INELASTIC ANALYSIS OF METAL STRUCTURES SOME APPLICATIONS

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SUMMARY

In the design of components for the power industry in general and the nuclear industry in particular, the engineer is called upon to perform inelastic analyses for the satisfaction of certain code and regulatory requirements. In other situations inelastic analysis is used to investigate the cause of failure of a component or to gain further insight of test results.

In all these cases a design limit or a failure criterion is set for evaluating the analytical results, but in most cases it is left up to the analyst to choose the appropriate material constitutive relations. Ideally, the constitutive relations should model the material behavior in the mode of failure or deformation under consideration. For example, yield stress, strain rate dependence could be of major importance in dynamic plasticity at high strain rates. Similarly, deformation plasticity theory was found to give better results than incremental theory in plastic buckling. In many cases however, the analyst, especially if he uses general purpose programs, has a very limited choice in the constitutive relations for use in the analysis. In addition to the above considerations, the analyst has to make sure that his analysis results could be used easily with the failure criteria adopted. For example, in evaluating creep-fatigue damage, currently there are several damage criteria, some of them make use of plastic and creep strains separately, while others use strains and strain rates without definite distinction between creep and plastic strains. In these cases the analyst must make sure to adopt the appropriate constitutive relation.

The other problems which the analyst faces are the amenability of the constitutive relation to computational procedures, available computer core and availability of experimental data in the environmental conditions imposed.

The following step is the mathematical modeling of the structure, and we are going to limit our discussion to the finite element method. In this case also the finite element model should represent the structural behavior and leads to results compatible with the mode of failure under investigation.

In this article several inelastic analyses are surveyed with reference to choice of constitutive relation, finite element model, and failure criteria. The survey covers analyses for creep fatigue evaluations at elevated temperature, inelastic fracture analyses, inelastic buckling, residual stresses due to fabrication processes and elastic-plastic dynamic analysis during accident condition.

1. Introduction

In recent years, there have been considerable developments in the fields of constitutive relations, finite element idealization, and failure criteria development. This paper will try to bring out the impact of the various developments from the point of view of the analyst. The paper is somewhat more extensive in creep-fatigue evaluation than in other areas as a result of the influence from the extensive development effort of the fast breeder reactor program. Inelastic fracture, inelastic buckling, and residual stresses due to fabrication processes, are briefly reviewed and an application in each case is discussed, with emphasis on the difficulties requiring additional work.

2. Inelastic Analysis for Creep-Fatigue Evaluations

The evaluation of elevated temperature creep-fatigue damage of a component that is subject to cyclic operation requires the calculation of stresses and strains during the life of the component. Design specifications give geometry and loading definitions and, in many situations, it is up to the analyst to choose the appropriate material behavior, modeling and failure criteria within the governing code and standards requirements.

Due to the inherent nature of the loading conditions and temperatures, inelastic strains have to be determined either by detailed inelastic analysis or simplified rules using elastic analysis. The resulting quantities are used with the failure criteria, which would be in this case: creep-fatigue damage, ratchetting and ductile or creep rupture, and excessive deformations leading to functional impairment. Here, the experimental data, the constitutive equations derivable from it, the structural modeling, and the failure criteria used in this type of analysis will be reviewed. In addition, other simplified methods that could be used in lieu of inelastic analysis will also be reviewed.

2.1 Material Behavior and Development of Constitutive Relations

The factors considered necessary for this type of analysis include cyclic effects, strain rate, environmental effects, weldment, longtime effects, aging and fabrication effects. This data, however, may be difficult to incorporate in a consistent theory. Some of the most commonly used constitutive relations and the latest developments will be reviewed below. Emphasis is placed on their appropriateness for this type of analysis, numerical computation, and failure criteria to be used.

2.1.1 Constitutive Relations Based on Separation of Inelastic Strain

These are most commonly used in inelastic analysis and are based on classical theories of plasticity and creep. In the classical theory of plasticity, a yield surface and its associated hardening and flow rules are assumed. These theories have been widely reviewed in the literature [1, 2, 3]. The von Mises yield surface is commonly used because of its simplicity and good agreement with material behavior at small strains. The use of the Tresca yield surface in general purpose programs is extremely limited. Both isotropic and kinematic hardening models are in most computer programs. Because

of the cyclic nature of the loading and in order to account for Bauschinger effect, the kinematic hardening model is usually adopted for this type of analysis.

Materials used in elevated temperature application, however, exhibit a peculiar behavior in which the yield surface moves in the stress space in a kinematic hardening manner and changes in size and shape. This is approximated by a model that combines both isotropic and kinematic hardening. Liu and Greenstreet [4] have examined this model for non-radial loading and achieved reasonable results.

Other models used in inelastic analysis are the mechanical sublayer model [5] and the Mroz Model [6]. Reference [7] gives a comparison of the results of the various models and their required computer storage.

