On the Multiaxial Behaviour of Concrete Exposed to High Temperature

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Abstract

An attempt is made to formulate a triaxial constitutive model for concrete in the temperature range up to 800°C. The model is based on

- Observed high temperature behaviour under uniaxial stress and in pure shear.
- Known multiaxial behaviour at normal temperature.
- Thermochemical and thermomechanical behaviour of the concrete constituents.

The proposed model can be characterized as isotropic, elastic-viscoplastic-plastic in the compression region. At the present stage of development either brittle failure or plastic behaviour can be assumed in the tensile region.

The thermal strain increment is assumed to be a function of both temperature and the current stress tensor. This assumption implies that the thermal strain may have a deviatoric component.

The volumetric thermal strain is used as a scalar damage measure instead of temperature itself. The corresponding softening function is obtained from isothermal, uniaxial tests. Also the elastic properties are taken as functions of the volumetric thermal strain.

The response of the model is illustrated and compared with experimental results.

1. INTRODUCTION

Mathematical modelling of the mechanical response of concrete at temperatures above 100°C is needed for analysis of hypothetical accident situations in nuclear reactors and other industrial plants. Another field of interest is the design of structural components with regard to fire resistance.

Various material models for uniaxial stress, describing the elastic-plastic response and time-dependent effects as influenced by thermal damage have been employed, see e.g. /1,2/.

To the authors knowledge, however, no attempt has been made to formulate models for the multiaxial behaviour of concrete at high temperatures. Neither has this problem been studied to any extent experimentally. (See however /3/).

A triaxial constitutive model for concrete in the temperature range up to 800°C is presented in this paper.

Due to lack of experimental evidence some features of the proposed model have to be based on rather crude assumptions. Still it is believed that the model can serve as a tool for analysis awaiting further data. The development of a triaxial model may also give valuable guidance for future experimental research.

2. CONSTITUTIVE MODEL

2.1 Basic assumptions

It is basically assumed that the total strain rate tensor $\dot{\underline{\epsilon}}$ can be subdivided as

$$\dot{\underline{\epsilon}} = \dot{\underline{\epsilon}}^M + \dot{\underline{\epsilon}}^\theta \quad (1)$$

where $\dot{\underline{\epsilon}}^M$ denotes mechanical strain rate and $\dot{\underline{\epsilon}}^\theta$ is temperature and moisture induced strain rate. The wavy underscore $\sim$ is used to indicate tensor quantities.

In general the component $\dot{\underline{\epsilon}}^M$ depends on temperature and moisture content since mechanical properties and creep behaviour vary with these parameters.

The strain rate tensor $\dot{\underline{\epsilon}}^\theta$ is commonly assumed to be a pure volume change, but is here proposed to be dependent on the current stress tensor $\underline{\sigma}$, see 2.2.

Time-dependent deformation of creep type is included on the model. Due to the nature of potential applications the time scale considered is of the order of hours and days rather than months and years. On the other hand it is assumed that strain rates are sufficiently slow to neglect dynamic-type rate dependence.

The proposed model is isotropic, which makes it unsuitable to describe cyclic behaviour.

2.2 Temperature induced strain rate $\dot{\epsilon}^\theta$

In the present paper we restrict ourselves to situations where the material response at temperatures well above 100°C is of primary interest. Furthermore it is assumed that moisture is not confined but can be driven out from the concrete as the temperature increases. In that case the strain rate component $\dot{\epsilon}^\theta$ can be regarded as a function of temperature change but not of the change in water content.

This does not mean that the influence of water-loss is disregarded. But the restriction is that the water content is assumed as a unique time-independent function of temperature. For unconfined concrete this is essentially true for temperatures above $\approx 150^\circ\text{C}$ [1]. Since the strains at lower temperatures in general are small compared to the strains developed at higher temperatures the moisture dependence can be neglected.

At first heating of concrete the aggregate will expand while the cement paste tends to shrink due to a continuous extraction of water. For unstressed concrete the resulting volume change is an expansion of the same order of magnitude as that of aggregate, but due to the incompatibility between aggregate and cement paste the thermal strain in the direction of loading is strongly affected by a prestress in the material during heating.

The influence of stress on the thermal volume change is demonstrated in figure 1 for unsealed concrete heated to 400°C [1]. The free expansion of the concrete is about 0.4% at 400°C, (Curve 1) but this expansion is completely restrained when a compressive stress equal to 0.45 f'_c is applied during the heating period (Curve 2).

To account for this behaviour the thermal strain rate $\dot{\epsilon}^\theta$ is assumed to be a function of both temperature and the current stress tensor and we can write

$$\dot{\epsilon}^\theta = [\alpha(\theta)\mathbf{I} + g(\theta, \underline{\sigma}, \text{sgn}\dot{\theta})]\dot{\theta} \quad (2)$$

where θ is the temperature, $\underline{\sigma}$ is the current stress tensor, $\alpha(\theta)$ is the coefficient of free thermal expansion and $\mathbf{I}$ is the unit tensor.

