**APPLICATIONS OF THE FUNDAMENTAL SOLUTION FOR
A THERMAL SHOCK ON A FINITE ORTHOTROPIC
CYLINDRICAL THIN SHELL**

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Abstract

The temperature variations in a thin cylindrical shell of graphite materials with finite length caused by an instantaneous thermal shock are investigated in this paper. Solutions for a line source and an area source of thermal shock are obtained. A quasi-linear theory for heat transfer is assumed. Grades ATJ and ZTA graphite are used in the numerical examples. As is expected, the orthotropically thermal properties significantly affect the temperature variations in the shell which are caused by the thermal shocks.

I. Introduction

Technological advances in aerospace and nuclear engineering in the past decades have made thermoelectricity a major branch of engineering practice. High temperatures can be generated in many structures and machine parts. Because these temperatures may be generated rapidly and failure can be disastrous, studies of thermal shock are of great importance for engineering safety and practice. Graphites have excellent thermal, nuclear, and thermal shock resistance, and are frequently used as components of nuclear reactors. Problems involving thermal shock on structural elements of anisotropic media have received little attention, except for a few fundamental problems [1,2,3] which have been studied recently by the authors.

The purpose of this paper is to apply the fundamental solution obtained in [1] to some practical engineering problems. Temperature variations, in an orthotropic cylindrical shell of finite length, induced by a thermal shock which consists of an instantaneous heat pulse acting on a finite surface area of the shell structure, are obtained. The quasi-linear theory of heat conduction is assumed. The material of the shells is assumed to be homogeneous and orthotropic in both mechanical and thermal properties. The material constants are taken to be independent of temperature. Graphites ATJ and ZTA are used in the numerical examples.

II. Fundamental Heat Conduction Solutions and Applications

Consider an orthotropic cylindrical shell of thickness h and radius of the middle surface

a. The material of the shell is thermally orthotropic in its conductivity and oriented in such a manner that the principal directions of conduction coincide with the coordinate axes x , ϕ and z . Linear heat conduction theory is used in the analysis.

The fundamental heat conduction solutions for a point thermal shock on a finite orthotropic cylindrical thin shell have been obtained by the writers for various boundary conditions [1]. For applications, only the case constrained by the thermal boundary condition that temperatures at both edges of the cylindrical shell remain the same as the ambient temperature will be considered in this paper. Shells with other thermal boundary conditions can be treated by the techniques outlined in this paper without difficulty. The fundamental solutions for an instantaneous point thermal shock on the surface of a finite shell with constant temperature at both ends are (see [1], Eqs. (24a) and (29a))

(1): For a plane hot-spot problem, ($G_{(0)} = 0$, at $x = 0, L$),

$$G_{(0)}(x, \phi, t) = \frac{e^{-\eta_0(t-t_0)/cp}}{4\pi ahcpL} \left\{ \theta_3\left(\frac{x-x_0}{2L}, \tau_x\right) - \theta_3\left(\frac{x+x_0}{2L}, \tau_x\right) \right\} \theta_3\left(\frac{\phi-\phi_0}{2\pi}, \tau_\phi\right) \quad (1)$$

(2): For a boundary hot-spot problem, ($G_{(1)} = 0$, at $x = 0, L$),

$$G_{(1)}(x, \phi, t) = \frac{3e^{-\eta_1(t-t_0)/cp}}{\pi h^3 L c p} \left\{ \theta_3\left(\frac{x-x_0}{2L}, \tau_x\right) - \theta_3\left(\frac{x+x_0}{2L}, \tau_x\right) \right\} \theta_3\left(\frac{\phi-\phi_0}{2\pi}, \tau_\phi\right) \quad (2)$$

where

$$\tau_x = \frac{k_x(t-t_0)}{L^2 cp}, \quad \tau_\phi = \frac{k_\phi(t-t_0)}{\pi^2 a^2 cp},$$

and

$$\theta_3(x, t) = 1 + 2 \sum_{m=1}^{\infty} e^{-m^2 \pi^2 t} \cos(2m\pi x), \quad t > 0$$

or

$$\theta_3(x, t) = \frac{1}{\sqrt{\pi t}} \sum_{n=-\infty}^{\infty} e^{-(n+x)^2/t}, \quad t > 0.$$

