COMPARISON OF MULTIPLE SUPPORT EXCITATION SOLUTION TECHNIQUES FOR PIPING SYSTEMS

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SUMMARY

Design and analysis of nuclear power plant piping systems exposed to a variety of dynamic loads often requires multiple support excitation analysis by modal or direct time integration methods. Both methods have recently been implemented in the computer program KUDROHR for static and dynamic analysis of piping systems, following the previous implementation of the multiple support excitation response spectrum method (see paper K6/15 and K6/15a of the SMIRT-4 Conference).

The extension of the time history analysis to cover multiple support excitations was prompted by discussions with licensing authorities about the validity of the mean value computations like root mean square, sum of the absolutes and superposition formulas in the U.S. Regulatory Guide for displacements, stresses and absolute accelerations, involving results from a multiple support response spectrum analysis. These results can only be examined by carrying out the equivalent time history analysis which do not distort the time phase relationship between the excitations at different support points.

Another point of discussion is multiple vs. single support excitation. A single support excitation analysis is computationally straight forward and tends to be on the conservative side as the numerical results show. On the other hand, a multiple support excitation analysis does not incur much more additional computer cost than the expenditure for an initial static solution involving three times the number, L , of excitation levels, that is, $3L$ static load cases. It gives, however, much more realistic results than a single support excitation analysis can accomplish.

A number of typical nuclear plant piping systems have been analyzed using single and multiple support excitation algorithms for (1) the response spectrum method, (2) the modal time history method via the Wilson, Newmark and Goldberg integration operators and (3) the direct time history method via the Wilson integration operator. Characteristic results are presented to compare the computational quality of all three methods.

1. INTRODUCTION

Support accelerations contribute considerably to the total loading environment of nuclear power plant piping systems. In fact, for most analyses using the response spectrum approach, support accelerations form the only dynamic loading condition to which the mathematical model of the structural system is subjected.

Piping systems of nuclear power plants are attached to the main structural systems at a large number of support points, each of which most likely travels at an acceleration different from the others. Any mathematical tool dealing with piping systems subjected to support motions should therefore allow for individual support point acceleration histories and support point acceleration spectra.

Most of the currently available computer programs deal with support accelerations on the basis of all support points traveling in phase and the same instant acceleration level. In the case of single support response spectrum analyses it has been shown that the results are overly conservative [1] as compared to a multiple support excitation response spectrum analysis. The present paper is intended to expand the multiple support excitation approach to time history analyses.

The computer program KWUROHR [2] for static and dynamic analysis of piping systems has recently been expanded to include multiple support acceleration time history analysis by both modal and direct integration. The corresponding response spectrum analysis with multiple support acceleration spectra has been part of the program for the past two years. Most of the existing program logic dealing with the generation and handling of influence functions of the support points has been applicable without change, for both of the two time history approaches.

While the response spectrum method computes only maximum amplitudes of the response for each mode, and finds the total maximum by superimposing the modal contributions, the time history methods predict the structural response at equal intervals on the time axis. Time history analyses thus offer an opportunity to evaluate the results of response spectrum analyses for accuracy. They can be more expensive in computer time consumption. Time history methods should therefore be used in a production environment only with proper engineering judgement from structural dynamics professionals.

2. EQUATIONS OF MOTION

The theory of multiple support excitation analysis has been outlined in the book by Clough and Penzien [3], and more recently in a paper by Wu et al. [4]. The equation of motion in its most general form is

$$\begin{bmatrix} \frac{M}{0} \\ \frac{0}{M_{gg}} \end{bmatrix} \begin{bmatrix} \ddot{\underline{v}}_t \\ \ddot{\underline{v}}_g(t) \end{bmatrix} + \begin{bmatrix} \underline{C} & \underline{C}_g \\ \underline{C}_g^T & \underline{C}_{gg} \end{bmatrix} \begin{bmatrix} \dot{\underline{v}}_t \\ \dot{\underline{v}}_g(t) \end{bmatrix} + \begin{bmatrix} \underline{K} & \underline{K}_g \\ \underline{K}_g^T & \underline{K}_{gg} \end{bmatrix} \begin{bmatrix} \underline{v}_t \\ \underline{v}_g(t) \end{bmatrix} = \begin{bmatrix} \underline{P}(t) \\ \underline{P}_g \end{bmatrix}. \quad (1)$$

