FAILURE OF TORISPHERICAL ENDS OF PRESSURE VESSELS DUE TO INSTABILITY AND PLASTIC DEFORMATION — AN EXPERIMENTAL INVESTIGATION

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SUMMARY

Twelve model pressure vessels with torispherical ends have been tested under internal pressure to investigate failure by instability and plastic deformation. The models covered combinations of three head heights and four thicknesses. All the thicker specimens with internal diameter/thickness ratios of 53, 106 and 212 failed by plastic deformation. The three thin specimens with internal diameter/thickness ratio of 530 failed by buckling of the torus due to the circumferential compressive stresses. The experimental results for limit pressure and instability failure are compared with theoretical values. The effect of change of geometry is significant particularly for the larger head heights. For these specimens the experimental limit pressure is higher relative to theoretical predictions than is the case for the smaller head heights.

A simple approximate theory is presented for predicting the pressure at which buckling occurs in the torus. The predictions are higher than the experimental values but are not as high as previously published predictions.

1. Experimental Work

The models were machined out of a solid bar of aluminium alloy (BS 1476 HE 30 WP). The material is very suitable for experimental investigations of limit pressure^{(1), (2)} because the strain hardening is negligible and the low value of Young's Modulus is useful for the instability studies. Fig.1 gives the nominal dimensions of the specimens. In general the variation from the nominal thickness and the geometrical accuracy was between + .025 mm and - .050 mm.

The material properties in the transverse plane of the billet most closely represented the material in the torispherical end and σ_y was taken as the 0.2% proof stress from test specimens cut from the billet in this plane.

The models were bolted to a base plate using a loose flange ring and an "O" ring seal and pressurised with oil.

Creep occurred beyond the elastic limit. Since no further oil was pumped into the specimen the pressure fell until the creep rate was below 10 micro strain per minute. Strain and deflection readings were recorded at this pressure.

About 40 electrical resistance strain gauges were fixed inside and outside the specimens to measure strains. Deflections were measured with Mercer deflection transducers. To detect buckling of the torus a displacement transducer was fixed to an arm which could be rotated round the circumference of the torus.

2. Results

Because of the effect of change of geometry it is difficult to define an experimental limit pressure. The procedure used was to examine the pressure-strain plots for every strain gauge and find the lowest pressure corresponding to a permanent plastic strain of 0.2% for one of the gauges. This has been defined as the experimental limit pressure $P_{0.2}$. The definition is probably conservative but has been used previously^{(1), (2)}. In addition, all the pressure-strain plots for each strain gauge were examined to find the lowest pressure at the limit of proportionality for one of the gauges and this is defined as P_{LOP} . Table 1 gives values of P_{LOP} and $P_{0.2}$ for all the specimens. The dimensionless experimental values P_{LOP}^* and $P_{0.2}^*$ are also given, obtained by dividing P_{LOP} and $P_{0.2}$ by P_Y where $P_Y = \frac{2 t \sigma_y}{D_i}$.

Table 1 gives the theoretical predictions of the limit pressure due to Shield and Drucker⁽³⁾ (upper and lower bounds) Calladine⁽⁴⁾, Townley et.al.⁽⁵⁾ and Taylor and Robinson⁽⁶⁾ made dimensionless with respect to $\frac{2 t \sigma_y}{D_i}$.

Fig.2 shows plots of hoop and meridional strains for specimen 3C. On this figure three curves are shown at three different pressures P_{LOP} , $P_{0.2}$ and higher pressures corresponding to a permanent plastic strain of about 0.4% ($P_{0.4}$) the purpose being to show the development and any redistribution of strain with increasing pressure. This figure also shows by an arrow head, the position of the strain gauge from which $P_{0.2}$ was found. Similar diagrams were plotted for all the other specimens.

Plots of the radial deflection for the corresponding pressures were plotted for all specimens but are not presented here.

For specimens 4A, 4B, 4C which failed by buckling of the torus, the buckles did not all form at the same time (see Fig.3). All buckles formed in an outward direction except

two on specimen 4A. Because of the high sensitivity of the measuring device for detecting the buckles, it was possible to note an incipient buckle before it was visible by eye. Fig.3 shows plots of radial deflection round the circumference of the torus for specimen 4B and the development of the buckles at various pressures. The first detection of the first buckle is shown on Fig.3 by the arrows. The half wave-length of a buckle was measured from these plots and the average values are given in Table 2. A photograph of specimens 4A, 4B, 4C after test is shown in Fig.4.

