AN ANALYTICAL MODEL FOR TRANSIENT GAS FLOW IN NUCLEAR FUEL RODS

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SUMMARY

The safety analysis of light water reactors considers creep deformation (Ballooning) of Zircaloy clad nuclear fuel rods. The driving force for clad ballooning is the internal pressure of the fuel rods relative to the coolant pressure. As ballooning occurs, the local internal driving pressure decreases unless there is gas flow to the region of deformation from a higher pressure region. If the gas flow is highly restricted then the cladding ballooning may be slowed by reduced local internal pressure. A realistic analysis of clad ballooning, such as the one presented in this paper, requires a space and time calculation of internal gas pressure. Our method is applied between all axial nodes rather than between two points (plenum and swelling node), employs a solution procedure believed to be new to transient LWR fuel rod analysis and offers an alternate to other gas flow models used in existing safety analysis codes, such as FRAP-T3 and SSYST.

The analytical model uses a porous media concept (permeability) to define the resistance to gas flow along the fuel rod. This concept is valid for laminar flow through cracked pellets and along the annular gap between the fuel and cladding. Flow in the annular gap is considered in this paper and the permeability is expressed as $\delta D^2/32$ where δD is the diametral gap.

The fuel rod is divided into axial segments (nodes) and the balance equations for mass and momentum for each node have the same form as the equations for conventional one-dimensional compressible gas flow. The momentum equation derivation, however, admits the possibility of a jump discontinuity of gas flow area and permeability between nodes. Gas temperature, pressure, and density are related by the perfect gas law.

An implicit equation solution approach is used to solve for a tentative flow field at new time by using the old time pressure field. A pressure field correction is then calculated such that the mass balance is satisfied. This is done by the direct solution of a tridiagonal matrix. A final velocity field correction is then calculated from the pressure field correction. Without cladding deformation, the method is unconditionally stable for any time step; however, with cladding deformation, there are time step limitations similar to the Courant limitation in explicit numerical methods.

The ability of the new model to calculate transient pressure response in a nuclear fuel rod was demonstrated by comparison with INEL experimental data for transient gas flow in irradiated PWR fuel rods. The experiments were run as a transient depressurization from an initial uniform pressure. A pressure transducer measured the plenum gas pressure during the discharge of gas from the other end of the rod. The results of the calculated plenum pressure decay were in good agreement with the data, especially since there was no optionization of the code input. The radial gap is the most significant parameter affecting the pressure decay. The selected gap of 30 μm was the recommended value developed by EG&G for the test rod and had an uncertainty on the order of 30 percent.

In summary the new gas flow model offers an alternate approach for detailed and accurate calculation of gas flow and pressure in nuclear fuel rods.

1. INTRODUCTION

The safety analysis of light water reactors (LWR) requires consideration of creep deformation (Ballooning) of Zircaloy clad nuclear fuel rods. The driving force for clad ballooning is the internal pressure of the fuel rods relative to the coolant pressure. As ballooning occurs, the local internal driving pressure decreases unless there is gas flow to the region of deformation from a higher pressure region, such as the plenum. If the gas flow is highly restricted then the cladding ballooning may be slowed by reduced local internal pressure. A realistic analysis of clad ballooning, therefore, requires a space and time calculation of internal gas pressure as part of the analysis. The analytical method presented in this paper accomplishes this calculation. It is applied between all adjacent axial nodes rather than between two points (plenum and swelling node), and employs a solution procedure believed to be new to transient LWR fuel rod analysis. Thus, it offers an alternate to other gas flow models used in existing safety analysis codes, such as FRAP-T3⁽¹⁾ and SSYST⁽²⁾.

2. TECHNICAL APPROACH

The analytical model uses a porous media concept (permeability to define resistance to gas flow along the fuel rod. This concept is valid for laminar flow through cracked pellets and along the annular gap between the fuel and cladding. Flow in the annular gap is considered in this paper and the permeability is expressed as $\delta D^2/32$ where δD is the diametral gap. The permeability is a key factor in the relationship between gas flow and pressure during deformation.

