STRUCTURALLY COMPATIBLE FLUID FINITE ELEMENT FOR SOLID FLUID INTERACTION STUDIES

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SUMMARY

This study develops a fluid finite element compatible with existing structural finite elements with the ultimate objective of analyzing solid fluid interaction problems. This type of problem is of interest in the design of nuclear reactor components involving geometric complexities and nonlinearities.

The differential equation governing the pressure distribution p in a laminar, compressible, two-dimensional flow subjected to small amplitude acoustical oscillations is:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{1}{c^2} \left(\frac{\partial^2 p}{\partial t^2} + k \frac{\partial p}{\partial t} \right) \quad (1)$$

in which c is the speed of sound and k is the viscous damping coefficient. Employing the Weighted Residual Method, this differential equation is discretized on plane triangular, plane quadrilateral, axisymmetric triangular, and axisymmetric quadrilateral finite element subdivisions of the fluid region. The elemental inertia, damping, and volumetric fluidity matrices are first derived. These matrices are then assembled for all system elements to obtain a set of coupled second order differential equations:

$$[G] \{\ddot{p}\} + [C] \{\dot{p}\} + [H] \{p\} = \{F\} \quad (2)$$

in which G , C , and H are the inertia, viscous damping, and volumetric fluidity matrices, p , $\dot{p}$, and $\ddot{p}$ are the nodal pressure array and its first and second time derivatives, and F is the contribution due to the pressure gradient at the boundary.

The structural dynamic capability of the WECAN (Westinghouse Electric Computer Analysis) digital computer program was expanded by incorporating the above fluid element model. This program uses the NASTRAN integration method and the wave front solution technique for the numerical solution of the above set of equations.

This finite element model is applied to the solution of a wave propagation flow problem consisting of fluid between two flat plates initially at rest and accelerated suddenly by applying a step pressure at one end. Three types of finite element grids (network) are considered: 1) 48 triangular element, uniform grid model; 2) 48 triangular element, non-uniform grid model; and 3) 108 quadrilateral element, uniform grid model. For each of the above networks, three cases of viscous damping are studied: inviscid, slightly viscous, and highly viscous flow. Response time histories, obtained numerically, are compared with the analytical solution derived by applying a Laplace Transform to the one-dimensional form of equation (1), and finding the inverse transform by the application of the Residue Theorem. A detailed examination of response time histories shows that the pressure and velocity time histories generally agree with the analytical solution except for certain spurious oscillations which have also been observed by other investigators in the numerical analysis of shock wave phenomena. These oscillations are very irregular for the non-uniform grid model and are more regular for the uniform grid. They have the highest frequency for nodes closest to elements with the smallest length and have the lowest frequency for nodes farthest from elements with the smallest length. This oscillatory pattern, caused by the finite element discretization, is much less predominant for highly viscous flow cases.

INTRODUCTION

Flow induced vibration is of particular interest in the design and analysis of nuclear reactor components. Interaction between fluid and solid has been recognized to be an important factor in the study of flow induced vibration and has been studied by many authors. Closed form solutions for simple geometries are available. Experimental studies have also been performed that confirm the importance of the solid-fluid interaction [1,2].

The hydrodynamic mass concept, frequently used in dynamic analyses, has the following limitations:

- (1) Nuclear components have complicated geometries and the hydrodynamic mass analyses are generally performed for simple geometries and have a limited range of applicability [3].
- (2) For problems involving impact, the use of hydrodynamic mass makes the response unrealistically conservative.

Representation of structures by the finite element technique has been well developed and documented in the literature [4,5]. During the recent years, finite element methods have been employed for the analysis of fluid flow problems [6]. Both incompressible and compressible flow problems, including wave propagation phenomena, have been investigated. It has been demonstrated that the finite element methodology has economic advantages for the computer-based solution of structural and fluid flow problems involving complicated space domains. Therefore, the solution of complex solid-fluid interaction problems would require the development of a compatible finite element approach for solid and fluid analyses.

Comprehensive, general-purpose digital computer programs for static and dynamic structural analyses by the finite element approach are presently available at many industrial organizations. These programs are equipped with many library subroutines for solid elements such as rods, beams, plates, shells, etc. These structural programs can be expanded to solve solid-fluid interaction problems by: (1) adding a number of fluid element library subroutines; and (2) establishing the interaction between the solid and fluid elements. The first step, in this direction, is the development of a fluid element model that can be incorporated in the structural programs. This development is the main purpose of this study.