In this equation $\ddot{\underline{v}}_t$, $\dot{\underline{v}}_t$, and $\underline{v}_t$ are the total accelerations, velocities and

displacements, respectively of the unconstrained degrees of freedom, and $\bar{V}_g(t)$, $\bar{V}_g(t)$, and $\bar{V}_g(t)$ are the prescribed time histories at the support points. The vector $\bar{P}(t)$ contains the time histories of the externally applied time-varying nodal forces, and $\bar{P}_g$ is the vector of the support reactions at the dynamically constrained nodes. The system mass, damping and stiffness matrices $\bar{M}$, $\bar{C}$, and $\bar{K}$, respectively, are augmented by the additional matrices corresponding to the dynamically constrained degrees of freedom of the support points.

From eq.(1) the first set of equations can be written as

$$\bar{M} \ddot{\bar{V}}_t + \bar{C} \dot{\bar{V}}_t + \bar{K} \bar{V}_t = \bar{P}(t) - \bar{C}_g \bar{V}_g(t) - \bar{K}_g \bar{V}_g(t) . \quad (2)$$

In eq.(2) all prescribed quantities are collected on the right-hand side.

The support acceleration time history of the k-th support level, acting in the global direction i, is given by $\bar{v}_g^{ki}(t)$. The vector $\bar{V}_g(t)$ is therefore composed as

$$\bar{V}_g(t) = [\bar{v}_g^{1,1}(t), \bar{v}_g^{1,2}(t), \bar{v}_g^{1,3}(t), \dots, \bar{v}_g^{k,i}(t), \dots]^T \quad (3)$$

$3n \times 1$

$i=1,2,3; k=1,\dots,n$

where n is the total number of support levels. A unit displacement in the global direction i at the k-th support level, with all other support degrees of freedom fixed, results in a displacement of the entire system. This displacement field is called an influence function, and will subsequently be denoted by η^{ki} . All 3n of these influence function vectors are combined into one matrix,

$$\eta = [\eta^{1,1}, \eta^{1,2}, \eta^{1,3}, \dots, \eta^{k,i}, \dots, \eta^{n,3}] , \quad (4)$$

$NEQ \times 3n$

where NEQ is the total number unconstrained degrees of freedom of the system. The combined influence of the support accelerations is

$$\ddot{\bar{V}}_s = \eta \ddot{\bar{V}}_g . \quad (5)$$

The computation of the matrix η is accomplished by applying 3n artificial load vectors, $\bar{p}_\eta^{ki}$, to the structural system. They are combined into one matrix,

$$\bar{p}_\eta = [\bar{p}_\eta^{1,1}, \bar{p}_\eta^{1,2}, \bar{p}_\eta^{1,3}, \dots, \bar{p}_\eta^{k,i}, \dots, \bar{p}_\eta^{n,3}] . \quad (6)$$

$NEQ \times 3n$

The load vectors are computed by converting the unit displacements into forces via the support springs,

$$\bar{p}_\eta = - \bar{K}_g \bar{I} , \quad (7)$$

where $\bar{I}$ is a $3n \times 3n$ unit matrix. The matrix of influence functions, η , is then the result of a static analysis

$$\bar{K} \eta = \bar{p}_\eta = - \bar{K}_g \bar{I} . \quad (8)$$

From the total accelerations, velocities and displacements the relative quantities, $\ddot{\bar{V}}$, $\dot{\bar{V}}$, and $\bar{V}$, respectively, are obtained by subtracting the combined influence $\ddot{\bar{V}}_s$, $\dot{\bar{V}}_s$, and $\bar{V}_s$ of all the support point motions

