EQUIPMENT RESPONSE SPECTRA FOR NUCLEAR POWER PLANT SYSTEMS

J. L. SACKMAN, J. M. KELLY

*University of California, Department of Civil Engineering,
733 Davis Hall, Berkeley, California 94720, U.S.A.*

ABSTRACT

An analytical method is developed whereby a simple estimate can be obtained of the maximum dynamic response of light equipment attached to a structure subjected to ground motion. The natural frequency of the equipment, modeled as a single-degree-of-freedom system, is considered to be close or equal to one of the natural frequencies of the N -degree-of-freedom structure. This estimate provides a convenient, rational basis for the structural design of the equipment and its installation.

The approach is based on the transient analysis of lightly damped tuned or slightly nontuned equipment-structure systems in which the mass of the equipment is much smaller than that of the structure. It is assumed that the information available to the designer is a design spectrum for the ground motion, fixed-base modal properties of the structure, and fixed-base properties of the equipment. The results obtained are simple estimates of the maximum acceleration and displacement of the equipment. The method can also be used to treat closely spaced modes in structural systems, where the square root of the sum of the squares procedure is known to be invalid.

This analytical method is also applied to nontuned equipment-structure systems for which the conventional floor spectrum method is mathematically valid. A closed-form solution is obtained which permits an estimate of the maximum equipment response to be obtained without the necessity of computing time histories as is required by the floor spectrum method.

1. INTRODUCTION

In this paper we present a response spectrum approach to the design of lightly damped relatively light equipment in nuclear plants subjected to seismic loading. The analysis uses as a model an N-degree-of-freedom structure to which is attached a single-degree-of-freedom component. It is known that significant interaction effects occur when the equipment frequency is close or equal to one of the natural frequencies of the structure, referred to as tuning. Otherwise, the conventional floor spectrum method which neglects interaction is valid.

In previous research the undamped equipment-structure problem for tuned systems [1] and slightly nontuned systems [2] was studied and it was shown that a design ground response spectrum together with fixed-base dynamic properties of the structure alone and of the equipment alone can be used to estimate the peak response of the equipment. The analysis took advantage of the mathematical structure of the equations and of asymptotic methods made possible by the smallness of the equipment mass in comparison with the mass of the structure to obtain simple results valid for tuned and nearly tuned systems. In this paper we extend the results to damped tuned, nearly tuned, and completely nontuned systems. For light equipment and small damping, the results obtained can be easily and efficiently implemented by a designer. The most important aspect of the analysis is its extreme simplicity; namely, if the response spectrum for the ground motion is available, the response spectrum for the equipment can be calculated merely by multiplying the former by an amplification factor.

This approach is in contrast to several earlier analyses of equipment response, such as the floor spectrum method in which the equipment is treated as a single-degree-of-freedom system subjected to a base motion that is taken to be that which the structure would experience at the attachment point in the absence of equipment. Not only does this method neglect interaction, it has the further disadvantage of requiring that an expensive time history analysis of the structure be conducted in order to determine a base motion [3]. Approximate techniques that bypass associated computational problems have been proposed whereby floor response spectra are developed from ground spectra, but these methods are ad hoc and their accuracy cannot be evaluated [4-10].

It is also possible to account for interaction by considering the system to be an N+1-degree-of-freedom model [11] which is then treated by a modal approach and spectral methods or by direct time history analysis using an ensemble of spectrum-consistent earthquake records. Neither approach is, however, ideal. If equipment-structure interaction is important, two closely spaced modes will appear, the contributions of which must be summed when a modal approach is used, but there exists no consensus as to an appropriate summation procedure. A disadvantage of the second approach -- time history analysis -- is that it is very expensive and for a given item of equipment must be carried out for a wide range of earthquake motion. If the equipment is light, the use of the N+1-degree-of-freedom model with standard structural dynamics computer codes may mask significant response.

The method developed here allows the design engineer to use directly the given ground motion design spectrum to determine the response of the equipment. The expense of time history analyses and the problems of closely spaced modes are thus avoided.

2. ANALYSIS

The equations of motion of the general N-degree-of-freedom structure to which the single-degree-of-freedom oscillator representing the equipment is attached, as shown in Fig. 1, are reduced by standard techniques to modal form (in terms of the N natural modes of the structure alone), and solved by the use of Laplace transforms, and combined with the transformed equation governing the motion of the equipment. The Laplace transform of the equipment acceleration $\ddot{u}$ can then be expressed in terms of the transformed ground acceleration $\ddot{u}_g$ by

