Comparison of Results of Soil-Structure Interaction Analyses Based on Time and Frequency Domain Approaches with Emphasis on Treatment of Damping

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SUMMARY

Comparison of results of soil-structure interaction analyses of the Reactor Building of a nuclear power plant using different analytical approaches and solution procedures is presented. The emphasis of the comparison was on the treatment of damping in these different approaches and procedures. An axisymmetric model of the Reactor Building was employed. The analyses were performed for the aircraft impact loadings. Two different locations were used for these loadings.

The following four different sets of analyses were performed.

1. Time-domain analysis using frequency-independent soil springs in conjunction with modal damping cut-off
2. Frequency-domain analysis using frequency-independent soil springs in conjunction with a complex modulus approach
3. Frequency-domain analysis using frequency-dependent soil-impedance coefficients in conjunction with a complex modulus approach
4. Frequency-domain analysis using frequency-dependent soil-impedance coefficients in conjunction with Rayleigh damping

The frequency-independent soil springs were computed using the standard approach based on rigid base supported on an elastic layered half-space. The frequency-dependent soil impedance coefficients were computed in the form of a soil substructure matrix which included the uncoupled as well as the coupled terms. The computations were based on the use of a "flexible" base mat supported on a layered half-space. Unit dynamic loads, for each frequency, were applied to the layered half-space corresponding to each degree of freedom and the displacements were computed corresponding to all degrees of freedom. The compliance matrix so computed was inverted to obtain the impedance matrix for each frequency. The computations were repeated for all frequencies of interest for the aircraft impact loading.

Floor response spectra were developed and compared at various floor elevations of the Reactor Building using the above four different sets of analyses. Conclusions were developed as a result of these comparisons.

1. Introduction

One of the procedures employed to determine the effects of the flexibility of soil-foundation on the response of the super-structure uses the equivalent springs and dampers to represent the soil-structure interaction impedances. These impedances, which are normally frequency-dependent, are determined by computing the response of a rigid (or flexible) footing on the surface of a layered visco-elastic (or elastic) half space, subjected to a unit harmonic excitation. For practical applications, simplified frequency-independent springs and dashpots are sometimes employed. In order to perform the soil-structure interaction analysis using the frequency-independent soil springs, several empirical methods are available for computing composite modal damping of the interactive system. The methods normally used are Bigg's method [1], Modified Bigg's method [1], and complex moduli method [2].

In this, paper a comparative study is performed using different analytical approaches (frequency-independent and frequency-dependent) and solution procedures (time-domain, and frequency-domain), with special emphasis on treatment of damping (composite modal damping approach, and complex moduli approach) in these analytical procedures. Analyses were performed using an axisymmetric lumped mass model of the Reactor Building of a nuclear power plant subjected to aircraft impact loading. The results obtained using different analytical approaches were compared. The degree of conservatism associated with the conventional analyses performed using frequency-independent springs was determined by computing the response of the Reactor Building subjected to aircraft crash load using frequency-independent soil springs in conjunction with the modal damping cut off values and comparing these results with those obtained using frequency-dependent soil impedance functions which treats the soil radiation damping effects in a more realistic manner.

2. Analytical Model, Loading and Material Properties

2.1 Analytical Model

An axisymmetric model of the super-structure of the Reactor Building was developed. A computer plot of the model, employed in the analyses, is shown in Figure 1. The Reactor Building consists of a dome shaped exterior concrete shell, interior steel shell and interior concrete structure.

2.2 Loading

Analyses were performed for the standard aircraft crash loading applied separately at two locations (Figure 1). The aircraft crash loading is 0.07 second duration and has a peak value of 11000mp [2].

2.3 Material Properties

The properties of the soil layer and bed rock below the Reactor Building were as follows:

	<u>Soil Layer</u>	<u>Bed Rock</u>
Shear Modulus	$2.10 \times 10^4 \text{ mp/m}^2$	$4.15 \times 10^4 \text{ mp/m}^2$
Poisson's Ratio	0.475	0.32
Mass Density	0.19 mp/m^2	0.23 mp/m^2

For concrete, steel and soil, material damping values of 7, 2 and 6 percent, respectively,

were used. The radiation damping in the soil was computed using different approaches depending on the type of analysis.

