ANALYSIS OF REINFORCED CONCRETE CONTAINMENT VESSELS CONSIDERING CONCRETE CRACKING

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SUMMARY

Cracking of concrete influences the stress analysis of concrete containment vessels. If cracking is ignored, the resulting shell analysis can be unconservative in some cases and extremely conservative in others. A cracked concrete shell is a structurally orthotropic one. That is, it does not have the same properties in membrane action and bending action.

The development of structurally orthotropic shell theory has progressed rapidly in all parts of the world and has been applied to stiffened shells (such as submarines, containment vessel liners, and underwater oil storage vessels), to stadium roofs (such as the Astrodome) and to the stability of reinforced concrete shells. It has also been accepted by the U.S. Structural Stability Research Council (formerly the Column Research Council).

The closed form solution for a cracked shell can be illustrated by resolving the differential equation of a cylindrical shell as given by Timoshenko for edge loadings. The equation is:

$$D \frac{d^4 w}{dx^4} + \frac{Eh}{a^2} w = Z,$$

where D is the shell bending rigidity and Eh is the shell membrane rigidity. Also, w is the deflection, a the radius and Z the loading. In the case of a cracked concrete shell, D is the modulus of elasticity times the moment of inertia per unit length of the cracked shell including reinforcing. Thus the shell action is dependent upon the amount of reinforcing steel as well as the thickness of the shell.

The above method has been used to solve for the forces and moments between a base slab and cylindrical shell on a reinforced concrete containment. Similarly the differential equations developed by Eringen, *et al.* for local effects due to openings have been resolved and have been used to analyze openings in containments for equipment hatches and personnel openings. The effects of internal pressure as well as other loads can be handled. Novozhilov's equations for local effects on shells have also been resolved using orthotropic shell theory. A more detailed derivation of the basic concepts and basic equation have been developed by Buchert.

Over the past few years, much interest has been generated within the universities by industry, to develop finite element programs capable of accurately predicting the behavior of reinforced concrete plates and shells, considering material nonlinearities. Although the programs thus far developed are for the most part, research tools, they are recently being considered for parametric studies to validate current design procedures. The finite element method presents a very straight-forward tool for considering material nonlinearities, structural irregularities and orthotropic behavior of concrete shells. Reinforcing can be accurately positioned and oriented within the thickness of the shell and partially cracked sections can be considered. — The method has been used successfully to predict the behavior of concrete shells by Scordelis.

Thus there are two general approaches that can be used to analyze reinforced concrete containment vessels considering concrete cracking. They are a) closed form solutions considering the structurally orthotropic shell properties and b) finite element programs that take into account the different membrane and bending properties of the element.

I. INTRODUCTION

Cracking of concrete influences the stress analysis of concrete containment vessels. If cracking is ignored, the resulting shell analysis can be unconservative in some cases and extremely conservative in others. A cracked concrete shell is a structurally orthotropic one. That is, it does not have the same properties in membrane action and bending action. The sketch below illustrates a typical situation in a concrete shell.

It is assumed that the shell is reinforced with #18 bars at 9 inches on centers each way on each face. Under relatively large membrane compression and bending the shell would be uncracked. The effective membrane thickness would be 42 inches plus the effective transformed steel area. Likewise the effective bending thickness would be 42 inches plus the effective transformed steel area. The detailed method of calculating the effective membrane and bending thicknesses will be illustrated later. In this case the relatively simple closed form shell equations or the available finite element methods could be used.

In the case of relatively large membrane tension, the concrete may be fully cracked and an entirely different situation exists. The effective membrane thickness is proportional to the transformed steel area and the effective bending thickness is a function of the moment of inertia of the transformed steel area. Thus there is no single thickness that can be used in the shell equations that will give correct results if both shell membrane and shell bending stresses exist. Fortunately, however, many of the shell equations have been resolved considering what is called split rigidity action and thus the designer can use the results in a similar way that has been done in isotropic shell design and analysis.

A similar situation exists in finite element technology. Some designers use uncracked finite element programs to calculate stresses in shells. In the case of thermal stresses, the uncracked finite element programs often give moments that are many times the actual moment when the section is cracked. If the shell section illustrated above is uncracked it has a transformed bending rigidity of 2.76×10^{10} lb-inches per inch. If it is fully cracked the bending rigidity is 0.233×10^{10} lb-inches per inch. Thus an uncracked section has about twelve times the bending rigidity of a fully cracked section. Similarly, the uncracked section has about nine times the axial rigidity of a fully cracked section. Some designers reduce the moment due to thermal stresses in accordance with code rules.

In order to develop more realistic stresses, cracked section finite element programs have been developed. In general these programs assume a number of elements through the thickness of the shell and vary the properties of each element (for example, assume that E is almost zero for concrete elements in tension). Usually some iterative process is involved.

The purpose of this paper is to illustrate some of the techniques that are available.