It should be stated, at the onset, that despite the ever-increasing interest in the finite element approach to fluid flow problems, the ultimate utility of this method in flow analysis has not yet been established. This study examines the accuracy of results, numerical stability, and convergence properties of the fluid finite element in a rigorous competition against analytical solutions.

Specifically, this paper presents plane, axisymmetric, triangular, and quadrilateral finite element fluid models for wave propagation phenomena in a two-dimensional flow field. This fluid element is verified analytically by solving a wave propagation flow problem consisting of water between two flat plates initially at rest and accelerated suddenly by applying a step pressure at one end.

MATHEMATICAL FORMULATION

The simplifying assumptions employed in the development of the fluid element model are:

- (1) The fluid flow is assumed to be compressible and two-dimensional.
- (2) The fluid pressure is used as the main dependent variable. The availability of experimentally-measured pressure oscillations provides an attractive incentive to use nodal pressure as the nodal degree-of-freedom.

- (3) The fluid oscillations are assumed to be of small amplitude.
- (4) The fluid shear stress is assumed to be proportional to its velocity.

The differential equation governing the pressure distribution p in a two-dimensional flow subjected to small amplitude acoustical oscillations is given by the wave equation:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} + k \frac{\partial p}{\partial t} \quad (1)$$

in which c is the velocity of the acoustic waves and k is the viscous damping coefficient. Discretization of the above equation on finite element subdivisions of the fluid region leads to:

$$[G_e] \{\ddot{p}_e\} + [C_e] \{\dot{p}_e\} + [H_e] \{p_e\} = \{F_e\} \quad (2)$$

where $[G_e]$, $[C_e]$, and $[H_e]$ are the inertia, viscous damping, and volumetric fluidity matrices, $\{p_e\}$, $\{\dot{p}_e\}$, and $\{\ddot{p}_e\}$ are the elemental nodal pressure arrays, its first and second time derivatives, and $\{F_e\}$ is the contribution due to boundary integrals corresponding to the prescribed motion.

Assembling the elemental discretized equations for all n system elements, the final differential equations for the flow field are obtained:

$$[G] \{\ddot{p}\} + [C] \{\dot{p}\} + [H] \{p\} = \{F\} \quad (3)$$

The boundary conditions for equation (3) are of two types:

- (1) Nodes with specified pressure
- (2) Solid boundaries for which $\partial p / \partial n = 0$

NUMERICAL SOLUTION

Application of the finite element method to structural dynamic problems has received widespread attention. A variety of digital computer programs have been developed for the static and dynamic structural analyses by the finite element approach. These programs are well suited to the analysis of flow problems provided that their library of finite elements (such as beams, plates, shells, etc.) are expanded to include a number of fluid finite elements (such as plane or axisymmetric triangular or quadrilateral fluid elements). The WE CAN (Westinghouse Electric Computer Analysis) digital computer program was used in this analysis. The dynamic capability of the program was extended by incorporating the following fluid elements based on the formulation presented herein (shown in Figure 1):

- (1) Plane, triangular fluid element.
- (2) Plane, quadrilateral fluid element.
- (3) Axisymmetric, triangular fluid element.
- (4). Axisymmetric, quadrilateral fluid element.

The computer program assembles the system inertia, volumetric fluidity, and damping matrices and boundary arrays and solves equation (3) using the NASTRAN integration method [7]. This integration scheme employs an elementary central difference formula for the acceleration and velocity terms and an implicit representation of the displacement term as derived by Chan, Cox, and Benfield [8] in their extension of the Newmark's β -method [9]:

$$\frac{1}{\Delta t^2} [G] \{p_{n+2} - 2p_{n+1} + p_n\} + \frac{1}{2\Delta t} [C] \{p_{n+2} - p_n\} + [H] \{\beta p_{n+2} + (1-2\beta) p_{n+1} + \beta p_n\} = \{\beta F_{n+2} + (1-2\beta) F_{n+1} + \beta F_n\} \quad (4)$$

where n , $n+1$, $n+2$ are the past, present, and future (updated) values of the variables and β

is a parameter to be selected on the basis of numerical stability and accuracy. With $\beta = \frac{1}{3}$ equation (4) reduces to:

$$\left[\frac{1}{\Delta t^2} [G] + \frac{1}{2\Delta t} [C] + \frac{1}{3} [H] \right] \{p_{n+2}\} = \frac{1}{3} \{F_{n+2} + F_{n+1} + F_n\} + \left[\frac{2}{\Delta t^2} [G] - \frac{1}{3} [H] \right] \{p_{n+1}\} + \left[\frac{1}{\Delta t^2} [G] + \frac{1}{2\Delta t} [C] - \frac{1}{3} [H] \right] \{p_n\} \quad (5)$$

which constitutes a set of linear algebraic equations and is solved by a variation of the Gaussian elimination method known as the wave front technique [10].