In eq (2) $\dot{\epsilon}^\theta$ consists of a purely volumetric part ($\alpha(\theta)\dot{\theta}\mathbf{I}$), which describes the free volume change, while the influence of stress is described by the tensor function g .

The function g can be given the following simple form.

$$g(\theta, \underline{\sigma}, \text{sgn}\dot{\theta}) = \begin{cases} \beta(\theta)\underline{\sigma} & \text{if } \dot{\theta} > 0 \text{ (first heating)} \\ 0 & \dot{\theta} < 0 \end{cases} \quad (3)$$

where $\beta(\theta)$ is a material property to be obtained from experimental data.

The form for g in eq (3) with g proportional to the current stress tensor is proposed on the basis of observed behaviour in uniaxial compression. In the absence of further experimental evidence it seems reasonable to use the form (3) also in multiaxial situations.

2.3 Mechanical strain rate

The material response is either elastic, elastic-viscoplastic or elastic-viscoplastic-plastic depending on the stress-state. For certain states of stress tensile cracking occurs.

A viscoplastic loading surface $F_{vp} = 0$ is defined such that the response is linearly elastic if $F_{vp} < 0$. The elastic response is temperature dependent.

A plastic loading function F_p is defined such that the response is elastic-viscoplastic if $F_{vp} > 0$ and $F_p < 0$. In this case we have

$$\dot{\underline{\epsilon}}^M = \dot{\underline{\epsilon}}^e + \psi(\theta, F_{vp}) \hat{n}_{vp} \quad (4)$$

where $\dot{\underline{\epsilon}}^e$ is the elastic strain and

$$\psi(\theta, F_{vp}) = \begin{cases} 0 & \text{if } F_{vp} < 0 \\ \gamma(\theta) \cdot \phi(F_{vp}) & \text{if } F_{vp} > 0 \end{cases} \quad (5)$$

The direction of the inelastic strain in this regime is defined by the unit vector

$$\hat{n}_{vp} = \frac{\frac{\partial F_{vp}}{\partial \underline{\sigma}}}{\left(\frac{\partial F_{vp}}{\partial \underline{\sigma}} \cdot \frac{\partial F_{vp}}{\partial \underline{\sigma}} \right)^{1/2}} \quad (6)$$

where an associated flow rule has been assumed. The strain rate $\psi \hat{n}_{vp}$ corresponds to the time-dependent deformation, which may be significant at high temperatures.

The plastic loading function is used to describe the instantaneous inelastic response and stress states outside the surface $F_p = 0$ are not admissible. For stress states on the plastic loading surface i.e. for $F_p = 0$ the response is given by

$$\dot{\underline{\epsilon}}^M = \dot{\underline{\epsilon}}^e + \psi(\theta, F_{vp}) \hat{n}_{vp} + \dot{\lambda} \frac{\partial F_p}{\partial \underline{\sigma}} \quad (7)$$

where $\dot{\underline{\epsilon}}^e$ is the elastic strain rate, ψ is given by eq (5) and the scalar function $\dot{\lambda}$ depends on the states of stress, strain and temperature. Again associated flow is assumed. Upon plastic flow it is required that $\dot{F}_p = 0$, which can be used to determine $\dot{\lambda}$. If $F_p < 0$ or if $F_p = 0$ and $\dot{F}_p < 0$ plastic flow does not occur and $\dot{\lambda} = 0$.

2.4 Loading functions F_{vp} and F_p

Both loading functions F_{vp} and F_p are assumed to be functions of the stress tensor $\underline{\sigma}$ and two hardening parameters H_1 and H_θ .

$$F_{vp} = F_{vp}(\underline{\sigma}, H_1, H_\theta) \quad (8)$$

$$F_p = F_p(\underline{\sigma}, H_1, H_\theta)$$

The same hardening parameters are used for both surfaces. H_1 is a measure of damage due to mechanical straining while H_θ describes the damage caused by thermal effects. Both $F_{vp} = 0$ and $F_p = 0$ are given the form of a generalized ellipsoidal surface suggested by Nilsson /4/, with

$$F = \left[\left(\frac{2\sigma_0 - \bar{\sigma}(H_1, H_\theta)(a_2(H_\theta) + a_1(H_\theta))}{a_2(H_\theta) - a_1(H_\theta)} \right)^2 + \left(\frac{\tau_0}{r(\alpha, H_\theta)} \right)^2 \right]^{1/2} - \bar{\sigma}(H_1, H_\theta) \quad (9)$$

where σ_0 and τ_0 are the octahedral normal and shear stress respectively, and α is the angle of similarity. The parameter $\bar{\sigma}(H_1, H_\theta)$ can be interpreted as equivalent stress in uniaxial compression. The nondimensional parameters a_1 , a_2 and r can be identified in figure 2.

A deviatoric section of $F = 0$ is shown in figure 3 and the corresponding analytical expression for $r(\alpha)$ is given in reference /5/.