The following sections present a derivation of the balance equations for mass and momentum, the numerical method used to solve the equations, and an assessment of the model using experimental data.

2.1 Computational Grid

The fuel rod is divided axially into an arbitrary number of computational cells. The first and last cells are reserved as phantom cells for boundary conditions. Pressure, temperature, and density are defined at cell centers and flow rates are defined at cell interfaces as shown in Figure 1. The variation of gas volume along the length of the rod is represented by a gas volume fraction (porosity) which is that fraction of gas volume within the cladding inside diameter. This gas fraction includes the annular gap volume.

2.2 Mass Balance

The mass balance is written for computational cell centered at k and Δz_k long.

$$\frac{\partial}{\partial t} \alpha_k \rho_k A_k \Delta z_k + \alpha_k \rho w_k A_k - \alpha_{k-1} \rho w_{k-1} A_{k-1} = 0 \quad (1)$$

It is convenient to introduce the concept of gas volume fraction as being that fraction of the area inside the cladding occupied by vapor; thus, $\alpha_k A_k$ represents the actual cross section area occupied by gas where A_k is the volume per unit length inside the cladding.

The finite difference analog to the mass balance is written as

$$\Delta z_k \frac{(\alpha \rho)_k^{n+1} A_k^{n+1} - (\alpha \rho)_k^n A_k^n}{\Delta t} + A_k^{n+1} [(\alpha \rho w)_k^{n+1} - (\alpha \rho w)_{k-1}^{n+1}] = 0 \quad (2)$$

where spatial changes of A_k along the axial length Δz_k are considered small. The first term can be factored such that equation (2) becomes

$$\Delta z_k A_k^{n+1} \frac{(\alpha \rho)_k^{n+1} - (\alpha \rho)_k^n}{\Delta t} + (\alpha \rho)^n \Delta z_k \frac{A_k^{n+1} - A_k^n}{\Delta t} + A_k^{n+1} [(\alpha \rho w)_k^{n+1} - (\alpha \rho w)_{k-1}^{n+1}] = 0 \quad (3)$$

Dividing by $\Delta z_k A_k^{n+1}$ give the mass balance equation

$$\frac{(\alpha \rho)_k^{n+1} - (\alpha \rho)_k^n}{\Delta t} + \frac{(\alpha \rho)_k^n}{A_k^{n+1}} \frac{A_k^{n+1} - A_k^n}{\Delta t} + \frac{(\alpha \rho w)_k^{n+1} - (\alpha \rho w)_{k-1}^{n+1}}{\Delta z_k} = 0 \quad (4)$$

2.3 Axial Momentum Balance

The axial momentum balance provides the relationship to couple computational cells. It is desired to obtain a relationship between flow across a cell boundary and pressure difference between cell centers. The approach taken admits the possibility of a jump discontinuity of flow area at a cell boundary; therefore, the momentum balance is first written for adjacent half cells. The equations are then combined to get the desired pressure-flow relationship. The following derivation assumes that the flow between cells is friction dominated; therefore, momentum flux and density differences with elevation are neglected.

For each half cell the momentum balance is

$$A_k \frac{\Delta z_k}{2} \frac{\partial (\rho w)_k}{\partial t} + A_k P' - A_k P_k = -K_k (\rho w)_k A_k \frac{\Delta z_k}{2} \quad (5)$$

$$A_{k+1} \frac{\Delta z_{k+1}}{2} \frac{\partial (\rho w)_{k+1}}{\partial t} + A_{k+1} P_{k+1} - A_{k+1} P' = -K_{k+1} (\rho w)_{k+1} A_{k+1} \frac{\Delta z_{k+1}}{2} \quad (6)$$