$$\ddot{\underline{V}} = \ddot{\underline{V}}_t - \ddot{\underline{V}}_s ; \dot{\underline{V}} = \dot{\underline{V}}_t - \dot{\underline{V}}_s ; \underline{V} = \underline{V}_t - \underline{V}_s , \quad (9)$$

where the vectors indexed with "s" are obtained from those indexed with "t" by means of the transformation of eq.(5). Substituting eqs.(9) into eq.(2), using the transformation relation of eq.(5), and collecting all terms involving prescribed quantities on the right-hand side, the equation of motion is rearranged as

$$\underline{M} \ddot{\underline{V}} + \underline{C} \dot{\underline{V}} + \underline{K} \underline{V} = \underline{\overline{P}}(t) - \underline{M} \ddot{\underline{V}}_g(t) - (\underline{C}_g + \underline{C} \eta) \dot{\underline{V}}_g(t) . \quad (10)$$

The velocity-dependent term is subsequently dropped from the right-hand side of eq.(10), because (a) the damping is assumed to be proportional to the system stiffness and mass matrices,

$$\underline{C} = \alpha \underline{M} + \beta \underline{K} , \quad (11)$$

with α and β being scalar quantities, and (b) the coupled support damping $\underline{C}_g$ is considered negligible. The simplified equation of motion in its final form is then

$$\underline{M} \ddot{\underline{V}} + \underline{C} \dot{\underline{V}} + \underline{K} \underline{V} = \underline{\overline{P}}(t) - \underline{M} \ddot{\underline{V}}_g(t) . \quad (12)$$

This equation will be solved by three different techniques: the modal response spectrum method, and both the modal and the direct time integration methods.

3. MODAL DECOMPOSITION

In modal analysis the first m modes $\phi_1, \phi_2, \dots, \phi_j, \dots, \phi_m$ and frequencies $\omega_1, \omega_2, \dots, \omega_j, \dots, \omega_m$, written in matrix form as

$$\underline{\Phi} = [\phi_1, \phi_2, \dots, \phi_j, \dots, \phi_m] ; \quad \text{NEQxm} \quad (13)$$

$$\underline{\Omega}^2 = \begin{bmatrix} \omega_1 & & & \\ & \omega_2 & & \\ & & \ddots & \\ & & & \omega_j & \\ & & & & \ddots \\ & & & & & \omega_m \end{bmatrix} , \quad \text{m \times m}$$

are computed for the eigenproblem of undamped free vibration,

$$\underline{M} \ddot{\underline{V}} + \underline{K} \underline{V} = \underline{0} . \quad (14)$$

The modes and frequencies are used to transform the equation of motion, eq.(12), into m individual, uncoupled equations, via the transformation

$$\underline{V} = \underline{\Phi} \underline{Y} . \quad (15)$$

With the above transformation the generalized mass, damping and stiffness matrices are given by

$$\underline{M}^* = \underline{\Phi}^T \underline{M} \underline{\Phi} = \underline{I} ; \underline{C}^* = \underline{\Phi}^T \underline{C} \underline{\Phi} = \underline{\Delta} ; \underline{K}^* = \underline{\Phi}^T \underline{K} \underline{\Phi} = \underline{\Omega}^2 , \quad (16)$$

where

$$\underline{\Delta} = \text{diag} [2 \xi_j \omega_j] , \quad (17)$$

$$\underline{\Omega}^2 = \text{diag} [\omega_j^2] ,$$

and ξ_j the j -th modal damping ratio. Correspondingly the generalized (modal) force vector is

$$\underline{P}^*(t) = \underline{\Phi}^T \underline{\bar{P}}(t), \quad (18)$$

and the diagonal matrix of modal participation factors is

$$\underline{L} = \underline{\Phi}^T \underline{M} \underline{\eta}. \quad (19)$$

