GENERATION OF ARTIFICIAL TIME-HISTORIES, RICH IN ALL FREQUENCIES, FROM GIVEN RESPONSE SPECTRA

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SUMMARY

In order to apply the time-history method of seismic analysis, it is often desirable to generate a suitable artificial time-history from a given response spectrum. The method to be described in this paper allows the generation of such a time-history that is also rich in all frequencies in the spectrum. This richness is achieved by choosing a large number of closely-spaced frequency points such that the adjacent frequencies have their half-power points overlap. The adjacent frequencies satisfy the condition that the frequency interval Δf near a given frequency f is such that

$$(\Delta f)/f < 2c/c_c$$

where c is the damping of the system and c_c is the critical damping. In developing an artificial time-history, it is desirable to specify the envelope and duration of the record, very often in such a manner as to reproduce the envelope property of a specific earthquake record, and such an option is available in the method described. Examples are given of the development of typical artificial time-histories from earthquake design response spectra and from floor response spectra.

1. Introduction

In the design of nuclear power plants, it has been found desirable in certain instances to use the time-history method of dynamic analysis to determine the plant response to seismic input. In the implementation of this method, it is necessary to determine an adequate representation of the excitation as a function of time. Because many design criteria are specified in terms of design response spectra, one is faced with the problem of generating a time-history whose own response spectrum approximates as closely as possible the originally specified design response spectrum. One objective of this paper is to present a method of synthesizing such time-histories from a given design response spectrum. The design response spectra may be descriptive of floor responses at a particular location in a plant, or they may be descriptive of seismic ground motions at a plant site. The spectra recommended by Newmark, Blume, and Kapur [1] are representative of the latter, and are shown in Figure 1.

In generating time-history representations of seismic ground motions, two points demand particular attention. First, the time-history should be recognizable by a designer as a realistic earthquake motion. While this view is subjective, it is quantified in our approach by the introduction of a modulating envelope. We ensure that the envelope of the final time-history follows the specified modulating envelope. Thus, the envelope shape of the synthesized time-history may be specified to be the envelope of a known earthquake record. We have used the envelope of the 1940 El Centro N-S record in the examples given in this paper.

The second point arises when we wish to excite the plant structural model with simultaneous seismic excitation components in two perpendicular horizontal directions and in the vertical direction. Usually, the specified vertical spectrum differs from the horizontal spectrum, so that the corresponding time-histories differ as well. However, it is desirable in any event to generate two horizontal time-histories, which should be as unrelated as earthquake records themselves are. Statistically, one desires that their correlation coefficients should be low. In the second section of this paper, we suggest a method of generating a second time-history from the first time-history such that they have low correlation, while at the same time have response spectra that approximate to a satisfactory degree the original design response spectrum. Finally, we inspect a number of alternative schemes of dual time-history generation, and discuss the resulting correlation functions and spectra.

2. Time-History Generation from Given Spectra

A number of methods have been devised for generating the time-history of an artificial earthquake whose spectra are replicas of given design response spectra. We cite the work of Vanmarcke and Cornell [2], Scanlan and Sacks [3], Tsai [4], and Rizzo et al [5]. The methods of Tsai and Rizzo et al. essentially depend on the manipulation of

the amplitude and phase of a Fourier representation of an existing accelerogram trace, for example of the 1940 El Centro N-S record. Using this record as input, they obtained a trial response spectrum. For frequencies where the trial response spectrum was higher than desired, a "filtering" action is applied to the initial time-history. For frequencies where the trial response was lower than desired, a damped sinusoid of that frequency was added to the initial time-history. The modified time-history was then used to compute a second response spectrum. The process of modifying the time-history and the computation of new response spectra was repeated until a satisfactory agreement between computed and desired response spectra was achieved. The last time-history used was taken as the desired time-history.