The common interface pressure between half-cells, P' , can be eliminated by dividing each equation by its respective area and then adding to give

$$\begin{aligned} \frac{\partial}{\partial t} [(\rho w)_k \frac{\Delta z_k}{2} + (\rho w)_{k+1} \frac{\Delta z_{k+1}}{2}] + P_{k+1} - P_k = \\ - [K_k (\rho w)_k \frac{\Delta z_k}{2} + K_{k+1} (\rho w)_{k+1} \frac{\Delta z_{k+1}}{2}] \end{aligned} \quad (7)$$

At the cell interface $(\rho w)_{k-} \neq (\rho w)_{k+}$; however, $(\alpha \rho w)_{k-} = (\alpha \rho w)_{k+}$. Multiplying top and bottom by the appropriate α , the final equation is

$$\frac{\partial (\alpha \rho w)_k}{\partial t} + \frac{P_{k+1} - P_k}{(\Delta z_+)_{k+1}} = -\bar{K}_k (\alpha \rho w)_k \quad (8)$$

$$\text{where } \bar{K}_k = (K_k \Delta z_k / 2 + K_{k+1} \Delta z_{k+1} / 2) / \Delta z_{+k} \quad (9)$$

$$(\Delta z_+)_{k+1} = 1/2 (\Delta z_k / \alpha_k + \Delta z_{k+1} / \alpha_{k+1}). \quad (10)$$

The above difference equation is identical in form to that of a porous media with uniform α . The special weighting of α is due to the jump discontinuity of α between cells. This is most important at the sudden area change at the plenum and of less importance for the gradual area changes along the rod.

2.4 Constitutive Relationships for Closure

Closure of the above equations requires an equation of state and a relationship for K , the flow resistance factor. A perfect gas is assumed for the equation of state; therefore,

$$P = \frac{\bar{R}}{M} \rho T = R_p T \quad (11)$$

where $\bar{R}$ is the universal gas constant and M is the molecular weight of the gas.

Frictional resistance to flow in the annulus is given by

$$\frac{\Delta P}{\Delta z} = - \frac{f}{D_e} \frac{\alpha \rho w^2}{2g_c} \quad (12)$$

assuming laminar flow

$$f = \frac{64}{Re} = \frac{64\mu}{D_e \rho w} \quad (13)$$

therefore,

$$\frac{\Delta P}{\Delta z} = K(\alpha \rho w) \quad (14)$$

where

$$K = \frac{32}{D_e^2} \frac{\mu}{\rho} \quad (15)$$

For the annulus

$$D_{e_k}^2 \equiv (D_{id} - D_{fuel})_k^2 \quad (16)$$

By using the concept of a porous media, K can also be expressed as

$$K = \frac{\mu}{\kappa \rho} \quad (17)$$

where κ is the permeability. For the annulus

$$\kappa = \frac{D_e^2}{32} \quad (18)$$

Other values of κ could be assigned for cracked fuel or other fuel forms. A closed gap can be approximated by limiting κ to a minimum value.

2.5 Solution Procedure

The mass and momentum equations are solved by using a method which has become common in computational fluid mechanics.⁽²⁾ The approach is fully implicit with respect to flow and pressure. The method is explicit with respect to cladding deformation. The method is also a direct calculation and does not require iteration on the flow and pressure field.

The finite difference form of the mass balance [eq. (4)] is written as

$$\frac{(\alpha \rho)_k^{n+1} - (\alpha \rho)_k^n}{\Delta t} + \frac{(\alpha \rho w)_k^{n+1} - (\alpha \rho w)_{k-1}^{n+1}}{\Delta z_k} + S_k^n = 0 \quad (19)$$

where S_k^n is a source term due to cladding deformation. The momentum balance [eq. (8)] is written as

$$\frac{(\alpha \rho w)_k^{n+1} - (\alpha \rho w)_k^n}{\Delta t} = \frac{p_{k+1}^{n+1} - p_k^{n+1}}{\Delta z_k} = - \bar{K}_k (\alpha \rho w)_k^{n+1} \quad (20)$$

where $\bar{K}_k$ is defined by eq. (9).