The classical treatment of creep deformation is by the equation-of-state approach. The creep strain rate is given as a function of the stress, creep strain, and temperature. In this case, it is referred to as strain hardening law. If instead of creep strain, time is used in the functional dependence, the creep law is a time hardening law. Materials in the LMFBR program [8] have been found to behave according to strain hardening laws. The decomposition of creep strains is in the direction of the stress deviators similar to plasticity. In the case of load reversals, a heuristic approach was taken by affecting a shift of the origin from which the creep strain in the creep law is measured. This approach is very similar to kinematic hardening. There is, however, some concern about representation of creep behavior using this approach for some materials whose creep curves are non-classical [9]. In addition, at temperatures close to the melting point, some of the above formulations lead to numerical instability in the course of analysis. In some cases, anisotropy in creep deformations could be of sizeable magnitude necessitating special constitutive law [10, 11]. Another approach that has been used in expressing creep deformations is the use of the hereditary theory [12] in which the creep strain is expressed in terms of a non-linear superposition integral equation. This theory is capable of expressing load reversals directly.

2.1.2 Constitutive Relations for Total Inelastic Strains

Recently, a number of constitutive relations have been developed. They are based on the fundamentals of thermodynamics, and dislocation theories. They do not include a yield surface and tend not to separate plastic and creep strains. The endochronic theory was developed by Valanis [13] from extensive experiments and a coupled thermomechanical theory of plasticity. It is based on introducing an intrinsic time scale common to all internal variables, which is a function of the history dependence, rate dependence, and material non-linearity. This functional dependence is evaluated experimentally. The theory is capable of predicting loading, unloading, reversed loading, cross hardening, cyclic hardening or softening, and many other effects. A similar theory was developed by Kremple [14]. It is, however, based on phenomenological input-output behavior for aging, rate dependence,

and history dependence.

Although these theories have been able to predict, with astonishing accuracy, effects that are not obtainable by other simple theories, their use by analysts has been extremely limited due to the requirements of unconventional material properties, large computer storage, and lack of three-dimensional data to substantiate their use in general cases. Another theory that holds a great deal of promise is referred to as the mechanical-equation-of-state developed by Hart [15]. It is a phenomenological theory based on experimental evidence, microstructural evidence, and dislocation motion. Essential to the theory is the introduction of a state variable, dependent on the stress and strain rate. Additional states are included to model complex material behavior. The theory promises to model very complex material behavior. Its application to analysis, however, is extremely limited at the moment.

2.2 Finite Element Modeling

Finite element modeling for evaluated temperature creep-fatigue evaluation has to be able to predict local strains that are largely caused by thermal gradients. Depending on the structure and loading, whether it is axisymmetric or three dimensional in nature, the usual models used in creep fatigue evaluation are plane strain, axisymmetric solid, three dimensional solid, or general shells.

In this type of analysis it is required to predict stresses and strains. Therefore, a more refined model is usually needed than in cases where displacements or modes of vibration are to be predicted. For the modeling, it is essential that the strains imposed by the thermal loads are not of a higher degree of variation than the strains implied by the displacement function of the element [16]. This requirement, however, could lead to unmanageable and extremely large meshes, especially in 3-D problems. If ignored, however, unrealistically high hydrostatic stresses arise. Tensile hydrostatic pressure is part of some failure criteria [17] as a ductibility reduction factor in the life prediction. And, thus, wrong life predictions could be made. On the other hand, if only effective stresses and strains or stress intensities are used in the failure criteria, these errors have minor effects on life.

The LMFBR program contains a large number of finite element analyses performed for creep fatigue evaluation [18-23]. References [18 and 20] analyze a nozzle-to-cylinder intersection of the same component. The former uses an axisymmetric solid simplified model and the latter uses a 3-D solid and shell model. In general, both models were subject to cyclic fast thermal down shock, a heat-up and hold period in the creep range. Mechanical loads were also cycled. The constitutive relations used in both cases were those in which the plastic and creep strains are separate (Section 2.1.1). The three-dimensional model was able to give insight to the effect of the actual geometry and constraints on the strains and also the effect of an out-of-plane bending moment. It has shown neutral and non-proportional loading in the most critical region. Some of these results have implications in the next section. It was also found that the ratchetting is quite significant along the longitudinal axis of

the cylinder. This overall behavior can only be obtained from the 3-D model. On the other hand, the refined axisymmetric model was able to predict the detailed thermal effects through the thickness. The next step is how to use these results in predicting the component life.

2.3 Material Properties and Development of Damage Criteria

Creep fatigue damage criteria are based on phenomenological evidence rather than atomic and microstructural results, although the latter are used in explaining some of the results. Mechanical data and metallographic results reveal that the environment is usually an important factor in accelerating the material damage. At elevated temperature, time-dependent fatigue or creep fatigue is usually predicted by one of the following theories.