$$\ddot{u} = \frac{\sum_{k=1}^N \frac{\Phi_i^k \sum_{l=1}^N \Phi_l^k \sum_{j=1}^N M_{ij} R_l (2B_k \Omega_k p + \Omega_k^2) (2\beta \omega p + \omega^2)}{M_k (p^2 + 2B_k \Omega_k p + \Omega_k^2)}}{(p^2 + 2\beta \omega p + \omega^2) + p^2 \sum_{k=1}^N \frac{m \Phi_i^{k2} (2\beta \omega p + \omega^2)}{M_k (p^2 + 2B_k \Omega_k p + \Omega_k^2)}} \ddot{u}_g \quad (1)$$

where M_{ij} is the structural mass matrix, Ω_k , Φ_i^k are the k^{th} natural frequency and mode shape of the structure, $M_k = \sum_{i=1}^N \sum_{j=1}^N \Phi_i^k \Phi_j^k M_{ij}$ is the generalized mass of the k^{th} mode, B_k is the fraction of critical damping in the k^{th} mode, m , ω , β are the mass, natural frequency, and damping factor for the equipment, and R_i is a vector of influence coefficients that couple the ground motion to the structure. The solution of eq. (1) will be obtained by residue theory. The poles of the transfer function relating $\ddot{u}$ to $\ddot{u}_g$ are simple poles and determined by the zeroes of the denominator of the expression in eq. (1). Two different situations arise, one when the equipment is tuned or nearly tuned and the other when the natural frequency of the equipment is well away from any of the structural frequencies (i.e., is grossly nontuned). In both cases the poles, for light equipment, appear near the natural frequencies of the structure alone and of the equipment alone. For tuned or nearly tuned equipment, two closely spaced poles (tuning poles) occur, located near the equipment frequency and the frequency of the structure to which the equipment is nearly tuned, one below those frequencies and one above them. For a perfectly tuned system these two poles coalesce into a double pole as the equipment mass approaches zero. This gives rise to a response in the time domain which grows without bound proportional to time regardless of the time history of the input. Thus the contribution to the sum of the residues at all poles is dominated by the residues at the tuning poles.

The contribution to the response of the nontuned poles is quite standard. The contribution of the tuned poles is different and must be obtained by a special approach. Asymptotic methods that take advantage of the relative lightness of the equipment and the smallness of the damping factors have been used in references 1, 2, and 12 to obtain rational forms of this result. The complete solution for the response of the equipment in the tuned or nearly tuned cases takes, to the order of the approximation, the form

$$\ddot{u}(t) = \int_0^t \ddot{u}_g(\tau) \left\{ \sum_{\substack{m=1 \\ m \neq n}}^N \frac{C_r^m}{1 - (\Omega_m/\omega)^2} \Omega_m e^{-B_m \Omega_m (t-\tau)} \sin \Omega_m (t-\tau) + \sum_{\substack{m=1 \\ m \neq n}}^N \frac{C_r^m}{1 - (\omega/\Omega_m)^2} \omega e^{-\beta \omega (t-\tau)} \sin \omega (t-\tau) + C_r^n \ddot{u}_g(t-\tau) \right\} d\tau \quad (2)$$

where r is the structural degree of freedom to which the equipment is attached, n is the structural mode to which the equipment is tuned, and $C_r^n = \frac{\phi_r^n \sum_{i=1}^N \sum_{j=1}^N \phi_{i,j}^n M_{i,j} R_j}{M_n}$ is an effective participation factor for the n^{th} mode at the r^{th} degree of freedom. In the above $\ddot{u}_G$ is the contribution to the Green's function for the equipment acceleration provided by the residues at the tuning poles. This takes the general form to dominant terms

$$\begin{aligned} \ddot{u}_G(t) = & \frac{\omega}{\lambda^2 + \mu^2} e^{-(\beta+B)\omega t/2} \left[\lambda \sinh \frac{\mu}{2} \omega t \cos \frac{\lambda}{2} \omega t \sin \left(1 + \frac{\xi}{2}\right) \omega t \right. \\ & - \lambda \cosh \frac{\mu}{2} \omega t \sin \frac{\lambda}{2} \omega t \cos \left(1 + \frac{\xi}{2}\right) \omega t \\ & - \mu \sinh \frac{\mu}{2} \omega t \cos \frac{\lambda}{2} \omega t \cos \left(1 + \frac{\xi}{2}\right) \omega t \\ & \left. - \mu \cosh \frac{\mu}{2} \omega t \sin \frac{\lambda}{2} \omega t \sin \left(1 + \frac{\xi}{2}\right) \omega t \right] \end{aligned} \quad (3)$$

where

$$\begin{aligned} \lambda = & \frac{1}{\sqrt{2}} \left\{ \left[(\gamma + \xi^2 - (\beta - B_n)^2)^2 + 4\xi^2(\beta - B_n)^2 \right]^{1/2} + [\gamma + \xi^2 - (\beta - B_n)^2] \right\}^{1/2} \\ \mu = & \frac{1}{\sqrt{2}} \left\{ \left[(\gamma + \xi^2 - (\beta - B_n)^2)^2 + 4\xi^2(\beta - B_n)^2 \right]^{1/2} - [\gamma + \xi^2 - (\beta - B_n)^2] \right\}^{1/2} \end{aligned}$$

and $\gamma^{\text{eff}} = m/M_n / (\phi_r^n)^2$ is the modal mass ratio; $\xi = (\Omega_n - \omega)/\omega$ is the detuning parameter.