Based on initial data at time n it is desired to obtain a solution for $\alpha \rho$, $\alpha \rho w$, and P at time $n+1$. Let the true solution be expressed as

$$\alpha\rho^{n+1} = \widetilde{\alpha\rho}^{n+1} + \delta(\alpha\rho) \quad (21)$$

$$\alpha\rho_w^{n+1} = \widetilde{\alpha\rho_w}^{n+1} + \delta(\alpha\rho_w) \quad (22)$$

$$p^{n+1} = \tilde{p} + \delta p \quad (23)$$

where the incremental values $\delta(\alpha\rho)$, $\delta(\alpha\rho_w)$, and δp must be added to the tentative solution. A consistent set of tentative values are $\tilde{p} = p_k^n$, $\tilde{\rho} = \tilde{p}/RT_k^{n+1}$ and $\widetilde{\alpha\rho_w}$ is calculated from the momentum equation by using $\tilde{p}$. The mass balance [eq.(19)] can be written as

$$\frac{\delta(\alpha\rho)_k}{\Delta t} + \frac{\delta(\alpha\rho_w)_k - \delta(\alpha\rho_w)_{k-1}}{\Delta z_k} = -D_k \quad (24)$$

where

$$D_k = S_k^n + \frac{(\widetilde{\alpha\rho})_k^{n+1} - (\alpha\rho)_k^n}{\Delta t} + \frac{(\widetilde{\alpha\rho_w})_k^{n+1} - (\alpha\rho_w)_{k-1}^{n+1}}{\Delta z_k} \quad (25)$$

From the equation of state [eq. (4)]

$$\delta(\alpha\rho)_k = \frac{\alpha_k}{RT_k^{n+1}} \delta p_k \quad (26)$$

and the momentum balance [eq. (20)]

$$\delta(\alpha\rho_w)_k = -\frac{\delta p_{k+1} - \delta p_k}{\left(\frac{1}{\Delta t} + R_k\right) (\Delta z_+)_k} \quad (27)$$

and

$$\delta(\alpha\rho_w)_{k-1} = -\frac{\delta p_k - \delta p_{k-1}}{\left(\frac{1}{\Delta t} + R_k\right) (\Delta z_+)_{k-1}} \quad (28)$$

The latter two equations can be written in the form

$$\delta(\alpha\rho_w)_k = a_k (\delta p_{k+1} - \delta p_k) \quad (29)$$

and

$$\delta(\alpha\rho_w)_{k-1} = c_k (\delta p_k - \delta p_{k-1}). \quad (30)$$

Substituting these into the continuity equation gives an equation of the form

$$a_k \delta p_{k+1} + b_k \delta p_k + c_k \delta p_{k-1} = -D_k \quad (31)$$

where

$$a_k = -1/\left[\left(\frac{1}{\Delta t} + R_k\right) (\Delta z_+)_k\right]$$

$$b_k = a_k + c_k + \frac{1}{\Delta t RT_k^{n+1}} \quad (32)$$

$$c_k = -1/\left[\left(\frac{1}{\Delta t} + R_k\right) (\Delta z_+)_{k-1}\right]$$

and where D_k is defined by equation (25).

This is a three point difference equation and is solved by using a standard tridiagonal matrix algorithm⁽³⁾. The boundary conditions for the problem assume no flow through the ends of the rods; therefore, $w = 0$ at each end. This is satisfied computationally by setting $c_2 = 0$ and $a_{k+1} = 0$.