For specimen 4B only, at a pressure of 0.378 MN/m^2 the pressure was reduced slightly but the buckle (although very small) did not disappear. The buckle was plastic and not elastic.

3. Discussion of Results

With a very large number of strain and deflection readings on each of twelve specimens, it is impossible to draw attention to all the features of the results, but the following general points about the behaviour are noteworthy:-

3.1 Radial deflection and the effect of change of geometry

In order to assess the effect of change of geometry, the original profile of each test specimen was drawn four times full size. On this diagram, the radial deflections at $P_{0.2}$ were plotted. Between the points on the meridian of zero radial deflection a circular arc was fitted to approximate as closely as possible to the new radius of the torus. Similarly a circular arc corresponding to the new sphere radius was fitted through the experimental points for the radial deflection for the sphere to join the new torus radius. Table 1 gives the upper and lower bound values of the dimensionless limit pressure from ⁽³⁾ calculated for the new geometry which may be compared with the corresponding values in the same table for the original geometry.

3.2 Position at which $P_{0.2}$ was measured

For the specimens with external gauges only (Series 1 & 2), $P_{0.2}$ was obtained from the highest measured strain which was in the meridional direction in the torus and sphere respectively for specimens 1A and 1B and in the hoop direction in the torus for specimens 1C, 2A, 2B, 2C. For the specimens with internal and external gauges (Series 3 & 4), $P_{0.2}$ was obtained from the highest measured strain which was on the inside surface in the meridional direction in the torus for all specimens 3A, 3B, 3C, 4A, 4B, 4C.

3.3 Comparison of $P_{0.2}^*$ with theoretical predictions

A comparison of $P_{0.2}^*$ with theoretical predictions can be made from Table 1 and Fig.5. The value of $P_{0.2}^*$ is greater than or equal to the upper bound p^u from Shield and Drucker ⁽³⁾ based on the original geometry. Fig.5 clearly shows that it would have been useful to have strain gauges on the inside surface of the thicker specimens. This might have given values of $P_{0.2}^*$ less than p^u for the specimens with $r_1 = 10.16 \text{ mm}$.

It is also evident from Fig.5 that the effect of change of geometry is more pronounced for the specimens with larger head heights and that even allowing for this change of geometry the experimental values of $P_{0.2}^*$ are still higher than the predictions from Shield and Drucker.

3.4 Distribution of Strains

It is not possible to discuss in detail the distribution of hoop and meridional strains in all the specimens although much information is available from them on the effect

of head height and thickness, and change of geometry.

3.5 Specimens failing by buckling in the torus

The buckles at the end of the test (see photograph Fig.4) appear uniformly spaced round the circumference, but closer examination shows that the pitch is not uniform and the sequence of formation does not suggest a regular pattern. The buckling mode is of a short wave length round the circumference. As the pressure and N_θ increases, one buckle becomes plastic and rapidly develops a large amplitude in the radial direction. Further buckles develop as the pressure and N_θ are further increased. The wide pressure range over which buckles develop suggests the influence of change of geometry. The short wave-length behaviour is similar to that of the behaviour of a strut on an elastic foundation and a simple analysis of the problem on these lines is given in the Appendix.

Table 2 gives the upper and lower bound limit pressures predicted by Shield and Drucker⁽³⁾, the buckling pressures predicted by Thurston and Holston⁽⁷⁾ and the predictions of the theory given in the Appendix for the buckling pressure and the half wave-length of the buckles.

In using the method given in the Appendix, the maximum value of N_θ in the torus in the linear elastic range was deduced from the experimental strain gauge readings on the inside and outside surfaces making allowance for the variation in thickness. The value of N_θ/P is given in Table 2. The value of $2\sqrt{\frac{k Et^5}{12}}$ varies with ϕ and the value in the Table is the lowest value which occurs at $\phi = \phi_0$.

As discussed by Kemper⁽⁸⁾ the predictions of a plastic limit pressure analysis⁽³⁾ are much closer to the actual buckling pressure than any elastic prediction. This is confirmed in Table 2 and Fig.5. At the experimental buckling pressure the structure is near its limit pressure and the stress resultants are near to yield. The buckling is a plastic and not an elastic phenomena.

It is also of interest to compare the measured wavelengths of the buckles (see Fig.3) with the theoretical predictions of the Appendix - see Table 2.

