown failure mode $N + 1$ does exist and
2. The shape parameter α_{N+1} of the assumed Weibull distribution, for the tff due only to this unknown mode.

It will be seen in the following sections that the shape parameter α_{N+1} has not necessarily to be assessed. It is also possible to adopt a worst case approach in the analysis and derive most important conclusions.

3 The likelihood of an unknown failure mode

It should be emphasized again that, if it exists, the unknown failure mode may be due to phenomenological uncertainty or to human error. In either case, the likelihood that mode $N + 1$ exists may be evaluated on the basis of expert opinion, on the relative frequency of failure of similar systems in modes previously unknown or accounted for, or on the combination of both. The first approach is discussed, for instance, by Singpurwalla and Song (1988), although with a somewhat different objective in mind.

Recognizing that a large number of structural failures have been caused by human errors committed during the design stage, Engelund and Rackwitz (1993) address the problem of quality assurance systems in design. In the process, relevant concepts related to the evaluation of the likelihood $\mathcal{L}_{N+1}$ due to human error are discussed. Stewart (1993) proposes employing the Human Reliability Analysis (HRA) to model the effect of human error in risk analysis, applying the technique to assess the reliability of a specific concrete construction. Note that, in the present context, human error in construction or operation might lead to a mode of failure $N + 1$ in the original reliability assessment, but should be dealt separately from design errors.

The likelihood $(1 - \mathcal{L}_{N+1})$ represents, in fact, the degree of confidence on the results of the reliability assessment and indirectly on those responsible for the assessment. It must be observed that the likelihood of an human error in the reliability analysis of a system may be as low as 10^{-5} or 10^{-6} . On the contrary, on the basis of past experience, it is judged that it is difficult to admit a value of $\mathcal{L}_{N+1}$ lower than 10^{-2} , associated with phenomenological uncertainty, whenever the analysis applies to any new material, device or technology.²

4 Application of Bayes' theorem

Now, let A denote the event "failure does not occur during an operating time t ", while E_1 and E_2 denote the events "failure mode $N + 1$ does exist" and "failure mode $N + 1$ does not exist", respectively. Clearly, $E_1 \cup E_2 = \Omega$, i.e. they are complementary events since they span the whole sample space. Let us further assume that an estimate of $\text{Prob}[E_1] = \mathcal{L}_{N+1}$, herein called "the likelihood that failure mode $N + 1$, characterized by α_{N+1} , does exist", can be obtained.

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²Witness the first applications of welding in the naval industry and the first applications of welding to high strength steels, leading to fracture under low temperatures, to vibration induced crack growth with subsequent fracture in new commercial aircraft, to steel corrosion caused by concrete additives, failures in the US and USSR space programs, and many others.

Then, according to Bayes' Theorem:

$$\text{Prob}[E_1|A] = \frac{\text{Prob}[A|E_1] \text{Prob}[E_1]}{\text{Prob}[A|E_1] \text{Prob}[E_1] + \text{Prob}[A|E_2] \text{Prob}[E_2]} \quad (8)$$

where:

$$\text{Prob}[A|E_1] = 1 - F^*(t) = R^*(t) \quad (9)$$

$$\text{Prob}[A|E_2] = 1 - F(t) = R(t) \quad (10)$$

$$\text{Prob}[E_1] = \mathcal{L}_{N+1} \quad (11)$$

$$\text{Prob}[E_2] = 1 - \mathcal{L}_{N+1} \quad (12)$$

so that:

$$\text{Prob}[E_1|A] = \frac{\mathcal{L}_{N+1}}{\mathcal{L}_{N+1} + (1 - \mathcal{L}_{N+1})R(t)/R^*(t)} \quad (13)$$

The updated estimate of $\mathcal{L}_{N+1}$, or the likelihood, after an operating time t , that mode $N + 1$ does exist is denoted $\mathcal{L}'_{N+1}(t)$:

$$\mathcal{L}'_{N+1}(t) = \text{Prob}[E_1|A] \quad (14)$$

Replacing eqs. 4 and 6 in eq. 13 yields:

$$\mathcal{L}'_{N+1}(t) = \frac{\mathcal{L}_{N+1}}{\mathcal{L}_{N+1} + (1 - \mathcal{L}_{N+1}) \exp \left[+ (t/\alpha_{N+1})^\beta \right]} \quad (15)$$

which means that $\mathcal{L}'_{N+1}(t)$ does not depend on the scale factors α_i but only on α_{N+1} .

It may be easily verified that:

$$\lim_{t \rightarrow 0} \mathcal{L}'_{N+1}(t) = \mathcal{L}_{N+1} \quad (16)$$

$$\lim_{t \rightarrow \infty} \mathcal{L}'_{N+1}(t) = 0 \quad (17)$$

i.e. initially, the $\mathcal{L}_{N+1}$ prior continues to be the best estimate of $\mathcal{L}'_{N+1}(t)$, while if time grows with no failure the likelihood of E_1 decreases monotonically up to zero.

Once $\mathcal{L}'_{N+1}(t)$ is calculated for time t , the best failure proneness estimate, $F'(t)$, is then given by:

$$F'(t) = [1 - \mathcal{L}'_{N+1}(t)] F(t) + \mathcal{L}'_{N+1} F^*(t) \quad (18)$$

according to the theorem of total probability. The implications of eq. 18 will be analysed in the following sections.