For the grossly nontuned case, standard analysis leads to a solution of the form

$$\begin{aligned} \ddot{u}(t) = & \int_0^t \ddot{u}_G(\tau) \left\{ \sum_{m=1}^N \frac{C_r^m}{1 - (\Omega_m/\omega)^2} \Omega_m e^{-B_m \Omega_m (t-\tau)} \sin \Omega_m (t-\tau) \right. \\ & \left. + \sum_{m=1}^N \frac{C_r^m}{1 - (\omega/\Omega_m)^2} \omega e^{-\beta \omega (t-\tau)} \sin \omega (t-\tau) \right\} d\tau \end{aligned} \quad (4)$$

We note that this equation is independent of the modal mass ratio, representing the general closed-form solution of the floor spectrum method.

3. APPROXIMATE RESPONSE SPECTRA RESULTS

The results given in the previous section can be applied to the design of equipment and equipment mounting. Typically the information available to the designer is a design spectrum. It was shown in references 1 and 2 that for the case of the undamped system, both tuned and nearly tuned, the peak acceleration in the equipment could be related to the response spectrum of the ground motion. The present paper is concerned with the generalization of these results to the damped tuned or nontuned cases.

For the grossly nontuned system where the equipment frequency is far away from all structural frequencies and these are well separated, conventional summation methods, e.g., the square root of the sum of the squares procedure applied to eq. (4), lead to an estimate for $|\ddot{u}|_{\text{max}}$ in the form

$$|\ddot{u}|_{\text{max}} = \left\{ \sum_{m=1}^N \left[\frac{C_r^m}{1 - (\Omega_m/\omega)^2} S_A(\Omega_m, B_m) \right]^2 + \left[\sum_{m=1}^N \frac{C_r^m}{1 - (\omega/\Omega_m)^2} \right]^2 S_i^2(\omega, \beta) \right\}^{1/2} \quad (5)$$

where $S_A(\Omega, B)$ is the acceleration design spectrum for the ground motion $\ddot{u}_g$ evaluated at frequency Ω and damping factor B . If maximum displacements are required, then S_D , the displacement design spectrum, may be substituted.

In the case of a tuned or nearly tuned system, the contributions to the peak accelera-

tion of the equipment from the nontuned modes and of the same order from the tuning poles is similarly obtained. Equation (2) is used in conjunction with the square root of the sum of the squares procedure; the appropriate estimate of the contributions to the maximum acceleration $|\ddot{u}|_{\max}$ is then given by

$$|\ddot{u}|_{\max} = \left\{ \sum_{\substack{m=1 \\ m \neq n}}^N \left[\frac{C_m^n}{1 - (\Omega_m/\omega)^2} S_A(\Omega_m, B_m) \right]^2 + \left[\sum_{\substack{m=1 \\ m \neq n}}^N \frac{C_m^n}{1 - (\omega/\Omega_m)^2} \right]^2 S_A^2(\omega, \beta) \right\}^{1/2} \quad (6)$$

when the equipment is tuned to the n^{th} structural frequency. The contribution to the maximum acceleration from the tuning poles is special and must be treated separately. The procedure is described in detail in reference 12. The approach is based on the recognition that the response predicted by eq. (3) involves a classic beat phenomenon. In some cases the beats are underdamped as shown in Fig. 2, but may be overdamped or critically damped. In each case the method used is to locate the peak value of the envelope curve. For light tuned equipment this peak occurs late, after many cycles of motion at the equipment frequency. This allows simple approximations to be made which result in an estimate of the tuning contribution to $|\ddot{u}|_{\max}$ in the form

$$|\ddot{u}|_{\max} = \frac{|C_n^n| e^{-\kappa}}{(\gamma + \xi^2 + 4\beta B_n)^{1/2}} S_A\left(\frac{\omega + \Omega_n}{2}, 0\right) \quad (7)$$

where

$$\kappa = (\arctan \xi)/\xi \quad \xi = [\gamma + \xi^2 - (\beta - B_n)^2]^{1/2}/(\beta + B_n) \quad (8)$$

The peak value given by eq. (6) occurs early after only a few cycles of equipment motion and that given by eq. (7) occurs late and thus in principle they should not be taken together. However, we have recently completed extensive numerical and physical experimentation the results of which suggest ways in which the results can be conveniently applied to equipment mounting design. The physical experiments were carried out at the Earthquake Simulator Laboratory of the Earthquake Engineering Research Center, University of California, Berkeley, and involved the use of a three-story steel frame model structure around one-third full scale. The response of the steel frame model is roughly that of a shear building with natural frequencies at around 2 Hz, 8 Hz, and 15 Hz. The total weight of the model was 39,500 lbs and its height was about 20 feet. Three single-degree-of-freedom oscillators were attached to the concrete blocks at the second and third floors to simulate equipment in a primary structure. The test structure was instrumented to measure displacements and accelerations at each floor and acceleration of the oscillators. The mechanical oscillators, each weighing about 20 lbs, were constructed to correspond to the first three natural frequencies of the model structure.