2.3.1 Linear Damage Rule [24, 25]

In this method time-dependent damage is assumed to be due to creep. The creep damage is summed on the basis of time fractions independent of the fatigue damage. The total damage is unity for some materials and in others it depends on the ratio of the two damages. The fatigue damage is evaluated from the equivalent strain range and the creep damage from the effective stresses and the rupture curves. Although the linear damage rule has been in use for a number of years, recent tests have shown its inadequacy to deal with complex cycle conditions. It is extremely conservative for some materials, while non-conservative for others [25].

2.3.2 Strain-Range Partitioning [27]

This method promises to deal with actual complex thermo-mechanical problems on the basis of true variation of service conditions. The concept underlying strain-range partitioning is basically that creep-fatigue interaction is related to the manner in which creep and fatigue reverse each other and that loop unbalance could have a significant effect on life. Depending on the wave shape of the cyclic inelastic strains, the strain range is partitioned into four components. These components are considered to have different damaging effects on the material. The results of the evaluations are improved if steady-state creep occurring during a given cycle is identified. This may not be possible in many computer programs.

The constitutive relations most appropriate for use with strain range partitioning are the classical ones that separate inelastic strains from plastic and creep strains. Others, such as the endochronic theory and the mechanical equation of state can also be used by placing a limit on the strain rate for separating the plastic strain from the creep strain.

Although guidelines have been given in order to deal with multiaxial conditions [28] these are still limited to radial loading cases. Strain-range partitioning also has incorporated a ductility reduction factor for triaxial tensile stress, and a modification for ratchetting strain per cycle.

2.3.3 Frequency Separation Method [29]

In this method, temperature and strain rate are considered as two basic and independent variables in the relation between inelastic strain range and the number of cycles to failure. The inelastic strain is not separated

into "creep" and "plastic" components. The loop unbalance is handled by separation of the loop into a slow-fast (tension going) and fast-slow (compression going) legs. All constitutive relations reviewed above can be used with this method. The method, however, has not offered any guidelines in dealing with multiaxial conditions.

The strain range partitioning and frequency separation methods are thoroughly reviewed in an ORNL report [28] and the fatigue crack growth method is explored. It is noted that the above three methods are concerned with the propagation stage. Although its use in the elevated temperature environment has been in explaining the process of time dependent fatigue, its application is limited because of its reliance on linear elastic fracture mechanics concepts that are not applicable in many cases.

2.4 Simplified Methods

Although inelastic analysis can be performed with great accuracy and, to an extent, can account for most material behavior, it still has several drawbacks. Most important among these is the high cost and long time needed to perform inelastic analysis, especially 3-D analysis. This causes the designer to consider only the worst loading case or histogram, material variability, etc., if he can determine that a priori. In the writer's opinion, bounding methods hold a great promise in providing means for evaluating the effects of various variables at a very low cost and in a short time. These methods are essential in the planning and decision making stages. In the U.S. [25] as well as the U.K. [30] more emphasis is now placed on simplified methods. It is essential that simplified methods be conservative, but not so pessimistic as to render them uneconomical, therefore, useless. Similarly, they should not be too sensitive to material variations so that expensive tests for obtaining constitutive relations can be eliminated. Also they should be easy to use and provide quick answers to enable the designer to modify the design or material selection early in the design process. This latter point is almost absent in current simplified methods.

The work of Leckie [31] and Ponter [32] is notable in this area. This work depends on elastic stress distribution and a minimization of a self-equilibrated stress field subject to inequality constraints for obtaining the failure condition. The work of O'Donnell et.al. [33] and Bree [34], who use a one-dimensional inelastic model for evaluating ratchetting strains has been incorporated in the ASME code [25] and other design standards. The method of reference stress [35] is also gaining considerable attention. In this method, the creep strain in a structure is evaluated using the reference stress (which has to be found from elastic and limit analysis) and the uniaxial creep data. Creep fatigue evaluation using elastic analysis is in a state of continuous change in the ASME code case 1592 [25]. Its use, however, is quite complex [36]. It has been shown to be overly conservative in some cases [21] while non-conservative in others.

3. Inelastic Fracture Analysis

Assessing the margin of safety of a flowed component during operation when

there is a general yielding, is in the forefront of fracture mechanics research [37, 38]. The developments in this field require analysts to have the ability to analyze a propagating crack and to develop a fracture criterion for a flow-growing crack.

