SEISMIC RESPONSE OF FLEXIBLE CYLINDRICAL LIQUID STORAGE TANKS

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SUMMARY

Liquid slosh and tank wall vibrations are studied experimentally in a flexible model storage tank subject to simulated earthquake environments. The program objective is to determine the validity of simple analytical models for representing this earthquake dynamics problem.

A similitude study is first performed to develop the important nondimensional parameters. It is found that simulation of both membrane stresses and bending stresses is difficult for geometric scales smaller than $1/3$, unless plastic-type materials are used for the model shell. However, an approach is given for studying each independently for $1/16$ and $1/7$ scale models. These scales are applied to an aluminum model tank of 62.87 cm diameter, 76.2 cm height, and 0.51 mm wall thickness, having flat rigid bottom and top cover. The tank is instrumented to measure liquid slosh amplitudes and pressures, and tank wall vibrations and strains.

Simulation of a scaled earthquake environment is provided by a biaxial shake table having the capability of simultaneous independent horizontal and vertical excitation. Ground motion time histories are developed from random signals, and are modified to produce specified different earthquake response spectrums along each axis. Both excitation displacements and accelerations are recorded on analog instrumentation tape, along with liquid and tank responses for subsequent processing. Data are acquired for motion of several magnitudes with horizontal only, vertical only, and combined horizontal and vertical excitation.

It is found that liquid slosh amplitudes are principally related to horizontal seismic displacements, while pressures and shell wall flexural vibrations are principally related to both horizontal and vertical seismic accelerations. Wall vibrations and strains are further found to be as sensitive to vertical accelerations as they are to horizontal accelerations. The wall response is comprised principally of the first several flexural or breathing modes which have multiple waves around the shell circumference. Influence of vertical excitation on this type response is surmised to occur through either geometric eccentricities or through parametric excitation mechanisms. Shell wall responses damp out rapidly after the seismic event has subsided. However, low frequency liquid slosh, which occurs principally in the first antisymmetric mode, persists for a long time duration after the seismic event. By comparison with predictions from a simple analytical model, liquid pressures are separated into impulsive, or acceleration loads, and convective, or slosh loads. The simple analytical model is shown to be inadequate for certain parameter ranges and earthquake conditions. Explanations for the discrepancies are offered.

1. Introduction

Inadequacy of existing procedures for aseismic design of storage tanks for potentially hazardous liquids has been recognized for some time, and has received special attention since the failure of several petrochemical tanks in the Alaska Earthquake of 1964 [1]. Most of these failures resulted from dynamic buckling caused by overturning moments produced by the seismically-induced liquid inertia and surface slosh waves. Simple analytical models [2] have been used in the past for aseismic design of this type structure. Generally, the models include assumptions of a rigid container, and the effects of horizontal ground motion only. Some update of the model has been accomplished in more recent work [3], wherein effects of both horizontal and vertical ground motion were studied for a rigid cylindrical tank. On the other hand, it is known that thin shells, with or without contained liquids, typically exhibit a multitude of vibration modes, for which multiple flexural waves occur in patterns that depend on the basic geometry. For cylindrical shells, the flexural waves appear along the longitudinal axis and around the circumference. Recent efforts [4,5,6] have probed the significance of these modes for earthquake response to horizontal ground motion. This paper presents some results of experiments to determine the significance of flexural responses in scale model cylindrical tanks subject to both horizontal and vertical ground motions, and comparisons of data with that predicted by the existing simple analytical models.

2. Design of Experiments

2.1 Similitude Concepts

Application of similitude concepts to the seismic slosh problem for a flexible cylinder has been outlined in previous work [3]. Briefly, a nondimensional prediction equation for liquid slosh height η , pressure p , and tank wall stress σ , in a flexible cylindrical tank can be formulated as

$$\left(\frac{\eta}{d}, \frac{p}{\rho_l d g}, \frac{\sigma}{\rho_l d g} \right) = \mathcal{F}_1 \left(\frac{l}{d}, \frac{h}{d}, \frac{l_i}{d}, \zeta, \frac{E}{\rho_l d g}, \frac{\rho_t}{\rho_l}, \frac{H}{d}, \frac{\mu}{\rho_l g d^2}, \frac{a_x}{g}, \frac{a_y}{g}, \frac{a_z}{g}, \frac{g T_e^2}{d}, \frac{w^2 d}{g} \right) \quad (1)$$

where the variables are defined as

d	= tank diameter (2R), L	η	= vertical response relative to
l	= tank length, L		coordinates r, θ , which are located
h	= tank wall thickness, L		at the quiescent liquid level, and
E	= elastic modulus, $ML^{-1}T^{-2}$		move with the tank, L
σ	= stress response at some point, $ML^{-1}T^{-2}$	ρ_l	= liquid density, ML^{-3}
		μ	= viscosity, $ML^{-1}T^{-1}$
l_i	= any other tank dimension, L	p	= pressure response at some point, $ML^{-1}T^{-2}$
ρ_t	= tank density, ML^{-3}		
ζ	= structural damping ratio (nondimensional)	a_x, a_y, a_z	= time-dependent components of base acceleration relative to a space-fixed coordinate system, LT^{-2}
H	= liquid depth, L	g	= steady acceleration of gravity, LT^{-2}
		T_e	= earthquake duration, T
		w	= frequency, T^{-1}