4. Conclusions

The values of the experimental limit pressure, $p_{0.2}^*$, are generally higher than the upper bound predicted by Shield and Drucker⁽³⁾ especially for the higher head heights where the effect of change of geometry is a major feature of the results. If inside gauges had been used on the thicker specimens with the smaller head heights, experimental values of limit pressure slightly lower than p^u due to Shield and Drucker might have been obtained.

For the specimens failing by buckling of the torus the failure pressure is much lower than the predicted values either by previously published theory⁽⁷⁾ or the theory presented here. Plastic limit pressure results give a better prediction of the buckling pressure except for a large head height.

5. Appendix - Buckling of the torus region of a torispherical end under internal pressure

The following is a crude engineering approximation and makes no pretence to theoretical rigour.

Assume points on the torus undergo a small radially outward displacement w (independent of θ). r = torus radius, r_2 is the principal radius of circumferential curvature.

Using Hooke's law, the displacement gives strains which cause membrane forces

$$\bar{N}_\theta = \frac{Et}{1-\nu^2} (\bar{\epsilon}_\theta + \nu \bar{\epsilon}_\phi) = \frac{Etw}{1-\nu^2} \left(\frac{1}{r_2} + \frac{\nu}{r} \right)$$

$$\bar{N}_\phi = \frac{Et}{1-\nu^2} (\bar{\epsilon}_\phi + \nu \bar{\epsilon}_\theta) = \frac{Etw}{1-\nu^2} \left(\frac{1}{r} + \frac{\nu}{r_2} \right)$$

These membrane forces cause a radially inward force per unit area of shell =

$$\frac{\bar{N}_\theta}{r_2} + \frac{\bar{N}_\phi}{r}$$

This may be looked on as a "restoring force" arising from the radial displacement w . A circumferential strip of the torus of unit width (subject to a compressive force due to the vessel internal pressure) may be considered as a strut on an elastic foundation of stiffness.

$$k = \frac{Et}{1-\nu^2} \left[\frac{r^2 + 2 \nu r r_2 + r_2^2}{r^2 r_2^2} \right] \quad \begin{array}{l} \text{force/unit radial displacement} \\ \text{/unit circumferential width of shell} \\ \text{/unit meridional width of shell} \end{array}$$

For a strut on an elastic foundation, the buckling load is given by $P = 2 \sqrt{k EI}$ ⁽⁹⁾ and the buckles have a half wave length $= \pi \sqrt[4]{\frac{EI}{k}}$. For a unit width of shell, I was taken as $\frac{t^3}{12}$.

Because r_2 is a function of ϕ , the value of k varies round the torus but the variation is fairly small (about 8% at most). The maximum value of N_θ for unit internal pressure was found from the measured strains.

The buckling pressure is then given by

$$p_{cr} = 2 \sqrt{\frac{k EI}{N_\theta}}$$

References

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TABLE 2
BUCKLING RESULTS

Specimen	Exp. buckling pressure (MN/m ²)	Shield & Drucker ref (3) (MN/m ²)		Thurston & Holston ref.(7)		Appendix			b mm	b exp + .25 mm
		LB	UB	°	Buckling pressure (MN/m ²)	$2\sqrt{\frac{kE}{12}} \frac{t^3}{12}$ (MN/m)	$\frac{N_{\theta}}{P}$ m	Buckling pressure (MN/m ²)		
4A	.241	.228	.292	27.33	1.49	.275	.217	1.27	2.59	2.5
4B	.378	.295	.368	25.86	1.34	.186	.144	1.30	3.15	3.0
4C	.758	.415	.516	23.44	2.61	.127	.076	1.67	3.81	3.8

Specimen	H_i	t	r_i	Strain gauges	
				Intern.	Extern.
1A	24.05	2.540	10.16	x	✓
2A	24.05	1.270	10.16	x	✓
3A	24.05	0.635	10.16	✓	✓
4A	24.05	0.254	10.16	✓	✓
1B	27.18	2.540	15.24	x	✓
2B	27.18	1.270	15.24	x	✓
3B	27.18	0.635	15.24	✓	✓
4B	27.18	0.254	15.24	✓	✓
1C	32.08	2.540	22.86	x	✓
2C	32.08	1.270	22.86	x	✓
3C	32.08	0.635	22.86	✓	✓
4C	32.08	0.254	22.86	✓	✓

Fig.1 Test Specimen

Fig.2

Specimen 3C strain distributions

Fig.3

Development of the buckles in Specimen 4B

Fig. 4

Fig. 5

Comparison of the theoretical and experimental limit pressures.