NON-LINEAR DYNAMIC RESPONSE OF REACTOR CONTAINMENT

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SUMMARY

In the seismic analysis of nuclear power plants, the time history method is used as one of the techniques of dynamic analysis.

Many problems such as an idealization of dynamic model, an assessment of damping coefficients and the soil-structure interaction are involved in this method. Since the time history method is usually carried out in the elastic range, the ultimate behavior of structure still remains unknown after those problems were solved to some extent.

Thus, a computer program was developed to investigate the elasto-plastic behavior of structures.

The purpose of this paper is to present the outline of this program and to discuss the problems of non-linear response of structures.

There are two theoretical methods in the time history, that is, the mode superposition method and the direct integration method.

Since the mode superposition method is only valid in an elastic analysis, the direct integration method was adopted here.

As the sample model, an actual reactor containment (reactor building) of PWR plant was adopted.

This building consists of three components, that is, a concrete internal structure, a steel containment vessel and a concrete outer shield wall. These components are resting on a rigid foundation mat. Therefore they were modeled with a lumped mass model respectively and coupled on the foundation.

The following assumptions were employed to establish the properties of dynamic model:

1. Rocking and swaying springs of soil can be obtained from an elastic half-space solution, and the hysteretic characteristic of springs is bi-linear.
2. Springs connecting each mass are dealt with shear beams so that both bending and shear deflections can be included. Hysteretic characteristics of springs are linear, bi-linear and tri-linear for the internal structure, the containment vessel and the outer shield wall, respectively.
3. Generally, each damping coefficient is given for each mode in modal superposition. However, a damping matrix must be made directly in a non-linear response. Therefore the damping matrix of the model was made by combining the damping matrices [C] of each component obtained by Caughey's method and a damping value of the rocking and swaying by the half-space solution.

On the basis of above conditions, the non-linear response of the structure was obtained and the difference between elastic and elasto-plastic analysis is presented.

1.0 Introduction

In the seismic analysis of nuclear power plant the time history method is used as one of the techniques of dynamic analysis.

Many problems such as an idealization of dynamic model, an assessment of damping coefficients and the soil-structure interaction are involved in this method.

Since the time history method is usually carried out in an elastic range, the ultimate behavior of structure still remains unknown after those problems were solved to some extent.

Thus, a computer program was developed to investigate the non-linear behavior of structure.

The purpose of this paper is to present the outline of this program and is to discuss the problems about non-linear response of structure.

2.0 Equations of Motion

Following equations of motion were employed in this program. And symbols used in these equations are illustrated in Fig. 1. When eliminate the damping which will be added to equations later, the elasto-plastic equation of motion of arbitrary mass m_i in a plastic range can be written as follows.

$$-m_i(\ddot{Z} + \ddot{X}_i + \ddot{\theta}h_i) + \{Q_{i+1} + K_{i+1}(X_{i+1} - X_i - \delta_{i+1})\} - \{Q_i + K_i(X_i - X_{i-1} - \delta_i)\} = 0 \quad (1)$$

When K_i and $K_{i+1} = 0$, and substitute $eK_i X_i$ and $eK_{i+1} X_{i+1}$ for Q_i and Q_{i+1} , eq.(1) becomes a elastic equation.

Equations of motion of swaying and rocking in an elastic range can be given by eq.(2) and eq.(3) respectively.

$$-m_s(\ddot{Z} + \ddot{X}_s) - K_s X_s + K_1(X_1 - X_s) = 0 \quad (2)$$

$$-I_r - \sum_{i=1}^n m_i(h_i \ddot{\theta} + \ddot{X}_i + \ddot{Z})h_i - K_r \theta = 0 \quad (3)$$

Eq.(2) and (3) can be easily transferred to the plastic equation like eq.(1).

Where $X, \ddot{X}$ = displacement and acceleration

$\theta, \ddot{\theta}$ = rotational angle and acceleration

$\ddot{Z}$ = ground motion

δ = yield displacement at which a significant change in spring constant occurs

Q = yield shear force at which a significant change in spring constant occurs

K = spring constant after yielding

I_r = rotatory inertia

h = height of each mass from foundation mat.

Suffixes of each symbol represent the nodal points. Equations of motion have been solved using a direct integration method, because the plastic deformations of springs are allowed. The intergration was carried out by the "linear acceleration method".