We wish to describe here another simple method which differs from the above in certain important respects. First, to ensure a full coverage of the frequency range, a large number of closely spaced frequencies are chosen such that they cover the frequency range of the desired response spectrum. Adjacent frequencies satisfy the condition

$$\frac{\Delta f}{f} < 2 \left(\frac{c}{c_c} \right) \quad (1)$$

where Δf is the frequency separation at frequency f and $\left(\frac{c}{c_c} \right)$ is the damping ratio. The choice is such that the half-power points of adjacent frequencies overlap. Since the desired time-history is considered to have inputs at each of these frequencies, it is apparent that it will be "rich" over the full range of frequency and no region of the desired frequency range will not be covered. This criterion leads to the conclusion that the number of frequency points within a frequency range f_0 to f_n is at least $\ln(f_n/f_0) / \ln(1 + 2c/c_c)$. Thus, typically, for 2% critical damping, there should be at least 86 separate frequency components within the frequency range from 1 Hz to 30 Hz. In general, we use a few more frequency components than this number in order to include particular frequencies specified in the input.

To take into account the time-severity characteristics of actual earthquakes, an envelope curve, $F(t)$, is used. For example, $F(t)$ could be the positive envelope to the El Centro N-S earthquake. In that case, $F(t)$ is at all times positive and piecewise linear, and has a value of about 0.3 g at about 2.5 seconds, a value of 0.1 g at about 4.0 seconds, a value of about 0.2 g at about 5 seconds, etc., with a final value of about 0.05 g at about 30 seconds.

The acceleration time-history $H(t)$ of the earthquake is expressed by

$$H(t) = F(t) \sum_{i=1}^N (-1)^i A_i \sin(2\pi f_i t) \quad (2)$$

where the coefficients A_i are to be determined, the frequencies f_i have already been specified to have a spacing such that successive frequencies have overlapping half-power points, and N is the total number of frequencies required to cover the frequency range.

The values of A_i 's are assumed to vary linearly in magnitude between frequencies at which the desired response spectrum is specified. The use of the factor $(-1)^i$ in eq.(2) improves the solution.

The final step in the procedure is to determine the coefficients A_i . These are found using an iteration process. Initially, the magnitudes of the A_i 's are taken to be proportional to the corresponding response spectrum g values at frequencies f_i on the desired response-spectrum curve. Using the initial trial time-history, a response spectrum curve is computed and compared with the desired response spectrum curve. For the second trial time-history, the new magnitudes of the A_i 's are obtained by multiplying the initial values of the A_i 's by the ratio of the desired response spectrum at frequency f_i to that computed in the first iteration. This iteration process is continued until the computed response spectrum is close enough to the desired one.

It has been found that some time-histories generated by this method have non-zero velocities and displacements at the end of the event. These quantities can be reduced to zero if desired, by the addition to the time-history of eq.(2) of a small acceleration term $At + Bt^2$, whose constants are adjusted in each iteration cycle.

The computation of the response spectrum curve involves the determination of the maximum relative displacement response of single degree-of-freedom systems having frequencies covering the desired frequency range. This computation is done using a numerical integration procedure. In particular, a solution is obtained for the equation of motion of a single degree-of-freedom system described by the equation

$$m \frac{d^2 (y + x)}{dt^2} + c \frac{dy}{dt} + ky = 0 \quad (3)$$

where x is the ground motion, y is the relative displacement, t is time, c is the damping, m is the mass and k is the stiffness. The maximum absolute value of (ky/gm) is taken as the pseudo-acceleration on the response spectrum curve corresponding to a frequency $(1/2\pi) \sqrt{k/m}$.

As an example of the results obtainable by this method, we illustrate in Figure 2 the convergence to the design response spectrum of Figure 1 for 2% critical damping. The first three iterations are shown in Figure 2. The time-history that is derived is shown in Figure 3. It was obtained using the 1940 El Centro N-S envelope described above. Naturally, it is not unique. There are many other time-histories (actually an infinite number) whose spectra could approximate the design response spectra. However, by using the envelope curve and the duration of a realistic earthquake, a time-history is generated that has characteristics that are in reasonable agreement with physical reality. The accuracy with which the spectra match may be improved either by using more iterations or by increasing the number of points specifying the input design spectrum. In this instance, we have used the twelve frequencies describing the design spectrum break points.