With the above abbreviations the uncoupled modal equations of motion are, in matrix form,

$$\ddot{\underline{Y}} + \underline{\Delta} \dot{\underline{Y}} + \underline{\Omega}^2 \underline{Y} = \underline{P}^*(t) - \underline{L} \underline{\bar{V}}_g(t). \quad (20)$$

4. MODAL TIME HISTORY ANALYSIS

In a modal time history analysis, eq.(20) is solved by applying a one-step time integration scheme to solve the modal response $Y_j(t)$ for each of the decoupled equations

$$\ddot{Y}_j + 2 \xi_j \omega_j \dot{Y}_j + \omega_j^2 Y_j = P_j^*(t) - \sum_k \sum_i L_j^{ki} \bar{v}_g^{ki}(t). \quad (21)$$

In the present analysis three different time integration operators are being employed, which were originated by (a) Wilson [5], (b) Newmark [6] and (c) Goldberg [7]. The one-step algorithms are summarized in matrix form as

$$\begin{bmatrix} \ddot{Y}_j \\ \dot{Y}_j \\ Y_j \end{bmatrix} (t+\Delta t) = \underline{A}_j \begin{bmatrix} \ddot{Y}_j \\ \dot{Y}_j \\ Y_j \end{bmatrix} (t) + \underline{B}_j \begin{bmatrix} \hat{P}_j^*(t+t_a) \\ \hat{P}_j^*(t+t_b) \end{bmatrix}, \quad (22)$$

where Δt is the solution time step, and the effective load is given by

$$\hat{P}_j^*(t) = \underline{P}_j^*(t) - \sum_k \sum_i L_j^{ki} \bar{v}_g^{ki}(t). \quad (23)$$

The quantities $t+t_a$, $t+t_b$ are the points on the time axis at which the effective load $\hat{P}_j^*$ is sampled. In the Wilson method this sampling is done only at one time point per interval, $t+1.4\Delta t$; in the Newmark method at $t+\Delta t$. The Goldberg method uses two sampling points, either $t_a=0.5\Delta t$ and $t_b=\Delta t$, or the Gauss-integration points $t_a=0.211... \Delta t$ and $t_b=0.788... \Delta t$. The Goldberg integration operator is given in the Appendix for completeness, the other two can be found in the book by Bathe and Wilson [8].

The relative displacements are obtained from the modal displacement time histories by superposition, following eq.(15),

$$\underline{V}(t) = \sum_j \underline{\Phi}_j Y_j(t). \quad (24)$$

While relative displacements are sufficient for a stress analysis, following the argument of Wu et al. [2], it is the total accelerations that are required for analysis of dynamic effects. These follow from eq.(9)

$$\ddot{\underline{V}}_t(t) = \sum_i \underline{\Phi}_i \ddot{Y}_i(t) + \underline{\eta} \ddot{\underline{V}}_g(t). \quad (25)$$

5. MODAL RESPONSE SPECTRUM ANALYSIS

The solution of eq.(12) in the absence of an external load $\underline{\bar{P}}(t)$ by the response-spectrum method [9] was presented ref.[1]. Eq.(12) is rewritten as

$$\ddot{Y}_j^{ki} + 2 \xi_j \omega_j \dot{Y}_j^{ki} + \omega_j^2 Y_j^{ki} = - L_j^{ki} \bar{v}_g^{ki}(t). \quad (26)$$

The maximum displacement of each of these $3mn$ uncoupled equations (three global directions i , m modes j and n support levels k) is

$$Y_{j\max}^{ki} = \text{abs} | Y_{j\max}^{ki} | = \left| \frac{L_j^{ki}}{\omega_j^2} S_a^{ki}(\xi_j, \omega_j) \right| \quad (27)$$

The value of $S_a^{ki}(\xi_j, \omega_j)$ is the acceleration taken from the spectrum table for a modal damping ratio ξ_j at the frequency ω_j , applied at support level k and global direction i . The total modal displacement response of the j -th mode is

$$Y_{j\max} = \left\{ \sum_k \sum_i (Y_{j\max}^{ki})^2 \right\}^{\frac{1}{2}} \quad (28)$$

The final result for each displacement component v and stress resultant s is obtained by individually summing the corresponding modal quantities, like for example using the root mean square summation rule

$$s_{\max} = \left\{ \sum_j (s_{j\max})^2 \right\}^{\frac{1}{2}} \quad (29)$$

or any other superposition law called for by the Regulatory Guide [10].