By this method the duration of the earthquake is divided into small intervals of time (Δt).

And Δt will be further divided into 10 equal intervals at stiffness changing points.

3.0 Properties of dynamic model

The dynamic model employed in the analysis is illustrated in Fig. 2 and its properties are shown in Table I and Table II.

3.1 Soil-Structure Interaction

The swaying spring can be obtained from eq.(4) of an elastic half-space solution.

$$K_s = 2\pi aG/(2-\nu) \quad (4)$$

The rocking spring in an elastic range can be written as follows

$$K_r = K_v' I_f = \pi a^3 G/4(1-\nu) \quad (5)$$

Where K_v' = vertical coefficient of soil = $G/a(1-\nu)$
 I_f = moment of inertia of foundation = $\pi a^4/4$
 a = radius of foundation 19.035 m
 G = shear modulus 2.02×10^6 t/m²
 ν = poisson's ratio 0.36

Non-linear rocking spring is considered in a following assumption. If a bearing capacity of subgrade is large enough not to be yielded, the rocking spring will be in a plastic range when a foundation mat detach from the subgrade.

The relationship between moment and rotational angle can be given by following equations and symbols in these equation are illustrated in Fig. 4.

$$N = \int_{l_x - a}^a Y K_v' dA \quad (6)$$

From eq.(6) the maximum settlement Y_m can be written as follows :

$$Y_m = N (2a - l_x) / 2K_v' \left[\frac{1}{2} A \left\{ \frac{\pi a^2}{2} + A \sqrt{B} - a^2 \sin^{-1} \left(-\frac{A}{a} \right) \right\} + \frac{B^2}{3} \right] \quad (7)$$

Thus moment will be:

$$M = \int_{l_x - a}^a Y K_v' x dA = \frac{2 \cdot Y_m K_v'}{2a - l_x} \left[\frac{1}{3} A B^{\frac{3}{2}} + \frac{1}{8} \left\{ \frac{\pi}{2} a^4 - 2 A B^{\frac{3}{2}} + a^2 A B^{\frac{1}{2}} - a^4 \sin^{-1} \left(\frac{A}{a} \right) \right\} \right] \quad (8)$$

And

$$\theta = Y_m / (2a - l_x) \quad (9)$$

From eqs. (8) and (9) the nonlinear rocking spring can be obtained as follows:

$$K_r = M/\theta \quad (10)$$

Where N = total weight imposed on the subgrade: given value

l_x = length detached from the subgrade

$A = a - l_x$

$B = a^2 - A^2$

$M - \theta$ relationships under different N are shown in Fig. 5.

Where a hatched area within circles represents a part of foundation mat which has touched to the subgrade. For the dynamic response the curve No. 2 was adopted and idealized as shown in Fig. 6. Two kinds of non-linear rocking spring, i.e., bilinear type and spring type as illustrated in Fig. 3 were considered in the response analysis.

3.2 Springs of Superstructure

As mentioned before this building consists of three component, i.e., a concrete internal structure (hereinafter called CIS), a steel containment vessel (SCV) and a concrete outer shield wall (OSW). The structural design for CIS and SCV would be carried out primarily to accident loads because the accident loads give larger effect to them. However seismic loads are significant for OSW, therefore OSW has been mainly studied and discussed in this paper. In the elastic range the springs connecting each mass were dealt with shear beams so that both bending and shear deflections could be included. The plastic springs of OSW were established by making the following assumptions or procedures;

- 1) Each member is dealt with a cylindrical column in spite of having been dealt with a cylindrical shell in actual design.
- 2) Stress-strain relationship are subjected to Fig. 7.
- 3) Reinforcing bar on tensile side yields first and the strain at yielding is 0.00167 and the stress σ_y at that time is 3,500 kg/cm².
The moment calculated under these conditions is symbolized M_{y1} .
- 4) Concrete on compressive side yields at its strain 0.002 and the stress σ_y at that time is 178.5 kg/cm² (0.85 F_c). The moment at this stage is symbolized M_{y2} .
- 5) Elastic moments M_e of each member were computed by modifying shear coefficients obtained from two earthquake waves which will be used for the nonlinear response.
- 6) Hysteresis of each member is determined by the ratio of M_{y1}/M_e and M_{y2}/M_e and deflections which were calculated using curvatures at the time of M_{y1} and M_{y2} occurred.