Having calculated the updated values of the nodal pressures $\{p_{n+2}\}$, the velocity components are computed by the time integration of the momentum equations.

VERIFICATION OF RESULTS

The finite element model, developed in this study, is verified for a two-dimensional channel, 24.5 feet long and 2 feet wide shown in Figure 2. Water initially at rest is accelerated suddenly by applying a step pressure p_0 at $x = 0$ while maintaining a zero pressure at $x = L$. All geometric dimensions and flow variables, used in the analysis, are nondimensionalized so that the final solution is valid for similar problems.

Three types of finite element grids are considered:

- (1) 48 triangular element, uniform grid model (Figure 3)
- (2) 48 triangular element, non-uniform grid model (Figure 4)
- (3) 108 quadrilateral element, uniform grid model (Figure 5)

Three values of viscous damping are studied:

- (1) Inviscid case, $k = 0$
- (2) Slightly viscous case, $k = 64.0816 \text{ sec}^{-1}$
- (3) Highly viscous case, $k = 1046.6666 \text{ sec}^{-1}$

Response time histories were obtained both numerically and analytically for all 9 cases indicated above. Typical nodal pressure and elemental velocity for one position upstream and one downstream were plotted and compared. In view of the element size difference among the various models, the upstream and downstream positions could not be selected to be the same as seen in the following table:

FIGURE NO.	UPSTREAM POSITION NODE NO. x/L		DOWNSTREAM POSITION NODE NO. x/L	
(3)	9	0.235	25	0.806
(4)	17	0.204	29	0.653
(5)	29	0.204	92	0.755

For the sake of brevity, only six sets of the above time histories are shown in this paper, in Figures 6 through 11. The numerical results are verified by comparison with the analytical solution represented by dashed lines in these figures.

The NASTRAN integration method, used in this study, is an implicit numerical integration scheme and as such it exhibits a high degree of numerical stability. However, if the integration time step exceeds the upper bound required for convergence, the numerical solution will diverge from the true solution. The integration time step is calculated on the basis of the following considerations.

In view of the symmetry of the flow with respect to the x -axis, pressure at all node having the same abscissa are equal and transversal oscillations will not be excited. The

minimum period for longitudinal oscillations is equal to the product of $2\sqrt{t}$ times the ratio of the smallest element length (0.5 feet) to the speed of sound (5000 ft/sec) i.e., 0.000628 sec. Examination of computer results showed that this period was indeed present in the nodal time histories on the element with the smallest length. The integration time step was then set equal to 1/50th of this minimum period, i.e., .0000125 sec. This time increment led to a convergent solution for all cases studied. The convergence of the solution was established when doubling the time interval produced no change in the pressure and velocity time histories.

A detailed examination of the response time histories reveal the following features:

(1) Inviscid flow time histories

The inviscid flow pressure and velocity time histories, typically shown in Figures 6 and 7, exhibit an oscillatory behavior about the analytical solution. The pressure and velocity responses for the upstream and downstream nodes follow the analytically-calculated rectangular and staircase wave form respectively, except for certain spurious oscillations. Comparison of the uniform and nonuniform grid model responses (not shown) indicate that these oscillations have the highest frequency for nodes closest to elements with smallest length and have the lowest frequency for nodes farthest from elements with smallest length. This pattern confirms the basis of the selection of the integration time increment, presented earlier, that the element with the smallest length imposes an upper bound on the integration step size.

This oscillatory behavior is caused by the finite element discretization and not by the lack of convergence in numerical integration. Similar fluctuations have been observed by other investigators in the study of wave propagation in fluids using finite difference [11] and finite element formulation [12], when shock waves are generated in the fluid, or at the interface of two fluids at different entropy (such as shock tubes). Von Neuman and Richtmeyer [11] have suggested that shock waves could be simulated mathematically by the addition of terms representing viscosity and conductivity to the equations of non-viscous and non-conducting flow. Lax [13] introduced the effect of viscosity and heat-conduction implicitly through the manner in which derivatives are replaced by finite difference for the non-viscous flow. These "pseudo-viscosity" methods are also not free from spurious information. They produce unrealistic pressure fluctuations which grow with respect to time and sometimes they generate unrealistic flow reversals. Jonsson et al [14] describe a hybrid method combining the best features of the fixed-grid approach and the method of characteristics. They state that their method leads to more realistic solutions free from spurious fluctuations and physical implausibilities. This technique is in its early stages of development and its implementation is beyond the scope of the present study.

