A Numerical Two-Dimensional Modelling of Eddy-Currents: An Application to Non-Destructive Test

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ABSTRACT

A way to detect cracks in steam-generator tubes, consists in using probes made of two coils of wire, concentric with the axis of the tube. Alternative currents of the same amount flow in the coils. As the probe is moved inside the tube, eddy currents, circulating in its wall, are created. Then, these eddy currents cause in the coils e.m.f.'s, opposing the initial currents. A crack in the tube offers resistance to the eddy currents circulation, and in so far as the coils are not equidistant from the crack, their corresponding e.m.f.'s are not equal.

The two coils are connected to a Wheatstone bridge, thus the e.m.f.'s difference, represented by a complex number, is visualized on a cathodic screen.

We decided to build up a mathematical model, allowing us to compute these e.m.f.'s differences in order to get a better knowledge of the relations between crack and visual signals on screens. We had to solve the following problem : knowing J_0 , current-density in the coils, determine the inducted current density J , in the tube, and then the e.m.f.'s in the coils. As currents unvary following a rotation, it appeared that a two-dimensional model would fit.

Starting from Maxwell's equations, a formulation in J and A (vector-potential), is obtained. An equation can be expressed in terms of J in the following integral form :

$$(1) \quad \frac{\partial}{\partial t} G.J + \rho J = - \frac{\partial}{\partial t} G.J_0 ,$$

where G is an integral operator and, ρ the tube's resistivity. As the phenomenon is time-periodic, we can consider J as a complex quantity ; $J(x, y, t) = J(x, y) e^{i\omega t}$, and solve a linear complex equation on a domain made of the intersection of the tube and a plane containing the tube's axis. This domain is divided in triangular elements. J will be approximated by a linear combination of step-functions, constant on each element.

One difficulty was the double-integration of a $\text{Log } |x - y|$ kernel on the same element (diagonal matrix terms). This leaded us to use a seven-points integration formula. Another difficulty consisted in the fact we had to get the difference between quantities of the same order of values. So, we needed the use of a regular and sufficiently refined mesh. Having to deal with a full and complex matrix, we needed a fairly big core size and c.p.u. time. So computing was run on a CRAY-1 computer (a full matrix lending itself easily, to a vectorizable inversion algorithm).

Results obtained on different patterns corresponding to different cracks will be presented, as comparaisons with screen-visualized signals.

I. Introduction

We are concerned here, with a problem of non destructive testing, we built a mathematical model of eddy-currents for the simulation of signals produced by a probe. In order to detect cracks in steam-generator tubes, a probe, made of two coils is moved along inside the tube. Alternative currents of the same amount flow in the coils, inducing eddy-currents in the tube. Thus, opposing the initial currents e.m.f.'s appear in the coils. As they are the same size and concentric with the axis of the tube, the induced e.m.f.'s are of the same magnitude. When the probe gets into the vicinity of a crack, offering resistance to the eddy current circulation, e.m.f.'s in the coils become no longer equal (in so far as the coils are not equidistant from the crack).

The coils are connected to a Wheatstonebridge, and the e.m.f.'s difference, represented by a complex number is visualized on a cathodic screen (see fig. (1) and (2)). A crack in the tube should be characterized by the signal on the screen, obtained when the probe has been moved along. Unfortunately, the signal is disturbed when the probe gets near the plate. Disturbed :

- i) by the plate
- ii) by the "dudgeonage" : buckling of the tube in order to fit to the plate (see fig. (3)).

To get a better knowledge of these signals, we decided to compute the induced e.m.f.'s in the coils.

II. Equations

We know the position of the coils and the current flowing within, whose density is J_0 . We have to calculate J , induced current in the tube and in the plate. Knowing J , it is straightforward to get the e.m.f.'s in the coils.

See Bossavit [1] for further details on the following. Let us note :

- ρ : the metal's resistivity
- H : the magnetic field
- B : the magnetic induction
- A : the vector potential
- J : the current density
- J_0 : the current density in the coils
- E : the electrical field
- μ : the magnetic permeability

Maxwell's equation is solved by :

$$\frac{\partial B(H)}{\partial t} + \text{curl } E = 0 \quad (1)$$

$$\text{With Ohm's law : } E = \rho \cdot J \quad (2)$$

$$A \text{ is defined by : } B = \text{curl } A \quad (3)$$

$$\text{Thus we have : } \text{curl} \left(\frac{\partial A}{\partial t} + \rho J \right) = 0 \quad (4)$$

Now, we must choose between using the variable A or J , to solve (4). This problem was solved in terms of A (see Lord and Palanisamy [3]). Using a finite element method leads to solving a sparse linear system of equations, but with a great number of variables, because A is defined in the whole space. We chose to solve (4) in terms of J , which exists only in metallic parts, so we had to deal with a smaller number of variables, but a full matrix as we shall see further on.

We must now, in (4), express A in terms of J.