The no flow boundary conditions cause the tridiagonal matrix to be singular. This is not difficult to overcome since it means that any solution can be obtained to within an additive constant. The pressure distribution, therefore, is calculated and then corrected by a constant to conserve the mass of gas within the rod. The procedure is to first calculate the total initial mass in the rod by

$$m_0 = \sum_{k=2}^{N-1} \rho_{0k} \Delta V_{0k} \quad (33)$$

where $\Delta V_{0k} \equiv A_{0k} \Delta z_{0k}$

After any time step the calculated pressure P may be in error by a constant ΔP . The correct mass is, therefore,

$$\sum_{k=2}^{N-1} \left[\frac{P + \Delta P}{RT} \right]_k \Delta V_k = m_0 \quad (33)$$

where $\Delta V_k = A_k \Delta z_k$

or

$$\Delta P \sum_{k=2}^{N-1} \frac{\Delta V_k}{RT_k} = m_0 - \sum_{k=2}^{N-1} \left(\frac{P}{RT} \right)_k \Delta V_k \quad (34)$$

and the required correction is

$$\Delta P = \frac{m_0 - \sum_{k=2}^{N-1} \left(\frac{P}{RT} \right)_k \Delta V_k}{\sum_{k=2}^{N-1} \frac{\Delta V_k}{RT_k}} \quad (35)$$

Each density must also be corrected. No correction to the velocity is required because the pressure gradient does not change.

In summary, the calculational procedure is as follows:

1. At time $t = 0$ calculate m_0 .
2. For each $t > 0$:
 - 1, calculate $\tilde{\alpha}_p$, $\tilde{\alpha}_{pw}$, D for each k
 - 2, calculate δP for each k
 - 3, calculate P , α_p , α_{pw} for each k
 - 4, correct P and α_p to conserve mass

The above procedure is a direct calculation requiring no iteration. It is stable with regard to pressure and flow; however, it is time step limited by the explicit coupling between pressure and cladding deformation.

3. MODEL ASSESSMENT

The ability of the new model to calculate transient pressure response in a nuclear fuel rod was demonstrated by comparison with INEL experimental data for transient gas flow in irradiated PWR fuel rods⁽⁵⁾. The data from Rod K-4 was arbitrarily selected for comparison with the calculations. Other rods tested had similar results. The experiments were run as a transient depressurization from an initial uniform pressure. A pressure transducer measured the plenum gas pressure during the discharge of gas from the other end of the rod. Rod K-4 was modelled by using the published nominal rod dimension and test conditions. The discharge end of the rod was modelled by using a large plenum at atmospheric

pressure. The rod was divided into ten (10) axial nodes. A time step of 5.0 sec was arbitrarily chosen for the calculation.

The results of the calculation are compared to test data for Rod K-4 in Figure 2. While the pressure decay is slightly faster than the data, the agreement is considered very good, especially since there was no optimization of the code input or model tuning by adjusting the resistance factor. It should be noted that the radial gap is the most significant parameter affecting the pressure decay. The selected gap of 30 μm was the recommended value developed by EG&G for Rod K-4 and has an uncertainty on the order of 30 percent. Gas temperature is also important since it affects the gas viscosity and; therefore, the flow rate. The calculations for 40 time steps required 0.9 seconds of execution time on a CDC 6600 computer.

In summary, the new gas flow model offers an alternate approach for detailed and accurate calculation of gas flow and pressure in nuclear fuel rods.

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NOMENCLATURE

Variables

A	Area
D	Diameter
D_e	Equivalent or hydraulic diameter
D_{id}	Inside cladding diameter
D_{fuel}	Diameter of fuel pellet
K	Flow resistance factor
L	Channel length
m	Mass
M	Molecular weight
N	Number of axial nodes into which the fuel rod is discretized
P	Pressure
R	Gas constant, /M
$\bar{R}$	Universal gas constant
Re	Reynolds number
t	Time
T	Temperature
V	Volume
w	Axial gas flow velocity
z	Distance along rod
α	Gas volume fraction inside cladding
κ	Permeability
μ	Viscosity
ρ	Density

Subscripts

k	Index along rod
o	original initial value

Superscripts

n	Time t
n+1	Time t+ Δt
$\sim$	Indicates tentative solution

Figure 1. Fuel Rod Noding for Gas Pressure and Flow

Figure 2. Calculated and Measured Pressure Decay for INEL Rod K-4