Once the fundamental solutions are obtained, the history of heat content in a finite orthotropic cylindrical shell can be calculated readily, i.e.,

$$T_{(0)} = H_0 G_{(0)}(x, \phi, t; x_0, \phi_0, t_0), \quad (3)$$

and

$$T_{(1)} = H_1 G_{(1)}(x, \phi, t; x_0, \phi_0, t_0). \quad (4)$$

The heat content stored in a finite cylindrical shell at time $t-t_0$ is

$$H = c\rho h a \int_{-\pi}^{\pi} \int_0^L T_{(0)} dx d\phi + \frac{c\rho h^3}{12} \int_{-\pi}^{\pi} \int_0^L T_{(1)} dx d\phi. \quad (5)$$

Substituting the Green's functions (1) and (2) into the Eqs. (3) and (4), respectively, and then into Eq. (5), one obtains the history curves of heat content in the shell with constant temperature at two end edges caused by an instantaneous hot-spot source.

The idea of using an instantaneous point source of heat has proved to be most powerful in the linear theory of conduction of heat. One great advantage is that it is based on a very simple physical idea from which the solution of a large number of important problems can be obtained by use of the principle of superposition. The solutions for the instantaneous line and area sources can be obtained by integrating the fundamental solution with respect to appropriate space variables. Furthermore, the integration of those solutions with respect to time coordinate will yield solutions for prolonging line and area sources.

In this paper, the case of the instantaneous circumferential line source acting on a thin graphite cylindrical shell with a finite length L will be studied. If the instantaneous line source is distributed along a line of circumferential direction from (x_0, ϕ_1) to (x_0, ϕ_2) , as shown in Fig. 1, the temperature on the middle surface of the shell T_c and the temperature gradient T_{c1} , obtained by directly integrating equations (3) and (4), respectively, are

$$\begin{aligned} (T_c, T_{c1}) &= \frac{1}{\phi_2 - \phi_1} \int_{\phi_1}^{\phi_2} (T_{(0)}, T_{(1)}) d\phi_0 \\ &= \left(\frac{H_0^c e^{-\eta_0(t-t_0)/c\rho}}{4\pi a h L c \rho}, \frac{3H_1^c e^{-\eta_1(t-t_0)/c\rho}}{\pi h^3 L c \rho} \right) \\ &\quad \times \left(\theta_3\left(\frac{x-x_0}{2L}, \tau_x\right) - \theta_3\left(\frac{x+x_0}{2L}, \tau_x\right) \right) I_c, \end{aligned} \quad (6)$$

where

$$I_c = 1 + \frac{4}{\phi_2 - \phi_1} \sum_{n=1}^{\infty} \frac{e^{-n^2 \pi^2 \tau}}{n} \cos\left[n\left(\phi - \frac{\phi_1 + \phi_2}{2}\right)\right] \sin\left[n\left(\frac{\phi_2 - \phi_1}{2}\right)\right], \quad (7)$$

H_0^c = heat content due to a plane circumferential line source,

and H_1^c = heat content due to a bending circumferential line source.

Introducing the non-dimensional quantities

$$x' = x/L,$$

$$x'_0 = x_0/L,$$

$$A_0 = L/a ,$$

$$\xi = h/a ,$$

$$T'_0 = T_{(0)}/(H_0/\pi h a^2 c\rho) ,$$

$$T'_1 = T_{(1)}/(H_1/\pi h a^3 c\rho) ,$$

$$Bi = h'h/k_0 ,$$

and

$$\tau_0 = k_0(t-t_0)/a^2 c\rho ,$$

where k_0 is the reference thermal conductivity, into Eqs. (3) and (4), for an orthotropic cylindrical shell of finite length with constant temperature at both edges, leads to