3. Analytical Approaches

Analyses of the soil-structure system were performed using the following approaches:

3.1 Time-domain Analysis Using Frequency-Independent Soil Springs in Conjunction with Modal-Damping Cut-Off

In this approach the frequency-independent soil springs obtained for a circular footing on an elastic half space were used. The radiation damping in the soil was taken into account by providing corresponding damping ratio. The composite modal damping values for the concrete/steel/soil system were computed by Bigg's Procedure [1]. The damping values were limited to 15 and 30 percent for horizontal and vertical modes, respectively. Frequencies and modes of the super-structure model in conjunction with the frequency-independent soil springs were determined for the first 11 harmonics. For each harmonic frequencies up to 80 Hz were included. Time history analyses of the model were performed for two aircraft crash load cases. The analyses were performed for each harmonic to determine acceleration response time histories. These acceleration time histories due to each harmonic were superimposed to obtain the combined acceleration response time histories for each load case. Floor response spectra for 2 percent damping were generated.

3.2 Frequency-Domain Analysis Using Frequency-Independent Soil Springs in Conjunction with Complex Modulus Approach

In order to include the damping properties of the soil in a more rational manner in low as well as high frequency ranges, and to form a basis for comparison for all frequency-domain analyses, soil-structure interaction analyses of the Reactor Building axisymmetric model, in conjunction with the frequency-independent soil springs, were performed in frequency-domain using a complex modulus approach. In this approach, the damping in the soil-structure system was introduced by using a complex modulus in the formation of the stiffness matrix. The complex moduli may be defined as:

Shear Modulus, $G^* = G (1-2\beta^2 + 2i\beta\sqrt{1-\beta^2})$ - - - - (1)

Young's Modulus, $E^* = E (1-2\beta^2 + 2i\beta\sqrt{1-\beta^2})$ - - - - (2)

where

G and E are shear and Young's moduli, and β is the uniform fraction of critical damping for each component of the system. The G, E and β values for each component of the system may be different.

The dynamic equilibrium equation in frequency-domain for input loading corresponding to each harmonic was expressed as

$(K^* - \omega^2 M) \ddot{U}(\omega) = \omega^2 P(\omega)$ - - - - - (3)

where

K^* = Global complex stiffness matrix of the system, M = Global mass matrix of the system, $P(\omega)$ = Fourier amplitudes of the applied load vector at frequency ω , $\ddot{U}(\omega)$ = Fourier amplitudes of the acceleration response vector at frequency ω ,

ω = Frequency at which responses are computed.

The global complex stiffness matrix, $\underline{K}^*$, for the complete soil-structure system was assembled from the component complex stiffness matrix $\underline{k}^*$ by direct stiffness assembly procedure. For each component of the system, the complex stiffness matrix was computed using the complex moduli given by equations (1) and (2).

The acceleration responses for each harmonic at each frequency step was computed by solving complex set of equations (3). The combined acceleration responses in time-domain for each harmonic was then obtained by superimposing and inverse-transforming the response in frequency-domain. The time histories of acceleration responses so obtained for each harmonic were superimposed for eleven harmonics to obtain the acceleration time history responses for each load case.

The response analyses of the Reactor Building model in conjunction with frequency-independent soil springs were performed in frequency-domain using the above procedures for two aircraft crash load cases. For each load case, the acceleration response time histories due to each harmonic were computed, and they were superimposed to obtain combined acceleration time history responses. Floor response spectra for 2 percent of critical damping for each of the two load cases in radial and vertical directions were generated.

3.3 Frequency-Domain Analysis Using Frequency-Dependent Soil Impedance Coefficients in Conjunction with Complex Modulus Approach

In this approach, frequency-dependent soil impedance coefficients were used in conjunction with the superstructure model and analyses were performed in frequency-domain using the complex moduli approach to determine the acceleration time history responses at several locations of the model.

3.3.1 Derivation of Frequency-Dependent Impedance Matrices for Soil-Foundation System

Due to a force vector, $\underline{P}(\underline{x}_0)$, acting on the surface of an elastic half-space at an arbitrary point $\underline{x}_0$, on the surface, the displacement vector $\underline{U}$, that results at another point $\underline{x}$ is given by

$$\underline{U}(\underline{x}) = \underline{G}(\omega, \underline{x} - \underline{x}_0) \underline{P}(\underline{x}_0)$$

where

$\underline{G}(\omega, \underline{x} - \underline{x}_0)$ is the Green's function matrix, whose elements are functions of Poisson's ratio, ν , and shear modulus, G , of the elastic medium, as provided in Reference 3.