2.5 Thermal softening behaviour

The damage parameter H_θ is assumed to be a function of the volumetric thermal strain ϵ_θ^0 such that

$$\dot{H}_\theta = \begin{cases} \dot{\epsilon}_\theta^0 & \text{if } \dot{\epsilon}_\theta^0 > 0 \\ 0 & \text{if } \dot{\epsilon}_\theta^0 < 0 \end{cases} \quad (10)$$

This choice of damage parameter is made since the thermal damage for temperatures between 100 and 600°C is mainly caused by differential thermal deformation of cement paste and aggregate. Many tests have shown that a compressive stress applied during heating of concrete leads to higher compressive strength in the prestress direction. This effect is modelled when $\dot{H}_\theta$ is chosen according to eq (10). Eq (10) implies that no further damage takes place under decreasing temperature. Tests show that the strength decreases somewhat under cooling, but this has been neglected here. Given reliable data however eq (10) could be refined to include this.

For temperatures above 500-600°C the main damage is caused by chemical decomposition of cement paste. This can not be described by the measure ϵ_θ^0 and the softening rule is modified for temperatures above this level.

2.6 Strain hardening

The hardening parameter H_1 is taken as a nondecreasing function of the nonelastic strain. We have

$$H_1 = \int \dot{H}_1 dt \quad \dot{H}_1 \geq 0 \quad (11)$$

The strain hardening behaviour is characterized by two distinct regimes. At low values of H_1 simple isotropic hardening is assumed for all stress states and both the plastic yield surface and the viscoplastic loading surface expand with increasing H_1 until the surface $F_p = 0$ reaches a certain limit $F_{p1} = 0$. This limit corresponds to $\bar{\sigma} = f'_c$ in eq (9), i.e. the ultimate stress limit in uniaxial compression.

Beyond this limit a critical state model is used in the form suggested by Nilsson /4/. In the critical state theory the yield surface comprises two distinct parts, the subcritical and the supercritical regions. The boundary between the two regions is defined in stress space by stress states which produce zero volumetric strain at plastic yielding. In the subcritical region, yielding is stable with continued strain hardening while in the supercritical region strain softening due to fracturing of the material occurs, see fig 2.

In the tensile region brittle failure is assumed when $F_{p1}(\sigma, H_1, H_0) = 0$ and at least one principal stress is positive (tension cut-off).

2.8 Tangential material stiffness and pseudo stress rate

The constitutive equations can be written on an incremental form suitable for Finite Element Analysis. Generally we can write

$$\dot{\underline{\sigma}} = D_{ep} \dot{\underline{\epsilon}} - \dot{\underline{\sigma}}_0 \quad (12)$$

where D_{ep} is the tangential stiffness tensor and $\dot{\underline{\sigma}}_0$ is the pseudo stress-rate vector. D_{ep} and $\dot{\underline{\sigma}}_0$ take different forms depending on the type of response.

3. PREDICTIONS OF THE MODEL

One of the main features of the model is that the thermal strain is taken as a function of the current stress. The importance of this assumption is illustrated in figure 4, where the material is heated to 800°C, while the strain in one direction is kept constant = 0. The figure shows the corresponding stress σ_3 as a function of time. The response of the proposed model is well in agreement with test-results /1/. If the influence of stress on thermal strain is disregarded ($g = 0$) however the restraint stress is strongly overestimated.

The consequence of the assumed thermal softening behaviour is illustrated in figure 5. The figure shows the ultimate failure stress in uniaxial compression as a function of temperature. Different states of prestress have been prescribed during the heating period up to respective temperature. It is seen that the strength of concrete at high temperatures increases with the degree of prestress.

4. CONCLUDING REMARKS

The main features of the proposed model are

- The thermal strain rate depends on the current stress.
- The volumetric thermal strain is used as a scalar measure of thermal damage.
- The mechanical damage is described by a model of the critical state type.
- Time-dependent creep type behaviour is modelled.
- Brittle fracture in the tensile region can be assumed.

A subject of further development is a better modelling of the effects of moisture. To improve the model in the tensile region it is desirable to account for softening due to strain localization.

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Figure 1 Measured total strain in concrete specimens heated to 400°C. Applied stress = $0.45 f'_c$, where f'_c is the uniaxial compressive strength at 20°C.

Figure 2 Hydrostatic section of the surface $F = 0$. The dotted line is the critical state line.

Figure 3 Deviatoric section of the surface $F = 0$.

Figure 4 Restraint stress in concrete heated to 800°C with the strain ϵ_3 in the 3-direction prescribed to zero. The lower figure shows the temperature history.

Figure 5 Ultimate stress f'_{c0} at failure in uniaxial compression as a function of temperature according to the proposed model. Prestress during the heating period: 1. No stress 2. Uniaxial stress $\sigma_3 = -0.2 f'_c$. 3. Biaxial stress, $\sigma_2 = \sigma_3 = -0.2 f'_c$. 4. Hydrostatic stress, $\sigma_1 = \sigma_2 = \sigma_3 = -0.2 f'_c$.