As one example the hysteresis of member No. 5 is shown in Fig. 8.

3.3 Damping

Since each component of structures has different damping the damping matrix would have been composed by superposing each damping matrix which could be obtained on each component.

Procedures to compose the damping matrix are as follows.

- 1) To make each damping matrix of each component using following equation.

$$[C] = a_0 [M] + a_1 [K] \quad (11)$$

Where $[C]$ is the damping matrix; $[M]$ is the mass matrix; and $[K]$ is the stiffness matrix.

a_0 and a_1 can be obtained from following equations.

$$2 h_1 \omega_1 = a_0 + a_1 \omega_1^2 \quad (12)$$

$$2 h_2 \omega_2 = a_0 + a_1 \omega_2^2 \quad (13)$$

Where ω_1 and ω_2 are the natural frequency, h_1 and h_2 are per-centage of critical damping in mode 1 and 2 respectively.

In this analysis 5 per cent of critical damping for ICS and OSW, 2 per cent for SCV were assumed in the first two modes.

2) To make the damping matrix of superstructure by composing each damping matrix given by eq. (11).

3) To add the damping of rocking and swaying to the damping matrix of superstructure.

Where the per-centage of critical damping of them are assumed to be 10 per cent. Equivalent damping coefficients could be calculated from the damping matrix produced by means of the above mentioned procedure and they are shown in Fig. 9. It can be apparently mentioned from eq. 12 that the damping matrix depends on how the natural frequencies are given. Therefore two kinds of natural frequencies were studied and discussed, i.e., one was those of coupled model including rocking and swaying springs, the other was those who were calculated in assuming each component to be independent structure with a fixed end. The former result is shown by curve (a) and the latter is shown by curve (b) in Fig. 9. Besides the damping coefficients in higher modes in case that any component of coupled model has 5 per cent of critical damping are shown by curve (c) and those in case of 2 per cent of critical damping are shown by curve (d) in Fig. 9. Considering the damping in each mode, curves (a) and (b) may have to approach to the curve (c) when the mode of ICS or OSW is predominant and they may have to approach to the curve (d) when the mode of SCV is predominant. These considerations have been employed in the dynamic response by means of mode superposition method. Thus, to investigate which curves are preferable the mode shapes were calculated and shown in Fig. 10. From Figs. 9 and 10 it may be deduced the curve (a) is to be applied for the response analysis.

4.0 Results of Response and Discussions

Two earthquake waves of El. Centro NS 40 and Golden Gate EW were used with modifying 300 gal, 450 gal and 600 gal. Periods of vibration are shown in Table III and results of the dynamic response are shown in Tables IV and V. The ratio of shear and moment presented in Table IV are the ratio of maximum shears and moments of each member, therefore, the times of which stresses occur do not agree necessarily.

According to these tables it becomes obvious the ratio of top response to the ground motion in OSW decreases with increase of ground acceleration in El Centro NS 40 and it is constant in Golden Gate EW.

To study on this phenomenon the natural periods of structure in which members Nos. 4 and 5 and the rocking spring had been yielded were calculated and are shown in Table III. The bilinear type and the spring type of rocking spring are used for 600 gal

earth-quake, however the spring type gives larger response than the bilinear. It is because a energy dissipation by hysteresis loop may be expected in the bilinear type. Hysteresis loop of member No. 5 subjected to 600 gal earthquake of Golden Gate is shown in Fig. 11 in which two types of rocking spring i.e., the bilinear type and the spring type are compared and illustrated. In Fig. 11 1 to 2 or 3 to 4 represent domains where the rocking spring are plastic. This loop shows the spring type of rocking gives a larger deflection which means a greater response to the superstructure because it has a longer plastic range. The maximum responses of swaying are proportional to input accelerations, however, those of rocking do not increase in proportion to increases of them, since the formers occurred before and the latters occurred after the yielding of superstructure.

5.0 Summary

From this study, it may be summarized as follows:

- 1) Hysteresis of spring type must be employed in the rocking spring, if the bearing capacity of subgrade is high enough to resist a overturning moment.
- 2) Even if a part of foundation mat detached from the subgrade, it would not give much effect to the superstructure.
- 3) When a damping matrix was composed by Caughey's method, it may be considered natural frequencies of entire structure should be adopted to a structure consisted of different kinds of material like this building.