On the other hand, the analysis of a stationary crack with a large plastic zone is relatively manageable with current methods. In reference [39], a finite element plain strain model was used to analyze a 3-point bend bar. The von Mises flow rule with the Romberg-Osgood stress strain relations and isotropic hardening were used because all theoretical developments for the strain singularity are based on power law hardening materials [37]. The crack tip elements used in this analysis were the quarter-point triangular elements [40, 41]. The results of this analysis were found to be in agreement with other finite element analyses using hybrid elements, simple triangular elements, etc. No experimental results, however, were used for comparison.

It was noticed that the finite element results gave large hydrostatic pressure near the crack tip when the plastic zone extended over the whole section. In addition, the results indicated no limit load. Reference [42] relates this problem to a high constraint in the element at or near the limit load due to the fact that the finite element formulation does not admit displacement fields that are strictly incompressible at the limit load. References [42 and 43] suggest a modified variational principle in which the dilatational strain increments are independent variables. On the other hand, tests performed with the quadratic isoparametric elements used in the analysis above were capable of reproducing the limit load for a beam in bending. Hence the problem above must be due to another representation error. Reference [44] suggests that large rotations occurring at the crack tip have to be accounted for in order to model the region accurately. As seen here, inelastic fracture applications are still limited to research, the use of constitutive equations is limited to simple models, and the failure criteria is in the development stage.

4. Plastic Buckling

The subject of plastic buckling is quite wide. The theoretical basis for plastic buckling is covered in reference [45]. The problem to be reviewed here is that of inelastic buckling of torispherical heads subject to internal pressure. Due to the compressive hoop stresses arising in the knuckle regions, heads with radius to thickness ratios greater than 500, exhibit plastic buckling as their controlling failure mode. The buckling mode is in the form of circumferential wrinkles.

References [46, 47] treat this problem as an axisymmetric shell using various higher order non-linear theories. The analysis is performed using harmonic superposition on an incremental basis. Reference [46] uses elastic analysis for evaluating the pre-buckling state and, then by assuming a buckled form, determines the smallest pressure for a nontrivial solution. If the stresses exceed yield, a reduced modulus is used. Though this method gives reasonable results, it ignores pre-buckling deformations, it does not account

for imperfections, and it cannot predict post buckling behavior. The second approach [47] accounts for pre-buckling deformations using an incremental elastic-plastic large deformation analysis. Because of the axisymmetric assumptions, however, it cannot deal with initial imperfections.

It was suggested [48] that imperfection sensitivity can be obtained from an elastic non-linear analysis by applying Koiter's method to obtain a reduction factor which is then applied to an analysis similar to that in Reference [47]. In an exploratory analysis [49], a general shell finite element elastic-plastic large deformation of a slightly imperfect shell was used to understand this complex behavior. The preliminary results lead to the following conclusions:

- a. Pre-buckling deformation must be included.
- b. The buckling load and number of wrinkles is dependent on the initial imperfections.
- c. Due to membrane tension in the shell, there exists essentially an infinite number of branching points that are stable. In order to find the buckling load, a perturbation method must be applied on the shell at various load increments.

In addition, it was felt that, due to the sensitivity of the problem, actual stress strain curves, anisotropic material behavior, and generation of corners in the yield surface, could have profound effect on the results. The latter point may be accounted for by the use of deformation plasticity theory, but which, unfortunately, is not available in most computer programs.

5. Residual Stresses due to Fabrication

The effect of residual stresses on the life of a component due to welding can be significant. The calculation of the thermomechanical history is quite complex. One of the most difficult problems here is obtaining material properties at these extreme temperatures and the rather unique changes in some of them at the transition temperature. Classical constitutive equations, generally used in these analyses, are a gross approximation of the actual behavior at these extreme temperatures. References [50-53] give various simplification to the problem. References [50, 51] found that the rate of metal depositing and welding technique have considerable effect on the results. Hibbitt [50] modeled the complete problem of melting and solidification. He found that an extremely high hydrostatic tension (about 3 times the yield stress) arises in the weld fillet. This was suspected to cause void growth and considerably affect incompressibility assumptions of plasticity and creep strains. As mentioned earlier [17], this hydrostatic tensile stress has considerable detrimental effect on the material ductility and its cyclic life. It was noted that upon modeling the stress relief process, most of the residual peak stresses are reduced considerably, whereas the hydrostatic tension changed slightly. This is expected since plastic and creep strains are only deviatoric.

Reference [52] gives an analysis of a bimetallic weld under cyclic loading. It considers the weld to be deposited as a hot ring and ignores the melting and

solidification process. The hydrostatic pressure predicted in this case was only 1/2 the yield stress. On the other hand, this analysis is concerned with the life under cyclic loading and, therefore, is quite justifiable in that current creep-fatigue data and procedures use only inelastic strains which are of the same order of magnitude as reported elsewhere. Reference [53] also gives an analysis of a bimetallic weld. It altogether ignores, however, the residual stresses due to welding and considers only the effect of the cyclic operation on the life.

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