II.1. Equation in J

We shall write :

$$B = \mu H \quad (\mu \text{ constant}) \quad (5)$$

which is true in the tube (made of a non-magnetic material where $\mu = \mu_0$, permeability in the vacuum). The plate is made of steel, but we will assume that (5) is true with $\mu = \mu_r \cdot \mu_0$, (μ_r = relative permeability). In fact, the current has a frequency of several hundred kilohertz, so the skin depth is of the same order of magnitude as the tube's width, and H in the plate is considered as small. Neglecting displacement currents, we consider the other Maxwell's equation :

$$J = \text{curl } H \quad (6)$$

Combining (3), (5) and (6), we get A as a solution of :

$$-\Delta A = \mu \cdot J \quad (7)$$

which can be written :

$$A = G(J) \quad (8)$$

The operator G will be detailed further on. Combining (4) and (8) we obtain :

$$\text{curl} \left(\frac{\partial GJ}{\partial t} + \rho J \right) = 0 \quad (9)$$

Thus there exists a function φ satisfying :

$$\frac{\partial GJ}{\partial t} + \rho J = \text{grad } \varphi \quad (10)$$

This equation is also satisfied in the coils, where J is known and equal to J_0 . It can be proved that, if we restrict (10) to the tube and the plate, it can be written (see Bos-savit [1]) :

$$\frac{\partial GJ}{\partial t} + \rho J = - \frac{\partial}{\partial t} G J_0 \quad (11)$$

We shall solve this equation to get J in the tube and in the plate.

II.2. E.m.f.'s in the coils

Each coil will be considered as a cylinder, whose cross section (5) has a radius equal to R (see fig. (4)).

The e.m.f. is equal to :

$$e = - \frac{\partial}{\partial t} \left(\int_S B \, ds \right), \text{ with } B = \text{curl } A$$

$$e = - \frac{\partial}{\partial t} \int_{\partial S} A \, d(\partial S) = 2 \cdot \Pi \cdot R \times \left(- \frac{\partial A}{\partial t} \right)$$

So

A will be defined, using $A = GJ$.

As we use cylindrical coordinates, (r, θ , z) to describe the domain, we suppose that the given current in the coil, J_0 , does not depend on θ . In fact, we solved (11), as a two-dimensional problem, instead of an axisymmetric one, the radius of the tube being much larger than the tube's width. Let (P), be a plane containing the axis of the tube, we shall solve (11) on the intersection of (P) and of the metallic part DUD' (tube and plate). At first, we shall test the program, only with the tube, the steel-plate being taken into account later on (see figures (5) and (6)).

As the problem is linear and time periodic, we shall use complex notation. We shall have:

$$J(x, t) = j(x) e^{i\omega t} \quad \text{and} \quad J_0(x, t) = j_0(x) e^{i\omega t}$$

with $x(x_1, x_2)$ point belonging to DUD' or D_0UD_0' and $\omega = 2\pi\nu$ (ν current's frequency).

Thus we have to solve

$$i\omega(Gj)(x) + \rho j(x) = -i\omega(Gj_0)(x) \quad \forall x \in DUD' \quad (12)$$

II.3. Operator G

In $\mathbb{R}^2$, A , vector potential is the solution of a potential problem :

- $\nabla^2 A = \mu j$, whose solution is

$$A(x) = \frac{1}{2\pi} \int_{DUD'} \mu(y) \text{Log } |x-y| j(y) dy \quad (13)$$

This is the expression of $A = G(J)$, G being an integral operator.

Therefore (12) becomes :

$$\frac{i\omega}{2\pi} \int_{DUD'} \mu(y) \text{Log } |x-y| j(y) dy + \rho j(x) = -\frac{i\omega}{2\pi} \int_{D_0UD_0'} \mu(y) \text{Log } |x-y| j_0(y) dy \quad (14)$$

Let y be a point of D , and y' its symmetric relative to the axis, we have $j(y) = -j(y')$.

Thus (14) becomes :

$$\frac{i\omega}{2\pi} \int_D \mu(y) \text{Log } \left| \frac{x-y}{x-y'} \right| j(y) dy + \rho j(x) = -\frac{i\omega}{2\pi} \int_{D_0} \mu(y) \text{Log } \left| \frac{x-y}{x-y'} \right| j_0(y) dy \quad (15)$$

(15) will be solved with an integral method. D will be divided into N triangles, for the approximation. Let η_l be a function equal to 1 on the l^{th} triangle, and vanishing elsewhere.

The approximated solution will be written :

$$j(x) = \sum_{l=1}^N j_l \eta_l(x) \quad (16)$$

The set of $(j_l, l = 1, N)$ will be the solution of a complex linear system written :

$$\sum_{l=1}^N A_{kl} j_l = b_k \quad \forall k \in [1, N] \quad (17)$$

A_{kl} is obtained in replacing (16) in (15), then by multiplying by η_k and integrating on D .

We get :

$$A_{kl} = \frac{i\omega}{2\pi} \int_{T_k} \int_{T_l} \mu(y) \text{Log } \left| \frac{x-y}{x-y'} \right| dx dy + \delta_{kl} \int_{T_k} dx \quad (18)$$

(where T_k is the triangle number k , and δ_{kl} , the Kronecker's symbol).