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TABLE - I MODEL PROPERTIES-1
(Spring Constant of Each Member)

	MEMBER	TYPE OF HISTERESIS	K_1 (t/cm, tcm/rad)	K_2	K_3	δ_y (cm, rad)	δ_u
O S W	1	Bilinear	14440.0	0.0		0.430	
	2	Bilinear	14709.0	0.0		0.518	
	3	Bilinear	15950.0	0.0		0.587	
	4	Trilinear	18271.0	1209.0	121.0	0.469	2.16
	5	Trilinear	22675.0	2377.0	238.0	0.371	1.20
	6	Bilinear	32099.0	10285.0		0.3659	
C I S	7	Linear	65020.0				
	8	Linear	85690.0				
	9	Linear	227946.0				
S C V	10	Bilinear	2915.0	163.0		9.87	
	11	Bilinear	3623.0	203.0		10.34	
	12	Bilinear	5405.0	303.0		5.02	
	13	Bilinear	9174.0	514.0		2.20	
F	Swaying	Linear	0.1473×10^7				
	Rocking	Bilinear, Spring	0.1710×10^{13}	0.2567×10^{12}		0.3216×10^{-4}	

$K_2 = 0$ means shear yield occurs before flexure yield.

TABLE - II MODEL PROPERTIES-2
(Weight at Each Node)

	NODE	WEIGHT (t, tcm ²)	HEIGHT (cm)
O S W	1	1069.77	6605.3
	2	2160.31	5653.8
	3	2181.19	4683.8
	4	2181.19	3713.8
	5	2181.19	2743.8
	6	2198.04	1773.8
C I S	7	3722.24	1963.8
	8	3347.39	1388.8
	9	5316.70	763.8
S C V	10	158.37	5953.8
	11	541.74	4632.8
	12	518.52	3083.8
	13	416.40	1773.8
F	Swaying	33597.0	
	Rocking	0.5231×10^{11}	

TABLE - III NATURAL PERIOD

	ELASTIC	PLASTIC
1	0.25405	0.67297
2	0.14583	0.16961
3	0.08882	0.15783
4	0.08403	0.11191
5	0.05683	0.08435
6	0.05490	0.06110
7	0.04383	0.05489
8	0.03934	0.04818
9	0.03906	0.04672
10	0.03567	0.03934
11	0.03400	0.03898
12	0.03218	0.03518
13	0.03122	0.03127
14	0.02705	0.02731
15	0.02197	0.02205

TABLE - IV MAXIMUM RESPONSE (MEMBER)