Floor response spectra may show several peaks and valleys. However, the generation of time-histories from given floor response spectra by the present method poses no difficulty. For example, in Figure 4 is shown the original of a floor response spectrum together with points obtained during the iteration to the final time-history. The fourth iteration cycle shows good agreement with the originally specified spectrum. Figure 5 shows the corresponding final time-history.

3. Generation of Statistically Independent Time-Histories

We wish to obtain two time-histories that are to represent the two perpendicular horizontal components of excitation. We are given a single design response spectrum, for example that of Figure 1. The two time-histories are to exhibit statistical properties similar to observed earthquake records, in that they are to have a low correlation. We start by generating as before a first time-history $H_1(t)$:

$$H_1(t) = F(t) \sum_{i=1}^N (-1)^i A_{1i} \sin(2\pi f_{1i}t) \quad (4)$$

While there are numerous ways of perturbing the coefficients A_{1i} and f_{1i} of this time-history in order to produce a second time-history, we have found by numerical experimentation that a satisfactory second time-history can be generated by choosing the frequencies f_{2i} and the amplitudes A_{2i} of the second time-history to be midway between those of the first time-history. Thus, we set

$$H_2(t) = F(t) \sum_{i=1}^N (-1)^i A_{2i} \sin(2\pi f_{2i}t) \quad (5)$$

where

$$f_{2i} = 1/2 (f_{1i} + f_{1(i+1)})$$

$$A_{2i} = 1/2 (A_{1i} + A_{1(i+1)})$$

We shall inspect the results of using other schemes in the next section.

An example of the results obtained is shown in Figure 6. In Figure 6a we show the original design response spectrum, from which was generated the time-history $H_1(t)$ shown in Figure 6b. Its response spectrum is given in Figure 6a, where it can be compared with the original spectrum. The dual time-history $H_2(t)$ generated by means of eq. (5) has the properties shown in Figures 6c and 6d. Its spectrum differs only slightly from that of $H_1(t)$.

We now inspect the correlation of $H_1(t)$ and $H_2(t)$. We first define the autocorrelation coefficient R_1 of $H_1(t)$ as

$$R_1 = \frac{1}{T} \int_0^T [H_1(t)]^2 dt \quad (6)$$

where T is the duration of the record (usually T is between 10 and 40 seconds; in this case, it is 30 seconds). A similar autocorrelation coefficient is defined for $H_2(t)$. The

correlation between $H_1(t)$ and $H_2(t)$ is given by the correlation function $\Psi(\tau)$:

$$\Psi(\tau) = \frac{1}{T \sqrt{R_1 R_2}} \int_{\tau}^T H_1(t) H_2(t - \tau) dt \quad (7)$$

where τ is the delay time between the start of H_1 and H_2 . In general, from a structural analysis viewpoint, we are interested in utilizing the time-histories as excitations with zero time delay. Consequently, we say that the two histories have low correlation when the correlation coefficient $\Psi(0)$ is small. However, it is of interest to inspect the properties of $\Psi(\tau)$ and this function is shown in Figure 6e. There, we observe a value of $\Psi(0) = 0.02$ while a peak of 0.3 occurs at $\tau = 0.46$ seconds.

An interesting survey has been made by Chen [6] of the correlation coefficients for the two horizontal components of strong motion accelerograms recorded at 104 sites. He shows that the mean correlation coefficient $\Psi(0)$ for these records is of the order of 0.16 with a minimum of 0.0014, a maximum of 0.68, and standard deviation of 0.21. With this observation as a guide, it is apparent that the present method can generate two time-histories that have satisfactorily small correlation, since the absolute mean value in Figure 6e is comparable to 0.16.