In the same way :

$$b_k = -\frac{i\omega}{2\pi} \int_{T_k} \int_{D_0} \mu(y) \text{Log } \left| \frac{x-y}{x-y'} \right| dx dy \quad (19)$$

There is no difficulty in computing these coefficients except for diagonal ones, where $T_k = T_l$. In fact, there is a singularity for $\text{Log } |x-y|$, on the same triangle. To solve this problem, the inner integral is calculated analytically, and the outer one computed with a numerical integration formula. This is a seven points formula, which can be found in [2].

III. Numerical results

In order to test the program, experiments were carried out. Three tubes were grooved, each with different depth defects. The length of the tubes was 30 mm, their width 1.26 mm, and the respective depth of the outside defects were 0.3, 0.6 and 0.9 mm, (see fig. (7)).

In these tubes, the probe was moved along 5 mm above to 5 mm below the crack. Figures (1) and (2) represent two photographs of the screen after the probe was moved along. Figure (1) represents the two signals corresponding to the 0.3 and 0.6 mm depth defect signals, and figure (2) represents the 0.9 mm depth defect signal.

A finite element mesh of the domain (D), seen above, was created. The resistivity of elements corresponding to the defect, was fixed at a very large magnitude. Having to compute the difference between two quantities of the same order of magnitude the mesh had to be sufficiently refined and regular. The mesh on figure (8) consists of 400 degrees of freedom (one for each two triangles). Later, modelling the plates will need around 1800 elements. The system's matrix being full and complex, the program was run on a CRAY-1. It should be noticed that the mesh does not depend on the coils position, thus each position of the coils only needs a new second-hand of the equation to be computed, the matrix having been inverted once already.

On figure (9) and (10), the numerical results corresponding to the three cracks were visualized. It must be observed that numerical and experiment results are identical except for a symmetry. We suppose it is due to the way (unknown by us), the coils were inserted into the Wheatstone bridge.

Time evolution of the signal

Figures (11a) and (11b) show the respective numerical and experiment signals when the probe is 2 mm from the defect. We have the same comparison on figures (12a) and (12b) at 1 mm from the crack.

Comparison between the modulus and the phase angles

Let M be the point of the curve such that $|\vec{OM}| = \text{Max } |\vec{OP}|$, P belonging to the curve. We will note M as the modulus of $\vec{OM}$, and α the phase angle ($\vec{OX}$, $\vec{OM}$) (see fig.(10)). We will note (α_3, M_3) , (α_6, M_6) , (α_9, M_9) , the phase angles and modulus corresponding to the signals obtained respectively with the 0.3, 0.6 and 0.9 mm depth defects. On table (1), comparisons between oscilloscope and numerical results are shown. These results are phase angles differences and modulus rates.

Conclusion

The results seem to us satisfactory, though the program is restricted to axisymmetric defects. Experiments were carried out on tubes having same depth defects, but covering 90°, 180° and 360° on the circumference. The signals we obtained on the oscilloscope screen had different amplitudes but the same phase angle, corresponding to the common depth of the tube defects.

New experiments have been started, taking into account cracks in the vicinity of the steam generator plate. On figure (13), we can see the mesh of the tube with dudgeonage, and the beginning of the plate. The signal obtained when the crack is 10 mm from the plate, can be seen on figure (14). On figure (15) we have the same signal, but when the crack is only 5 mm from the plate. In this case, comparisons with experiments have not yet been carried out.

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TABLE 1

Comparisons between oscilloscope
and numerical results

	Oscilloscope	Computer
$\alpha_9 - \alpha_6$	21°	21°
$\alpha_6 - \alpha_3$	22°	20 °
A_9 / A_3	4.67	4.37
A_6 / A_3	2.47	2.40

Fig. 1 : Oscilloscope signals corresponding to the 0.3 mm depth defect (below), and the 0.6 mm depth defect (above)

Fig. 2 : Oscilloscope signal corresponding to the 0.9 mm depth signal (Scale multiplied by 4)

Fig. 3 : Dudgeonage of the steam generator tube

Fig. 4 : Cylindric coil - Induction B and vector potential A

Fig. 5 : J_0 , current density in the coils, and J induced current in the tube

Fig. 6 : Two-dimensional domain, on which the equation is solved

Fig. 7 : Three outside defects in the tubes

Fig. 8 : Part of the mesh used in the approximation

Fig. 9 : Visualization of the computed e.m.f.'s differences
below : 0.3 mm depth defect
above : 0.6 mm depth defect

Fig. 10 : Visualization of the computed e.m.f.'s differences for the 0.9 mm depth defect

Fig. 13 : Mesh of the tube with dudgeonage and plate

Fig. 11.a and 11.b : Time evolution of the signals : when the probe is 2 mm from the crack. Above : computed signal
Below : oscilloscope signal

Fig. 12.a and 12.b : Same as 11.a and 11.b, the probe being 1 mm from the crack

Fig. 14 : Signal obtained when the crack is 10 mm from the plate

Fig. 15 : Signal obtained when the crack is 5 mm from the plate