Once the Green's function matrix of an elastic half-space is obtained as described above, the compliance matrix (with uncoupled and coupled terms) of the soil-foundation system is computed by the numerical procedure described below.

Consider a rigid circular footing, supported on the surface of an elastic half-space. The footing is subdivided into elements by radial and circumferential lines, as shown in Figure 2. Each of the circumferential lines on the footing is identified as a nodal circle. The expression for the dynamic displacements of nodal

circles in terms of the corresponding dynamic force amplitudes may be expressed as

$$\{d\} = [F] \{f\}$$

where

$$\begin{aligned} \{d\} &= \text{Dynamic displacement vector of the nodal circles} \\ \{f\} &= \text{Corresponding force vector at the nodal circles} \\ [F] &= \text{Compliance matrix of the footing} \end{aligned}$$

The numerical procedure employed for the computation of the compliance matrix basically consists of evaluations of the displacements resulting from unit dynamic loads at frequency, ω , corresponding to each degree of freedom (radial, tangential and vertical) of a nodal circle. Under the application of a unit line load at one of the nodal circles of the footing (in radial, tangential, or vertical direction), the corresponding displacements at the nodal circle where the load is applied and at the remaining nodal circles are computed. The nodal circle displacement vectors at each degree-of-freedom per nodal circle, due to force applied at each nodal circle, are assembled to form the compliance matrix of the footing.

The complex compliance matrix of the soil-structure system so obtained at frequency ω is inverted to determine the impedance matrix of the system at frequency ω . The frequency-dependent impedance matrices, so obtained, are input as sub-structure matrices at the corresponding degrees-of-freedom of the nodal circles at the base mat of the superstructure model.

3.3.2 Inclusion of Damping of the Superstructure Using Complex Modulus Approach and Computation of Response

The variation of damping in different components of the superstructure model was taken into account by using complex modulus approach. In this approach, the complex stiffness matrices were computed for each element of the superstructure finite element model by using the complex modulus

$$E^* = E (1 - 2\beta^2 + 2i\beta \sqrt{1 - \beta^2}) \text{ --- (4)}$$

where E is the elastic modulus of the corresponding material. The complex element stiffness matrices, so obtained, were assembled by direct stiffness approach to obtain the complex global stiffness matrix of the superstructure, K^* .

The dynamic equilibrium equations of the complex soil structure system were expressed at frequency ω as

$$K^* + K_s(\omega) - \omega^2 M \ddot{U}(\omega) = \omega^2 P(\omega) \text{ --- (5)}$$

where

K^* = Complex stiffness matrix of the superstructure, $K_s(\omega)$ = Complex sub-structure impedance matrix of the soil-foundation system at frequency ω , M = Mass matrix of the superstructure, $\ddot{U}(\omega)$ = Fourier amplitudes of the acceleration response vector at frequency ω , $P(\omega)$ = Fourier amplitudes of the applied load vector at frequency ω , ω = Frequency at which responses are computed.

Equation (5) was solved at each frequency step for each harmonic to determine the amplitudes of acceleration responses in the frequency-domain. The time histories of acceleration responses were obtained for each harmonic by inverse transformation. These acceleration time histories were then superimposed for all eleven harmonics to determine the combined response due to each load case. Floor response spectra for 2 percent of critical damping for each of the two load cases in the radial and vertical direction were generated.

3.4 Frequency-Domain Analysis Using Frequency-Dependent Soil Impedance Coefficients in Conjunction with Rayleigh Damping Approach

In this approach, the frequency-dependent soil impedance coefficients were used in conjunction with the superstructure model, and analyses were performed in the frequency-domain to determine acceleration responses at several locations of the model. The damping values in the superstructure model were specified by Rayleigh Damping (mass proportional damping α and stiffness proportional damping β).