4. Correlation Studies for Other Methods of Generation

We have inspected other methods of generating dual time-histories, three of which will be described here. They consist of first, the imposition of a 90° phase shift on all the frequency components; second, the introduction of a random phase with uniform probability distribution in each frequency component; and third, the use of a random shift in the frequency f_i with a uniform probability distribution between the adjacent mid-frequencies $1/2 (f_i - f_{i-1})$ and $1/2 (f_{i+1} - f_i)$. We note that the use of random phase shifts have been previously discussed by others, notably by Vanmarcke and Cornell [2] and by Scanlan and Sacks [3].

The results of a sample series of calculations are shown in Figure 7. There, we show for each method of generation, the spectra of H_1 and H_2 , together with the original design spectrum, the resulting time-history $H_2(t)$, and the correlation function $\Psi(\tau)$ as defined by eq.(7). A number of observations can be made through the inspection of Figures 6 and 7.

First, the spectra of the time-histories determined from the mid-frequency shift and from the 90° phase shift are not far different from the spectrum of the original time-history. However, the spectra of the time-histories generated by random phase and frequency shifts are significantly different -- different enough for these particular samples that the methods may be rejected on these grounds as being unsatisfactory.

Second, all methods appear to give satisfactorily low correlations at zero time delay. Peaks in the correlation functions occur at other delay times: in particular, a peak of

0.6 occurs for the 90° phase shift for a time delay of 0.1 second. This is the only peak for which significant correlation exists, and may preclude the use of this method. The correlation coefficients $\Psi(0)$ of the time-histories generated by a random phase shift and by a random frequency shift are, for these particular samples, larger than those generated by the two deterministic algorithms.

Finally, we note that the time-histories determined by the mid-frequency shift and the 90° phase shift have correlation coefficients $\Psi(0)$ that are very small -- in the order of 0.01. However, it might be argued that the correlation should also be low for time delays τ longer than any memory possessed by the structure. In linear systems, this memory would derive from the structural damping. Thus, as a rough measure, the required delay span should be larger than the time necessary for the structural amplitude to decay to a fraction $1/a$ of its original amplitude. Assuming free vibrational response, we obtain the approximation

$$\tau \lesssim \frac{2na}{2\pi(c/c)I} \quad (8)$$

where f is the lowest frequency component. If we require that the structure decays to $1/\sqrt{2}$ of its original excitation, and if f is 1 Hz, then $\tau \lesssim 2.7$ seconds. Thus, the correlation functions exhibited in Figures 6 and 7 should be of long enough duration to determine if the time-histories are adequately uncorrelated.

5. Summary

In this paper, we have presented a method for generating artificial time-histories from given design response spectra. A criterion was given for the determination of the spacing of the frequency components necessary for the synthesis of the time-history. An envelope curve was introduced to provide modulation so that the final time-history may resemble a known earthquake record. In order to generate a dual time-history whose spectrum closely resembles that of the first time-history, and yet which shows little correlation with the first time-history, we introduced a method based on shifting the frequencies of the first record to the adjoining mid-frequencies. Several other methods were discussed based on other shifting algorithms. They did not, however, yield completely satisfactory results either in one or both of the correlation or spectrum requirements. Based on the example calculations, we recommend the use of the mid-frequency shift for the production of adequately uncorrelated dual time-histories whose spectrum is close enough to the originally specified design spectrum.

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1 Seismic Design Response Spectra¹

2 Response Spectrum Curves for 2% Critical Damping

3 Time-History for Design Response Spectrum --
1940 El Centro N-S Envelope, at Iteration 3

4 Floor Response Spectrum -- Input Spectrum and
Computed Spectrum

5 Time-History for Floor Response Spectrum

6 Response spectra time histories, and correlation function of two time histories. The original design spectrum is denoted by D , while H_1 and H_2 denote the two independent time histories.¹

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Response spectra, time histories and correlation functions. The properties are generated by a 90° phase shift (a - c); a random phase shift (d - f); and a random frequency shift (g - i).