The computation of the maximum relative modal accelerations are computed from

$$\ddot{Y}_{j\max}^{ki} = | -\omega_j^2 Y_{j\max}^{ki} | = \left| -L_j^{ki} S_a^{ki}(\xi_j, \omega_j) \right| \quad (30)$$

Again as in the time history analysis, the maximum absolute accelerations are of interest. Following the argument of Marguerre et al. [11] the absolute accelerations are computed from two parts. The first part is the maximum relative acceleration

$$\ddot{Y}_{\max} = \left\{ \sum_j (\phi_j \ddot{Y}_{j\max})^2 \right\}^{\frac{1}{2}} \quad (31)$$

where the relative modal accelerations $\ddot{Y}_{j\max}$ are formed by superposition from the $Y_{j\max}^{ki}$ according to eq.(28). The second part is the so-called rigid body contribution,

$$\ddot{Y}_{\text{rigid}} = \sum_k \sum_i [\eta^{ki} - \sum_j (\phi_j L_j^{ki})] S_a^{ki}(\xi_r, \omega_r) \quad (32)$$

in which $S_a^{ki}(\xi_r, \omega_r)$ is the value taken from the acceleration spectrum for a modal damping ratio ξ_r at the frequency value ω_r at which the spectrum curve for support level k and global direction i flattens out. The maximum total acceleration is then

$$\ddot{Y}_{\text{tmax}} = \left\{ (\ddot{Y}_{\max})^2 + (\ddot{Y}_{\text{rigid}})^2 \right\}^{\frac{1}{2}} \quad (33)$$

The computations of eq.(33) are understood to be carried out separately for each component of the acceleration vectors.

6. DIRECT TIME HISTORY ANALYSIS

In the direct integration of the equations of motion, eq.(12), the well-known Wilson time integration scheme [5] is being used. The effective load vector $\underline{P}(t)$ is given by

$$\hat{\underline{P}}(t) = \underline{P}(t) - \underline{M} \underline{\eta} \ddot{\underline{V}}_g(t) \quad (34)$$

The analysis for multiple excitation differs from the one for single support excitation only by the matrix $\underline{\eta}$, as is true for the other two analysis techniques discussed previously.

7. NUMERICAL RESULTS

7.1 VERIFICATION OF ONE-STEP OPERATORS

Two single degree of freedom systems were analyzed to compare the one-step operators used in the modal time history analysis. The first system has the following properties: $m=1$, $f=0.318309$ cps, $\dot{v}(0)=2$ and $\Delta t=0.09817477s$, no load. The second system is the one of ref. [3], p.104, with $m=3$, $T=0.209s$ and a triangular load of duration $t_g=0.050s$ with a peak $P(0.025)=96.6K$. The time step is $\Delta t=0.005s$. Both problems were analyzed with no damping and with 5% critical damping. The results are summarized in Table 1.

Table 1 - Verification of One-Step-Operators

Method		Problem 1		Problem 2	
		0 %	5 %	0 %	5 %
		$v(4.5T)$	$\ln \frac{v(.5T)}{v(1.5T)}$	$v(.05)$	$(.05)$
Wilson	$\theta = 1.4$	.99219	.3115	.01680	.01610
Newmark	$\delta = 1/2$ $\alpha = 1/4$	.99716	.3117	.01736	.01664
Goldberg	$(.5\Delta t \text{ and } \Delta t)$	.99999	.3142	.01745	.01673
Goldberg	(Gauss Pts.)	.99999	.3143	.01745	.01673
Analytical Solution		1.000	.3146	.0176	.0169

7.2 ANALYSIS OF TWO PIPING SYSTEMS

Two piping systems were analyzed by multiple and equivalent single support excitations. The systems are shown on Fig.1,2 and 3, and a set of input spectra is presented for system 2 on Fig.4. The results of the analysis are shown in Table 2.