For a flexible cylindrical shell tank there are two very significant nondimensional parameters that can be further formed to replace the second and fifth parameters on the right side of eq. (1). These two parameters result in the requirement that

$$\left(\frac{Eh}{\rho \ell d^2 g} \right)_m \bigg/ \left(\frac{Eh}{\rho \ell d^2 g} \right)_p = 1$$

for membrane stress simulations, and

$$\left(\frac{Eh^3}{\rho \ell d^2 g \ell^2} \right)_m \bigg/ \left(\frac{Eh^3}{\rho \ell d^2 g \ell^2} \right)_p = 1$$

for bending stress simulations. Note that the subscripts *m* and *p* refer to model and prototype, respectively. Satisfaction of both these relationships simultaneously in small models is practically impossible for large storage tanks (i.e., greater than 36 ft diameter). This results from limitations on the physical size of available experimental facilities, and the elastic modulus ratio of most useful materials. However, by concentrating on the bending stress requirement, which will provide the simulation of dominant flexural motions experienced in wall responses, a useful physical model can be developed.

2.2 Apparatus and Procedures

A cylindrical aluminum tank with flat rigid bottom and top was used for experimentation with water as a liquid. Characteristics of this tank, and location of various instrumentation transducers are shown in Figure 1. This model was mounted on a biaxial seismic simulator which has the capability of simultaneous independent horizontal and vertical excitation. Details of this simulator have been described in Reference [7].

Both liquid and shell wall responses were studied experimentally for 1/16 and 1/7 scale earthquake excitation. This was accomplished by producing a simulator table motion according to a realistic (although scaled) earthquake time history. The computed response spectrum of the actual motion was compared with that prescribed by the U.S. NRC [8] for typical ground motion. An example of this parameter for the horizontal axis is shown in Figure 2 for the 1/16 scale earthquake at a magnitude of 0.3g. A similar plot occurs for 1/7 scale, although it is shifted lower in frequency by a factor equal to $(7/16)^{1/2}$. Note also that the frequency range of various flexural modes for the model tank at 3/4 full liquid level is indicated on this figure. They can be seen to be in the upper part of the range where earthquake excitation occurs. This range for flexural modes is typical for tanks with ℓ/d ratios > 1.0 . For short tanks of large diameter, the flexural modes will occur in even lower parts of the frequency range, and correspondingly more modes can be excited by typical earthquake motions.

3. Discussion of Results

Figures 3 and 4 show typical time histories, respectively, for combined excitation and vertical only excitation for 1/16 scale. It is important to recognize that peak pressures are mostly inertial, i.e., they are related to the acceleration of the excitation, and dominate over those which result from the lower frequency slosh motion. (Note that in a short tank, the two are probably more nearly the same amplitude.) Furthermore, liquid slosh motions continue long after the earthquake event has subsided. From Figure 4, it can be seen that no slosh occurs for vertical only excitation, although midwall accelerations

and pressures were found to depend very strongly on vertical ground motion. It was determined that the wall responses resulted principally from flexural modes in the 30 to 100 Hz frequency range (see Figure 2). The mechanism for such strong excitation of these modes by vertical excitation was not determined. Data runs similar to Figures 3 and 4 were also taken for horizontal only excitation, but are not included here for the sake of brevity.

A summary of peak liquid wave response results for both 1/16 and 1/7 scale experiments is shown in Figure 5a, for simulated earthquakes of increasing magnitude. It can be seen from the prediction equations that the model defined in Reference [2] is invalid for describing the results for excitation amplitudes beyond $x_H/R = 0.03$. On the other hand, the linear prediction equation developed in Reference [3] continues to be valid for amplitudes as large as $x_H/R = 0.065$. It can be concluded that the latter model is a better representation for the problem, and will be used for prediction of the remaining results.

A similar response plot is also given for peak inertial pressures at the tank bottom near the wall. The predicted values based on the existing simple analytical model [3] appear to be somewhat low in the flexible container for horizontal excitation. Furthermore, a rational basis for combining the effects of both horizontal and vertical motions needs to be explained. This will be done, and further results will follow.

4. Time Average Nonstationary Random Processes

In Figure 5, the square root of the squared sum (SRSS) has been employed to predict peak inertial pressure values for combined vertical and horizontal excitation. This will be justified by considering each time history in Figures 3 and 4 to be a nonstationary random process, in which the time trends are very slow compared to the instantaneous fluctuation of the data [9].