The first three natural frequencies of the frame were determined accurately and the oscillators were tuned to the structural frequencies. They were then bolted to the floors of the model structure. The Pacoima Dam (1971) and Taft (1950) earthquake signals as well as time-scaled Pacoima Dam, Taft, and El Centro (1940) signals and some approximate square waves were used as input to the shaking table. Figure 5 shows the time history of acceleration in a typical run. The top trace shows the input table acceleration, the second the response of the third floor of the structure, and the next three the accelerations of the three tuned

oscillators. The very large magnification of accelerations experienced by the oscillators is immediately obvious. The beat phenomenon is clear. The peak acceleration in the first two oscillators is achieved considerably after the peak acceleration in the input. These two oscillators are obviously responding at the coupled frequencies governed by equipment-structure interaction.

The response of the third oscillator is quite different; a highly irregular pattern appears and local maxima occur during as well as after excitation. Fourier transforms of the acceleration time histories of the three oscillators have been taken to clarify this response (Fig. 4). It is clear from these transforms that the first mode oscillator responded only at the first mode frequency, around 2 Hz; the second mode oscillator responded predominantly at the second mode frequency, around 8 Hz, with a small contribution from the first mode. The dominant contribution to the response of the third mode oscillator was from frequencies around the third mode frequency of 15 Hz, but significant contributions also appeared from the lower modes. Thus the pure damped beat response predicted by the theory was obscured by the contributions from lower modes.

These observations are supported by the numerical experiments (to be described in detail elsewhere) and have implications for the application of the theoretical results to design. First, only the structural modes up to and around that of the equipment under consideration need be considered and, second, the late peak acceleration given by eq. (7) should be summed with the early peak acceleration, eq. (6), by an appropriate summation rule. For instance, if the conventional square root of the sum of the squares procedure is used, the estimate is

$$\begin{aligned} |\ddot{u}|_{\max} = & \left\{ \sum_{m=1}^N \left[\frac{C_i^m}{1 - (\Omega_m/\omega)^2} S_i(\Omega_m, B_m) \right]^2 + \left[\sum_{m=1}^N \frac{C_i^m}{1 - (\omega/\Omega_m)^2} \right]^2 S_i^2(\omega, \beta) \right. \\ & \left. + \left[\frac{|C_i^N| e^{-\gamma}}{(\gamma^2 + 4\beta B_N)^2} S_i \left[\frac{\omega + \Omega_N}{2}, 0 \right] \right]^2 \right\}^{1/2} \end{aligned} \quad (9)$$

If the late peak occurs well after the peak of the excitation (as in the first mode oscillator) it will be the dominant term and the result will be nearly the same as if it alone were considered. If on the other hand the late peak occurs during the excitation (as in the third mode oscillator) then it should be superposed with the other modal contributions that may occur around the same time and this will be effected by the formula.

The advantages of this approach are its simplicity and adaptability for practical application. A great deal of computational effort is avoided since time history analyses need not be performed. The equipment and structure need not be analyzed as an N+1-degree-of-freedom system either by modal or matrix-time-marching methods, and errors in estimates of peak response due to the possible unreliability of numerical time integration schemes, or to uncertainty as to the appropriate procedure for summing the contributions of the two closely spaced modes, are thereby avoided. For tuned and nearly tuned systems the method accounts for the important effect of equipment-structure interaction, which effect is completely neglected in the floor spectrum method. The method advanced here does not require that new information be generated. Data available from the building design alone (M_{ij} , R_j , Ω_m , ϕ_i^m , B_m),

the equipment alone (m, ω, β), and the seismic response spectra (S_D or S_V or S_A) are used. The estimates of peak response have been obtained by rational analysis and are easily evaluated and conveniently used during the design process.

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FIGURE 1 EQUIPMENT-STRUCTURE SYSTEM

FIGURE 2 EQUIPMENT RESPONSE HISTORY IN THE CASE OF DAMPED BEATS

FIGURE 3 TIME HISTORY OF ACCELERATION FOR SHAKING TABLE, THIRD FLOOR, AND OSCILLATORS

FIGURE 4 FOURIER TRANSFORMS OF OSCILLATOR TIME HISTORY ACCELERATIONS