II. CLOSED FORM SOLUTIONS

In general, the closed form solutions of isotropic shells can be resolved for structurally orthotropic shells by considering an effective membrane and an effective bending thickness (See reference [1]). Timoshenko [2] as well as others have shown the development of the differential equation for cylindrical shells loaded symmetrically with respect to its axis. Using the equations of equilibrium, it is shown that

$$D \frac{d^4 w}{dx^4} + \frac{Eh}{a^2} w = Z \quad (1)$$

where

D is the flexural rigidity of the shell

Eh is the axial or shear rigidity of the shell

a is the radius

w is the deflection

Z is the lateral load.

This equation can be rewritten as follows:

$$\frac{Et_B^3}{12(1-\nu^2)} \frac{d^4 w}{dx^4} + \frac{Et_m}{a^2} w = Z \quad (2)$$

where

t_B is the effective bending thickness

t_m is the effective membrane thickness

For a reinforced concrete shell

$$\frac{1}{12} t_B^3 = \frac{I}{d} \quad (3)$$

$$t_m = \frac{A}{d} \quad (4)$$

$$t_B = \left(\frac{12I}{d} \right)^{1/3} \quad (5)$$

where

I is the effective transformed moment of inertia over a length of shell d

A is the effective transformed area over a length of shell d

Usually in solving the differential equation there is a quantity defined as:

$$\beta^4 = \frac{1}{\lambda^4} = \frac{Eh}{4a^2 D} = \frac{Et_m 12(1-\nu^2)}{4a^2 Et_B^3} \quad (6)$$

Using this substitution, one can find the deflections, moments and shears at any point in the shell as a function of β or λ . Thus one only needs to compute t_m and t_B for an orthotropic shell and can then use all the solutions for this type of shell. This simple relationship has been used to find the cracked section moments and shears for a cylindrical reinforced concrete shell.

It is interesting to note some of the classical parameters for the shell details described in the introduction. These are given in the following:

Parameter	Uncracked Section	Fully Cracked Section	Half Cracked Section
t_m	42 in.+steel	0.66 in.	3.66 in.
t_B	42 in.+steel	9.57 in.	11.65 in.
λ	0.0072	0.0077	0.0102
D conc. + steel	2.76×10^{10}	0.233×10^{10}	0.420×10^{10}
$\frac{3\pi}{4\lambda}$	27 ft.	25.4 ft.	19.2 ft.
$\frac{D \text{ conc. + steel}}{D \text{ actual}}$	1	12	6.6
$\frac{Et_m \text{ conc. + steel}}{Et_m \text{ actual}}$	1	8.9	1.61

These quantities illustrate that the solutions for a cracked shell can vary considerably from the solutions of an uncracked shell. They do illustrate, however, that the characteristic lengths $(3\pi/4\lambda)$ do not vary by a significant amount.

This same type of solution can be used to evaluate the effects of local loads on shells as well as the effects of openings. In addition, the critical buckling loads of cylinders, spherical shells and hypars have been calculated [3].

Bijlaard [4] has calculated the deflections, moments, shears and membrane loads due to local forces and moments on spheres. The equations have been resolved [3] and one can readily find the desired quantities by using the modified Bijlaard relationships

$$w = C_2 \frac{RP}{E(t_B^3 t_m)^{1/2}} \quad (7)$$

$$s = 1.82 \frac{r}{R} \left(\frac{R}{t_B} \right)^{1/2} \left(\frac{t_m}{t_B} \right)^{1/4} \quad (8)$$

$$u = 1.82 \frac{r_o}{R} \left(\frac{R}{t_B} \right)^{1/2} \left(\frac{t_m}{t_B} \right)^{1/4} \quad (9)$$

$$N_{\phi} = C_5 \frac{P}{t_B} \left(\frac{t_m}{t_B} \right)^{1/2} \quad (10)$$

$$N_{\theta} = C_6 \frac{P}{t_B} \left(\frac{t_m}{t_B} \right)^{1/2} \quad (11)$$

Similarly the equations and results developed by Koh, Thiel and Eringen [5] for the stresses at openings in cylindrical shells have been resolved using split rigidity concepts. All of the results can be used if

$$\beta = \frac{1}{2} \left[\frac{3(1-\nu^2)}{R^2 t_B^2} \right]^{1/4} \left(\frac{t_m}{t_B} \right)^{1/4} \quad (12)$$

$$KB = \left[\frac{1}{12(1-\nu^2)} \right]^{1/2} t_B \left(\frac{t_B}{t_m} \right)^{1/2} \quad (13)$$

The practical use of the above equations generally involves estimating the effective amount of cracking and then using a sensitivity study to bound the final results.

III. FINITE ELEMENT SOLUTION

The orthotropic behavior of reinforced concrete shells can be addressed using the finite element technique. The layered shell concept has been used by Hand, et.al. [6] and others [7] to successfully predict the behavior of concrete plates and shells subjected to static loads. Reinforcing steel is modeled as a discrete layer with orthotropic material properties, coupled to the middle surface of the shell. Cracking is accomplished by modifying the material constitutive properties of the appropriate layer and recycling the solution until convergence is achieved. Using these techniques, fairly complicated R/C shell structures have been analyzed.