$$(1): (T'_0 = 0, \text{ at } x' = 0,1)$$

$$T'_0 = e \frac{-2Bi\tau_0/\xi^2}{4} \frac{1}{A_0} \left\{ \theta_3\left(\frac{x'-x'_0}{2}, \frac{k_x\tau_0}{k_0 A_0 t}\right) - \theta_3\left(\frac{x'+x'_0}{2}, \frac{k_x\tau_0}{k_0 A_0^2}\right) \right\} \times \theta_3\left(\frac{\phi-\phi_0}{2\pi}, \frac{k_\phi\tau_0}{k_0 \pi^2}\right) \quad (8)$$

$$\text{and } (2): (T'_1 = 0, \text{ at } x = 0,1)$$

$$T'_1 = 3e \frac{-6(Bi + \frac{2k_z}{k_0})\tau_0/\xi^2}{\xi^2 A_0} \left\{ \theta_3\left(\frac{x'-x'_0}{2}, \frac{k_x\tau_0}{k_0 A_0^2}\right) - \theta_3\left(\frac{x'+x'_0}{2}, \frac{k_x\tau_0}{k_0 A_0^2}\right) \right\} \times \theta_3\left(\frac{\phi-\phi_0}{2\pi}, \frac{k_\phi\tau_0}{k_0 \pi^2}\right) , \quad (9)$$

for cases corresponding to Eqs. (1) and (2), respectively.

III. Numerical Examples

The numerical examples are given for an orthotropic cylindrical shell subjected to an instantaneous circumferential line source at time $t = t_0$. The line source is distributed along the circumferential direction from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ at the section $x' = 0.5$. It is assumed that the temperatures at both edges are constant and equal to the ambient temperature. A short cylindrical shell is selected here. Its dimensions are $L/a = 2$, $h/a = 0.05$ and $a = 2$ ft. Grade ZTA graphite is assumed to be the material used in the shell. Its thermal properties are given in Table 1 [4].

Table 1. Thermal Conductivities for Grade ZTA Graphite

	Type of material	
	C(ZTA)	D(ZTA)
k_x/k_0	104.02	38.7
k_ϕ/k_0	38.7	104.02
k_z/k_0	38.7	104.02

$$k_0 = 1 \text{ BTU/(hr., ft., } ^\circ\text{F)}$$

Graphites have two principal directions, one with the grain, the other across the grain. From the table, it is obvious that the direction with the grain possesses a higher Young's modulus, higher thermal conductivity, and a lower coefficient of thermal expansion than

the principal direction across the grain [4]. In the following numerical calculation, Biot's number, Bi , is taken to be 0.2.

From the results of previous investigations [2], we have concluded that the effects on the thermal responses due to a bending hot-spot are very localized and of short duration. Therefore, only the case of a plane hot-spot is of interest and only this case will be discussed in the following numerical example. Figs. 1 and 2 show the history of the temperature variation along the axial and circumferential directions after the application of the circumferential line source of the plane hot-spot thermal shock. The value of time parameter τ_0 is chosen within the most significant time period, i.e., $0.0003 \leq \tau_0 \leq 0.005$. It is of interest to note that, for the orthotropic shells described here, the higher thermal conductivity in the axial direction, as in the material type C, results in a higher rate of heat dissipation within the shell. This can be explained by the fact that a greater quantity of heat loss occurs at the edges and lateral surfaces of the shell. Therefore, in engineering design, the orthotropic properties of materials must be included.

With the principle of superposition presented in this paper, the displacements and stresses in the shell subjected to the thermal shock can be obtained readily from the fundamental solutions for displacements and stresses presented in [3].

V. References

- [1] Huang, C. L. and Woo, H. K., The Fundamental Solution for a Thermal Shock on a Finite Orthotropic Thin Shell, *Nuclear Engineering and Design*, 45, 1978, pp. 67-74.
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Fig. 1. Variations of Middle Surface Temperature for ZAT

Fig. 2. Variations of Middle Surface Temperature for ZAT.