5 Extension to many operating units

It is commonly accepted that the reliability of new technologies does increase if no failure is observed in M similar units, rather than in a single one, during a certain time t of simultaneous operation. We should now extend eq. 15 to account also for this information, namely the number of operating units.

Let B denote the event "failure does not occur in any of M similar systems during a simultaneous operating time t ". Events E_1 and E_2 are defined as in the previous section. Eq. 8 still applies by replacing the event A by B and:

$$\text{Prob}[B|E_1] = 1 - F_M^*(t) = R_M^*(t) \quad (19)$$

$$\text{Prob}[B|E_2] = 1 - F_M(t) = R_M(t) \quad (20)$$

where:

$$F_M(t) = 1 - \exp \left[-M (t/\alpha)^\beta \right], \quad t > 0 \quad (21)$$

$$F_M^*(t) = 1 - \exp \left[-M (t/\alpha^*)^\beta \right], \quad t > 0 \quad (22)$$

which leads to the following extensions of eq. 15:

$$\mathcal{L}'_{N+1}(t, M) = \frac{\mathcal{L}_{N+1}}{\mathcal{L}_{N+1} + (1 - \mathcal{L}_{N+1}) \exp \left[+M (t/\alpha_{N+1})^\beta \right]} \quad (23)$$

and of eq. 18:

$$F'_M(t) = [1 - \mathcal{L}'_{N+1}(t, M)] F_M(t) + \mathcal{L}'_{N+1}(t, M) F_M^*(t) \quad (24)$$

Hence, to account for a number M of similar operating units the time must be scaled by a factor $M^{1/\beta}$. It can be immediately observed that, if no failure occurs, the larger M is, the faster will increase the likelihood that the failure mode $N + 1$ does not exist.

6 A lower bound for the proneness to failure

The updated failure proneness estimate, $F'_M(t)$, as given by eq. 24, has an important property that can be used for deriving some conclusions about the original tff distribution, $F_M(t)$. It can be observed that for operating times up to $\approx 0.2\alpha M^{-1/\beta}$, $F'_M(t)$ attains a maximum for values of α_{N+1} quite close to $tM^{1/\beta}$. Hence, *if nothing is known about* α_{N+1} , we can write:

$$\mathcal{L}'_{N+1} = \frac{\mathcal{L}_{N+1}}{\mathcal{L}_{N+1} + (1 - \mathcal{L}_{N+1})e} \quad (25)$$

and:

$$(1/\alpha^*)^\beta = (1/\alpha)^\beta + (1/t)^\beta \quad (26)$$

Inserting eqs. 25 and 26 into eq. 24 leads to:

$$[F'_M(t)]_{\max} \approx 1 - g(\mathcal{L}_{N+1}) \exp \left[-M (t/\alpha)^\beta \right], \quad t > 0. \quad (27)$$

where:

$$g(\mathcal{L}_{N+1}) = \frac{(1 - \mathcal{L}_{N+1}) + e^{-2}\mathcal{L}_{N+1}}{(1 - \mathcal{L}_{N+1}) + e^{-1}\mathcal{L}_{N+1}} \quad (28)$$

so that:

$$\lim_{\mathcal{L}_{N+1} \rightarrow 0} g(\mathcal{L}_{N+1}) = 1 \quad (29)$$

$$\lim_{\mathcal{L}_{N+1} \rightarrow 1} g(\mathcal{L}_{N+1}) = e^{-1} \quad (30)$$

Note that eq. 27 is the same as eq. 21 provided with a correcting factor $g(\mathcal{L}_{N+1})$. It yields then a worst case estimate of the proneness to failure, given that only the likelihood $\mathcal{L}_{N+1}$ is available. For very low values of $F'_M(t)$ it results:

$$[F'_M(t)]_{\max} \approx 1 - g(\mathcal{L}_{N+1}) \quad (31)$$

Table 1: Lower bounds for $F'_M(t)$ as function of $\mathcal{L}_{N+1}$.

$\mathcal{L}_{N+1}$	0.5	0.1	0.01	0.001
$[F'_M(t)]_{\max}$	0.17	0.025	0.0023	0.00023

which represents a lower bound for the updated proneness to failure estimate. Alternatively, the function $g(\mathcal{L}_{N+1})$ represents an upper bound for the reliability estimate in the same case.

Some values calculated with eq. 31 are presented in Table 1. For example, if there is maximum uncertainty whether or not mode $N+1$ exists, then $\mathcal{L}_{N+1} = 0.5$ and $g(\mathcal{L}_{N+1}) = 0.83$. This means that, during operation, the proneness to failure ought not to be assessed as smaller than 0.17, no matter how small the estimate of $F_M(t)$ is.

It must be again emphasized that the surprisingly high lower bounds for the proneness to failure indicated in Table 1 come from the assumption of absolute ignorance on the value of α_{N+1} . If the worst case condition $\alpha_{N+1} = tM^{1/\beta}$ is judged not applicable, smaller values of $[F'_M(t)]_{\max}$ may result.

7 Conclusions

Although efforts are underway to account for human errors in structural and construction codes and regulations, the authors are not aware of similar attempts to quantify the influence of phenomenological uncertainty in reliability estimates. A possible way of dealing with the problem is suggested.

It is shown that an estimate of the likelihood that a relevant unknown failure mode exists would be a governing factor in the final reliability assessment. This likelihood might be introduced in codified design, and should be applied whenever innovative techniques or technologies are being employed.

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