Nuclear containment vessels and internal structures are generally very simple structures geometrically and the loads that cause cracking and major redistribution of forces and moments are mostly axisymmetric in nature. Internal pressure loads, prestressing forces, and thermal gradients are of major concern for the design of these structures. A highly complex non-linear finite element solution is not justified for the basic shell design.

The layered shell concept was extended to shells of revolution [8, 9] and included in a computer program written for analyzing axisymmetric structures for non-axisymmetric loads [10]. This code has been very useful for general analysis of containment structures and the first harmonic solution or axisymmetric displacement solution is easily extended to the analysis for cracked concrete sections.

The orthotropic material behavior and its non-linear behavior can be easily modeled to predict the redistribution of the forces and moments and to directly calculate reinforcing and concrete section resultants.

Reinforcing layers are offset from the shells middle surface through transformations and material constitutive relations expressed as:

$$\begin{pmatrix} N'_{\phi} \\ N'_{\theta} \\ N'_{\phi\theta} \end{pmatrix} = A_{\phi} \begin{bmatrix} E & 0 & 0 \\ 0 & mE & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \epsilon'_{\phi} \\ \epsilon'_{\theta} \\ \gamma'_{\theta} \end{pmatrix} \quad (14)$$

$$m = \frac{A_{\theta}}{A_{\phi}}$$

where

A_{ϕ} is the reinforcing area in the meridional direction
 A_{θ} is the reinforcing area in the hoop direction
 The primed terms represent layer resultants.

The offset transformation alters the constitutive matrix, coupling the reinforcing layer to the shells middle surface. Thus:

$$\begin{pmatrix} N'_{\phi} \\ N'_{\theta} \\ N'_{\phi\theta} \end{pmatrix} = A_{\phi} \begin{bmatrix} E & 0 & 0 & hE & 0 & 0 \\ 0 & mE & 0 & 0 & mhE & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \epsilon_{\phi} \\ \epsilon_{\theta} \\ \gamma_{\theta} \\ \kappa_{\phi} \\ \kappa_{\theta} \\ \kappa_{\phi\theta} \end{pmatrix} \quad (15)$$

In equation (15), h is the distance from the middle surface of the shell to the centroid of the reinforcing steel. If the centroids of the hoop and meridional reinforcing differ by a large amount, m can be modified to account for this or a separate shell can be used for each direction, eliminating the proper terms in the constitutive matrix.

The concrete shell material model for an uncracked section is simply:

$$\begin{pmatrix} N'_{\phi} \\ N'_{\theta} \\ N'_{\phi\theta} \\ M'_{\phi} \\ M'_{\theta} \\ M'_{\phi\theta} \end{pmatrix} = t \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 & 0 \\ & C_{22} & 0 & 0 & 0 & 0 \\ & & C_{33} & 0 & 0 & 0 \\ & & & dC_{11} & dC_{12} & 0 \\ & & & & dC_{22} & 0 \\ & & & & & dC_{33} \end{bmatrix} \begin{pmatrix} \epsilon_{\phi} \\ \epsilon_{\theta} \\ \gamma_{\theta} \\ \kappa_{\phi} \\ \kappa_{\theta} \\ \kappa_{\phi\theta} \end{pmatrix} \quad (16)$$

$$d = \frac{t^2}{12}$$

$$C_{11} = \frac{E}{1-\nu^2} \quad C_{22} = C_{11}$$

$$C_{12} = \nu C_{11} \quad C_{33} = G$$

This material model is used to determine the initial force and moment distribution for pressure and prestressing loads. Following the first cycle or prior to analyzing for thermal gradients, the section is cracked and a new material model is formulated. Thus:

$$\begin{Bmatrix} N'_{\phi} \\ N'_{\theta} \\ N'_{\phi\theta} \\ M'_{\phi} \\ M'_{\theta} \\ M'_{\phi\theta} \end{Bmatrix} = t_{\phi} \begin{bmatrix} C_{11} & C_{12} & 0 & hC_{11} & hC_{12} & 0 \\ \text{sym.} & C_{22} & 0 & hC_{12} & hC_{22} & 0 \\ & & C_{33} & 0 & 0 & hC_{33} \\ \hline \text{symmetric} & & & dC_{11} & dC_{12} & 0 \\ & & & \text{sym.} & dC_{22} & 0 \\ & & & & & dC_{33} \end{bmatrix} \begin{Bmatrix} \epsilon_{\phi} \\ \epsilon_{\theta} \\ \gamma_{\phi\theta} \\ \kappa_{\phi} \\ \kappa_{\theta} \\ \kappa_{\phi\theta} \end{Bmatrix}$$

$$d = \frac{t^3}{12} + h^2$$

$$C_{11} = \frac{E}{1-\nu^2} \quad C_{22} = mC_{11}$$

$$C_{12} = \nu C_{11} \quad C_{33} = G$$

$$m = \frac{t_{\theta}}{t_{\phi}}$$

The term h is determined by the amount of cracking that has taken place. Again, error is introduced if hoop and meridional cracking are not the same, but this is overcome in the same manner that unequal reinforcing centroids are handled.

The approach presented here for predicting the distribution of forces and moments for a concrete shell of revolution is not as rigorous or exact as many that have been suggested, but is sufficient for this type of design work.

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