(2) Slightly viscous flow time histories

The pressure and velocity time histories for slightly viscous flow typically shown in Figures 8 and 9, exhibit a pattern similar to inviscid flow case and oscillate about the analytical solution. However, the pressure time history fluctuations cease to approach closely to the time axis indicating that ultimately a steady state pressure distribution will be reached. Similarly, for the velocity time histories, the staircase wave form changes its shape, the distance between the steps becomes shorter and the slope of each step becomes negative, again indicating the approach toward a fixed steady-state value.

(3) Highly viscous flow time histories

The pressure and velocity histories for highly viscous flow, typically shown in Figures 10 and 11, exhibit a highly damped behavior and closely follows the analytical solution. However, some remnants of the spurious fluctuations, observed in the inviscid and slight viscous cases, still are present in the finite element solutions. All pressure and velocity responses approach their final steady-state values.

CONCLUSIONS

Available finite element computer programs for structural dynamic analyses can be expanded to analyze fluid flow problems. This purpose may be achieved by adding a number of fluid finite element subroutines to the structural finite element library. This approach is also ultimately useful for analyzing solid-fluid interaction problems.

A fluid finite element model for plane triangular and quadrilateral elements as well as for axisymmetric triangular and quadrilateral elements is developed and verified analytically. Application of this finite element model to a wave propagation flow problem, using different grid configurations, demonstrates that the pressure and velocity time histories generally agree with the analytical solution except for certain spurious oscillations also observed by other investigators. These oscillations are very irregular for the non-uniform grid model and are more regular for the uniform grid. They have the highest frequency for nodes closest to elements with smallest length and have the lowest frequency for nodes farthest from elements with smallest length. This oscillatory pattern, caused by the finite element discretization, is much less predominant for highly viscous flow problems.

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Fig. 1. Plane, Axisymmetric, Triangular, and Quadrilateral Fluid Finite Element.

Fig. 2. Flow Configuration.

Fig. 3. 48 Triangular Element, Uniform Grid Model for Flow Between Two Parallel Plates.

Fig. 4. 48 Triangular Elements, Non-Uniform Grid Model for Flow Between Two Parallel Plates.

Fig. 5. 108 Quadrilateral Element, Uniform Grid Model for Flow Between Two Parallel Plates.

48 TRIANGULAR ELEMENT, UNIFORM GRID MODEL,
INVISCID FLOW

Fig. 6. Pressure and Velocity Time Histories for 48 Triangular Element, Uniform Grid Model for Inviscid Flow Between Two Parallel Plates at $x/L=0.235$.

48 TRIANGULAR ELEMENT, UNIFORM GRID MODEL,
INVISCID FLOW

Fig. 7. Pressure and Velocity Time Histories for 48 Triangular Element, Uniform Grid Model for Inviscid Flow Between Two Parallel Plates at $x/L=0.806$.

48 TRIANGULAR ELEMENT, NON-UNIFORM GRID MODEL,
SLIGHTLY VISCOUS FLOW

Fig. 9. Pressure and Velocity Time Histories for 48 Triangular Element, Nonuniform Grid Model for Slightly Viscous Flow Between Two Parallel Plates at $x/L=0.653$.

48 TRIANGULAR ELEMENT, NON-UNIFORM GRID MODEL,
SLIGHTLY VISCOUS FLOW

Fig. 8. Pressure and Velocity Time Histories for 48 Triangular Element, Nonuniform Grid Model for Slightly Viscous Flow Between Two Parallel Plates at $x/L=0.204$.

108 QUADRILATERAL ELEMENT, UNIFORM GRID MODEL,
HIGHLY VISCOUS FLOW

108 QUADRILATERAL ELEMENT, UNIFORM GRID MODEL,
HIGHLY VISCOUS FLOW

Fig. 11. Pressure and Velocity Time Histories for 108 Quadrilateral Element, Uniform Grid Model for Highly Viscous Flow Between Two Parallel Plates at $x/L=0.755$.

108 QUADRILATERAL ELEMENT, UNIFORM GRID MODEL,
HIGHLY VISCOUS FLOW

108 QUADRILATERAL ELEMENT, UNIFORM GRID MODEL,
HIGHLY VISCOUS FLOW

Fig. 10. Pressure and Velocity Time Histories for 108 Quadrilateral Element, Uniform Grid Model for Highly Viscous Flow Between Two Parallel Plates at $x/L=0.204$.