Let $a_H(t)$ be the nonstationary random horizontal excitation, and $p_{BH}(t)$ be the corresponding bottom pressure response. We can write

$$p_{BH}(t) = B_H a_H(t) \quad (2)$$

where B_H is an inertial constant. If this expression is squared, integrated and averaged over time, we have

$$\overline{\sigma_{PH}}^2 = B_H^2 \overline{\sigma_{aH}}^2 \quad (3)$$

That is, the time averaged mean square pressure is proportional to the time averaged mean square acceleration. For independent horizontal and vertical inertial pressures acting simultaneously, we have

$$(\overline{\sigma_P})_{RMS} = [\overline{\sigma_{PH}}^2 + \overline{\sigma_{PV}}^2]^{\frac{1}{2}} \quad (4)$$

If both the horizontal and vertical acceleration processes are described by the same time average probability density function, then the combination also is described by this density function, and plotting of peak pressure values versus peak acceleration values for the combined process is justified at the same probability of occurrence for a given peak level. However, one should expect that RMS values should be less sensitive to statistical sample variations than are peak values, and should, therefore, plot with less scatter. Hereafter, RMS values are used.

For more general responses of elastic systems, the transfer function will vary with frequency, and the form of eq. (2) is no longer valid. Consider the midwall acceleration

for the present tank as an example. In this case, a relationship [9] between response and excitation can be written as

$$\bar{S}_m(f) = |H_{ma}(f)|^2 \bar{S}_a(f) \quad (5)$$

where $\bar{S}_m(f)$ is the time averaged power spectral density of the nonstationary random response, $H_{ma}(f)$ is the time-independent transfer function, and $\bar{S}_a(f)$ is the time averaged power spectrum of the acceleration input. One can stipulate that all samples of excitation to be considered have the identical normalized power spectrum shape, but the magnitude is proportional to the time-averaged mean square of the acceleration, that is

$$\bar{S}_a(f) = \bar{S}_{an}(f) \times \bar{\sigma}_a^2$$

where $\bar{\sigma}_a^2$ is simply a number. Substitute this expression into eq. (5), integrate over frequency, and take the square root to obtain

$$(\bar{\sigma}_m)_{\text{RMS}} = A (\bar{\sigma}_a)_{\text{RMS}} \quad (6)$$

where A is an amplification factor.

5. Additional Results

Figure 6a shows a plot of the bottom inertial pressure in terms of RMS values for the 1/16 scale only. These data, and also the peak values of Figure 5b, show that measured bottom pressures due to horizontal excitation are somewhat higher than those predicted by the simple (rigid wall) analytical model. Results in Figure 6b show that measured midwall pressure responses which result from both horizontal and vertical components of excitation exceed predictions using the rigid model. Thus, the effect of elastic flexural wall motion is to increase local midwall pressure. On the other hand, midwall circumferential strain, shown in Figure 7a, tends to follow the predictions of the simple model reasonably well. Note that for horizontal excitation, the same pressure equation is used in membrane shell theory for predicting midwall strain, while only hoop strain is predicted for vertical excitation. At the same time, however, local flexural motions occur quite readily, as is shown by the midwall horizontal acceleration response in Figure 7b. In fact, these motions appear to be more sensitive to vertical than to horizontal excitation.

6. Concluding Remarks

The above results indicate that midwall circumferential strain is more dependent on hoop and lateral rigid body pressure, rather than on local variations caused by the shorter wavelength flexural wall motions. Therefore, the existing simple analytical model [3] appears to be useful for prediction purposes in an elastic tank of the type investigated. Its application to tanks of other geometries and wall thicknesses remains to be verified. On the other hand, the simple model defined in Reference [2] appears to have shortcomings, and should be used only with caution.

Sensitivity of lateral flexural wall motion to vertical excitation is a particularly interesting result. Some direct excitation undoubtedly occurs through longitudinal excitation of the shell and its top mass. However, some lateral flexural responses may also result from parametric excitation in the combined liquid and shell system [10]. Verification of this assertion remains to be accomplished.

Consideration of earthquake excitations and associated structural responses as non-stationary random processes, and employment of corresponding time average parameters for their description, appears to provide an extremely useful approach. Of course, peak responses, rather than RMS responses are of most concern, but they can always be established from corresponding probability density functions. A more complete application of this approach to general seismic response problems would be appropriate.

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Tank material - 6061-T6 aluminum
 Diameter - 62.87 cm (24.75 in)
 Height - 76.20 cm (30.00 in)
 Wall thickness - 0.51 mm (0.020 in)

1. Location of Instrumentation Transducers

2. Response Spectrum for 1/16 Scale Simulated Earthquake

3. Combined 0.36g Earthquake for 1/16 Scale Liquid Storage Tank

4. Vertical Only - 0.37g Earthquake for 1/16 Scale Liquid Storage Tank

5. Peak Liquid Responses in Model Storage Tank
 - a. Liquid Wave Height
 - b. Bottom Pressure

6. Time Averaged Pressure Responses in 1/16 Scale Tank
- Bottom Pressure
 - Midwall Pressure

7. Time Averaged Midwall Responses in 1/16 Scale Tank

- a. Circumferential Strain
- b. Horizontal Acceleration