The frequency-dependent impedance coefficients for a circular footing on the layered soil were obtained as sub-structure matrices using the procedures described in Section 3.3.1. The damping in the superstructure model was specified by Rayleigh damping (mass proportional damping α and stiffness proportional damping β), i.e.,

$$\underline{C} = \alpha \underline{M} + \beta \underline{K} \text{ --- (6)}$$

where

$\underline{C}$ is the damping matrix, $\underline{M}$ is the global mass matrix, and $\underline{K}$ is the global stiffness matrix. For any frequency, ω_n , the damping ratio ξ_n , is related to α and β by

$$\xi_n = \frac{\alpha}{2\omega_n} + \frac{\beta\omega_n}{2} \text{ --- (7)}$$

For each harmonic, the damping ratios at two frequencies were assumed to be constant. Using Equation (7), α and β were computed. For superstructure model, the damping ratios ranged from 3 percent (steel) to 7 percent (concrete). The values of α and β were computed so that the minimum damping ratio fell in between the above ranges of the damping ratios. The dynamic equilibrium equations of the complex soil-structure system at frequency ω were expressed as

$$\underline{K} + \underline{K}_s(\omega) - \omega^2 \underline{M} + i\omega (\alpha \underline{M} + \beta \underline{K}) \underline{\ddot{U}}(\omega) = \omega^2 \underline{P}(\omega) \text{ --- (8)}$$

where

$\underline{K}$ = Global stiffness matrix of the superstructure, $\underline{M}$ = Global mass matrix of the superstructure, $\underline{K}_s(\omega)$ = Complex superstructure impedance matrix of the soil-foundation system at frequency ω , $\underline{P}(\omega)$ = Fourier amplitudes of the applied load vector at frequency ω , $\underline{\ddot{U}}(\omega)$ = Fourier amplitudes of the acceleration response vector at frequency ω , ω = Frequency at which responses are computed.

Equation (8) was solved at each frequency step for each harmonic to determine the amplitudes of acceleration response in frequency-domain. The time histories of acceleration response were obtained for each harmonic by inverse transformation. The

acceleration time histories were then superimposed for all 11 harmonics to determine the combined response due to each load case, from which floor response spectra for 2 percent damping were generated.

4. Comparison of Spectra Generated Using the Above Four Approaches

The floor response spectra at typical locations of the Reactor Building model generated using different approaches were compared. Figures 3 through 6 show plots of typical floor response spectra in horizontal and vertical directions at several locations of the Reactor Building model obtained for 2 percent damping for two aircraft crash loads. These figures show the comparison of the spectra obtained using conventional frequency-independent soil spring approach in time-domain with modal damping cut-off values, frequency-independent soil springs in frequency-domain with the complex modulus approach, frequency-dependent soil impedance matrices using the complex modulus and Rayleigh damping approaches.

5. Summary and Conclusions

Soil-structure interaction analyses of the Reactor Building of a nuclear power plant using different analytical approaches and solution procedures were performed for aircraft impact loading. The results using different analytical approaches with emphasis on treatment of damping in these approaches were compared. Based on these comparisons the following conclusions were drawn.

- In general, the conventional frequency-independent soil springs approach with arbitrary modal damping cut-off values showed higher spectral accelerations as compared to other approaches based on frequency-dependent impedance functions, especially in the lower frequency range.
- In general, the frequency-dependent impedance function approach with Rayleigh damping showed the lowest spectral accelerations.
- In general, the frequency-dependent impedance function approach with complex modulus showed spectral accelerations close to (but lower than) the frequency-independent impedance function approach with complex modulus. The results for these approaches (with complex modulus) were somewhere in between the results for frequency-independent approach with modal damping cut-off and the frequency-dependent approach with Rayleigh damping.

It can, therefore, be concluded on the basis of this study that the frequency-independent approach with arbitrary modal damping cut-off is usually very conservative. The use of Rayleigh damping, especially in conjunction with frequency-dependent impedance functions may, however, be unconservative at times and should be used with caution. The frequency-dependent approach with complex modulus is the most realistic approach to use for soil structure interaction analyses. This approach can provide realistic and rational designs which are not only safe but economical.

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Figure 1. Analytical Model of the Reactor Building

Figure 2. Mathematical Model of the Foundation to Compute Soil-Compliance Matrices

Figure 3. Comparison of Floor Response Spectra (Node 39-R, Load Case 1)

Figure 4. Comparison of Floor Response Spectra (Node 7-V, Load Case 1)

Figure 5. Comparison of Floor Response Spectra (Node 75-V, Load Case 2)

Figure 6. Comparison of Floor Response Spectra (Node 82-R, Load Case 2)