	MEMBER	300 gal			450 gal			600 gal - 1			600 gal - 2		
		DEFL. (cm, rad)	SHEAR (ton)	MOMENT (t.m)	6450 /6300	Q450 /Q300	M450 /M300	6600 /6300	Q600 /Q300	M600 /M300	6600 /6300	Q600 /Q300	M600 /M300
EL. CENTRO	OSV	1	0.100	1445.	13753.	1.37	1.37	1.37	1.53	1.53	1.53	1.57	1.57
		2	0.276	4060.	53135.	1.38	1.38	1.38	1.50	1.50	1.51	1.51	1.53
		3	0.383	6112.	112425.	1.31	1.31	1.34	1.38	1.38	1.44	1.38	1.45
		4	0.405	7391.	184119.	1.23	1.16	1.27	1.67	1.17	1.33	1.67	1.17
		5	0.360	8167.	263341.	2.08	1.06	1.21	2.57	1.08	1.26	4.53	1.14
		6	0.264	8484.	346910.	1.14	1.14	1.19	1.22	1.22	1.25	1.22	1.22
	CIS	7	0.044	2830.	16980.	1.50	1.51	1.51	2.06	2.06	2.06	2.13	2.13
		8	0.050	4296.	43831.	1.52	1.52	1.52	2.07	2.07	2.07	2.14	2.14
		9	0.024	5464.	68693.	1.50	1.50	1.51	2.04	2.04	2.04	2.08	2.08
	SCV	10	0.076	222.	2936.	1.50	1.50	1.50	2.00	2.00	2.00	1.97	1.97
		11	0.255	924.	17244.	1.50	1.50	1.50	2.00	2.00	2.00	1.95	1.95
		12	0.255	1378.	35290.	1.50	1.50	1.50	2.00	2.00	2.00	1.95	1.95
		13	0.169	1550.	53270.	1.50	1.50	1.50	2.00	2.00	2.00	1.95	1.95
GOLDENGATE	P	Swaying	0.013	19236.		1.50	1.50		2.00	2.00		2.00	2.00
		Rocking	0.266 x10 ⁻⁴	0.454 x10 ⁶	1.20		1.20	1.48		1.25	1.50		1.26
	OSV	1	0.080	1160.	11037.	1.51	1.51	1.51	1.98	1.98	1.98	2.07	2.07
		2	0.223	3273.	42787.	1.50	1.50	1.50	1.81	1.91	1.91	1.90	1.91
		3	0.303	4838.	89713.	1.50	1.50	1.50	1.72	1.72	1.72	1.73	1.83
		4	0.332	6059.	148489.	1.39	1.39	1.46	1.68	1.42	1.65	1.81	1.42
		5	0.295	6699.	213466.	2.07	1.28	1.40	3.24	1.32	1.55	3.46	1.32
		6	0.209	6721.	279668.	1.40	1.40	1.40	1.52	1.52	1.53	1.53	1.55
	CIS	7	0.030	1973.	11836.	1.50	1.50	1.50	2.00	2.00	2.00	2.00	2.00
		8	0.042	3578.	34198.	1.50	1.50	1.50	2.00	2.00	2.00	2.00	2.00
		9	0.025	5662.	59959.	1.50	1.50	1.50	2.00	2.00	2.00	2.00	2.00
	SCV	10	0.108	315.	4157.	1.50	1.50	1.50	1.97	1.97	1.97	2.09	2.09
		11	0.355	1287.	24088.	1.50	1.50	1.50	1.95	1.95	1.95	2.02	2.02
		12	0.345	1865.	48522.	1.50	1.50	1.50	1.96	1.96	1.96	2.04	2.04
		13	0.227	2080.	72646.	1.50	1.50	1.50	1.96	1.96	1.96	2.00	2.03
	P	Swaying	0.013	19034.		1.50	1.50		2.00	2.00		2.00	2.00
		Rocking	0.212 x10 ⁻⁴	0.362 x10 ⁶	1.41		1.41	2.22		1.62	2.22		1.62

HISTERESIS OF ROCKING

600 gal - 1 : Bilinear
600 gal - 2 : Soft Spring

TABLE - V MAXIMUM RESPONSE (TOP NODE)

	NODE	300 gal		450 gal		600 gal - 1		600 gal - 2	
		DISP. (cm)	Acc max /Z _{max}	D450 /D300	Acc max /Z _{max}	D600 /D300	Acc max /Z _{max}	D600 /D300	Acc max /Z _{max}
EL. C.	1 (O S W)	1.951	4.49	1.36	4.10	1.56	3.44	1.55	3.53
	7 (C I S)	0.129	2.53	1.49	2.56	2.07	2.62	2.07	2.70
	10 (S C V)	0.806	4.58	1.48	4.60	1.99	4.60	1.89	4.51
G.G.	1 (O S W)	1.561	3.54	1.49	3.56	1.79	3.54	1.87	3.68
	7 (C I S)	0.128	1.73	1.51	1.73	2.01	1.74	2.01	1.74
	10 (S C V)	1.121	6.49	1.50	6.52	1.89	6.40	2.00	6.84

Fig.-1. Explanation of Symbols used in eq. (1) (3)

Fig.-2. Dynamic Model

Fig.-3. Hysteresis of Rocking

Fig.-4. Relation ship between Moment and Rotational angle.

Fig.-5. M-θ Curve of Rocking

Fig.-6. Idealized Elasto-Plastic Rocking Spring (N = 5959.0 Ton)

Fig.-7. Assumed Strain-Stressblock

Fig.-8. Skelton Curve of Member 5.

Fig.-9. Equivalent Damping Coefficient

Fig.-10. Mode Shapes of Vibration

Fig.-11. Hysteresis Loop of Member 5.