Table 2 - Results of Piping Analysis

Method (S=Single, M=Multiple Support)		System 1		System 2	
		NY Node 6	MY Node 1	MT Node 5	MZ Node 21
Response-Spectrum-Method	S	33926705	345157151	235359245338	2703677909
	M	21872204	224928906	1072385508740	141170264
Modal Time History Method					
Wilson	S	1432	208200	19550000	32280000
	M	1704	113900	19050000	30380000
Newmark	S	1478	209900	-	-
	M	1698	115300	-	-
Goldberg(.5 Δt and Δt)	S	1495	210700	-	-
	M	1730	116500	-	-
Goldberg(Gauss Pts.)	S	1501	210800	19330000	32140000
	M	1734	116600	18910000	30800000
Direct Time History Method	S	4295	633900	53570000	101700000
	M	1700	113600	18120000	29750000

Note: At time of writing the results were not complete. The complete results will be given at the conference.

8. CONCLUSIONS

The multiple support excitation analysis of piping systems offers an increase in accuracy at a small increase in computational costs. The extension of the method from response spectrum to time history analysis has been accomplished by using the same algorithms for all methods. In modal time history analysis the Goldberg-Method appears to be most promising.

9. REFERENCES

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10. APPENDIX: ANALYSIS OF A SINGLE-DEGREE-OF-FREEDOM-SYSTEM BY THE GOLDBERG-METHOD

Figure A-1 Goldberg Time-Integration Scheme

$$\underline{A} = \begin{bmatrix} \tau_a(m + \frac{1}{2}c t_a + \frac{1}{6}k t_a^2) & \tau_a^2(m + \frac{1}{3}c t_a + \frac{1}{12}k t_a^2) \\ \tau_b(m + \frac{1}{2}c t_b + \frac{1}{6}k t_b^2) & \tau_b^2(m + \frac{1}{3}c t_b + \frac{1}{12}k t_b^2) \end{bmatrix}_{2 \times 2}$$

$$\underline{B} = \begin{bmatrix} (m + c t_a + \frac{1}{2}k t_a^2) & (c + k t_a) & k \\ (m + c t_b + \frac{1}{2}k t_b^2) & (c + k t_b) & k \end{bmatrix}_{2 \times 3}$$

$$\underline{C} = \begin{bmatrix} 1 & 1 \\ \frac{1}{2}\Delta t & \frac{1}{3}\Delta t \\ \frac{1}{6}\Delta t^2 & \frac{1}{12}\Delta t^2 \end{bmatrix}_{3 \times 2} \quad \underline{D} = \begin{bmatrix} 1 & 0 & 0 \\ \Delta t & 1 & 0 \\ \frac{1}{2}\Delta t^2 & \Delta t & 1 \end{bmatrix}_{3 \times 3}$$

$$\underline{G} = \underline{D} - \underline{C} \underline{A}^{-1} \underline{B} \quad \underline{H} = \underline{C} \underline{A}^{-1}$$

$$\begin{bmatrix} \ddot{Y}_{t+\Delta t} \\ \dot{Y}_{t+\Delta t} \\ Y_{t+\Delta t} \end{bmatrix} = \underline{G} \begin{bmatrix} \ddot{Y}_t \\ \dot{Y}_t \\ Y_t \end{bmatrix} + \underline{H} \begin{bmatrix} p_a \\ p_b \end{bmatrix}$$

Fig. 2 - Example 2, Feedwater Line

Fig. 1 - Example 1, Structural System (A) and Mathematical Model (B)

Fig. 4 - Example 2, Input Spectra

Fig. 3 - Example 2, Mathematical Model