A THEORETICAL AND EXPERIMENTAL INVESTIGATION OF CREEP PROBLEMS WITH VARIABLE TEMPERATURE

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SUMMARY

Although there has been extensive investigation of the creep behaviour of structures which are subject to isothermal conditions and constant load, the behaviour when temperatures vary spatially and both load and temperature vary with time has received relatively little attention. Numerical solutions are extremely difficult to produce under these circumstances and appropriate constitutive relationships have yet to be evolved.

This paper attempts to delineate the principal features of the behaviour when load level are maintained at those appropriate to many design situations and when the temperature and loading histories are cyclic with relatively short cycle times. We are concerned with structures which accumulate creep strains of less than 1% per year and have cycle times of the order of a few days at most.

In the first section the behaviour of a few simple structures are investigated for spatially varying temperature fields which remain constant in time. Adopting an appropriate form of Norton's Law we show that the deformation of the structure may be related to a single reference material test conducted at a reference stress and a reference temperature, which is independent of material constants, thereby providing a generalization of the reference stress method for isothermal structures. A sequence of experiments on a simple beam structure indicates that the co-relation between structural behaviour and material tests provides an acceptably accurate design method. In all cases considered the reference temperature remains close to the lowest temperature in the structure indicating that locally high temperature may sometimes be tolerated without excessive structural deformation.

The last section discussed a preliminary experimental investigation of a two-bar structure subject to variable temperature. It is shown that the residual stress field varies quite slowly in time and remains effectively constant after a few cycles. The theoretical consequences of the result are discussed and it is shown that constitutive relationships with differing physical assumptions can yield quite sharply contrasting deformation rates.

The results of the paper show that the behaviour of structures subject to a time constant temperature distribution may be related to material behaviour without difficulty. When temperature and load vary with time, the more important feature of the structural behaviour may be understood, although certain features of the material behaviour remain ill-defined.

1. Introduction

The analysis of the creep deformation behaviour of structures subject to variable loading and temperature remains amongst the least tractable problems of structural mechanics. A number of phenomena interact with each other. Changes in temperature induce thermal expansion and have an ill-defined effect upon the material behaviour. The continuum problem requires the evaluation of a continuously varying stress history. Even when computed to an acceptable accuracy such solutions remains only as accurate as that of the constitutive relationship.

This paper attempts to shed some light on the importance of these various effects by the analysis of a very simple two bar structure, by means of a method of structural analysis which arises from certain bounding theorems. In a number of previous papers [1,2,3] a theory was derived for a non-linear viscous material which allows the evaluation of upper and lower bounds on the energy dissipated in a cyclically loaded structure. These solutions correspond to the exact solution when the cycle time is either very short (upper bound) or very long (lower bound) compared with a characteristic time scale of the average deformation rate. This reference time scale may be taken as the time for the creep strain, in the steady state, to be equal to the elastic strain at either an average or maximum stress in the structure [1]. Consideration of typical time scales indicates that in most applications cycle time may be considered to be very short and hence the upper bound solution may be expected to provide a relevant solution which should closely approximate the exact solution. A full description of these arguments may be found in the references cited above.

If we accept the relevance of the upper bound solution, we may compute corresponding solutions for any constitutive relationship, and this theory is described in reference [4]. Here we compute these rapid cycling solutions for three constitutive relationships, non-linear viscous, strain hardening and the Bailey-Orowan model. The objective is to see if any general modes of behaviour are discernable. We find that the solution for the viscous and strain hardening materials exhibit similar behaviour. For distant ranges of stress and temperature the solution may be understood as resulting from the dominance of the strains occurring during either that part of the cycle when the stress is greatest or when the temperature is greatest. A material parameter β is introduced in terms of which regions may be defined where a reference stress may be defined which is independent of other material parameters. This result indicates that a reference stress approach is applicable to variable temperature problems, but that the relevant reference stress depends upon the range of values of this quantity β .

However when the solution for the Bailey-Orowan model is investigated, no such regions occur and a distinctly different pattern of deformation is exhibited due to the presence of recovery. This difference occurs as the model predicts a much greater strain rate for a history of stress and temperature which involves a high stress at a lower temperature followed by a low stress at a high temperature.

Experiments on the simulation of a two-bar model involving a coupled pair of uniaxial testing machines are described in the full paper. Tests on aluminium indicate that the rapid cycle solution is achieved within a few cycles, and that recovery is present, but of a magnitude which is much less than that predicted by the Bailey-Orowan model.

2. The Two-Bar Structure

Consider the model exhibited in Fig. 1. Two bars of equal initial length are restrained to remain of equal length under the action of a constant load $P = 2\sigma_p A$, where A denotes the cross-sectional area of each bar. One bar remains at a constant temperature $\theta_2 = \theta_0$ whereas the second bar is subject to a temperature history

$$\theta_1 = \theta_0 + \Delta\theta, \quad 0 \leq t \leq \Delta t/2$$

$$\theta_1 = \theta_0 - \Delta\theta, \quad \Delta t/2 \leq t \leq \Delta t$$

The thermo-elastic solution is given by

$$\left. \begin{aligned} \hat{\sigma}_1 &= \sigma_p - \sigma_t \\ \hat{\sigma}_2 &= \sigma_p + \sigma_t \end{aligned} \right\} \quad 0 \leq t < \Delta t/2$$

$$\left. \begin{aligned} \hat{\sigma}_1 &= \sigma_p + \sigma_t \\ \hat{\sigma}_2 &= \sigma_p - \sigma_t \end{aligned} \right\} \quad \Delta t/2 \leq t < \Delta t$$

where $\sigma_t = E\alpha\Delta\theta/2$ and α and E denotes the coefficient of linear expansion and Young's modulus respectively.

Consider a non-linear viscous material,

$$\dot{\epsilon} = \frac{\dot{\sigma}}{E} + \dot{v}, \quad \dot{v}(\sigma, \theta) = k \sigma^n \exp(-\Delta H/R\theta) \quad (1)$$

where k denotes a material constant, n an odd integer, ΔH an activation energy and R the universal gas constant. For sufficiently small changes in θ we may write

$$\dot{v} = k' \sigma^n \exp(\gamma(\theta - \theta_0)) \quad (2)$$

where $k' = k \exp(\Delta H/R\theta_0)$ and $\gamma = \Delta H/R\theta_0^2$

When the cycle time Δt is small, the cyclic solution is given by

$$\sigma_1 = \hat{\sigma}_1 + \rho, \quad \sigma_2 = \hat{\sigma}_2 - \rho \quad (3)$$

where ρ denotes a constant residual stress which is determined from the compatibility condition over a cycle

$$\Delta v = \frac{\Delta u}{l} = \int_0^{\Delta t} \dot{v}_1 dt = \int_0^{\Delta t} \dot{v}_2 dt, \quad (4)$$

where l denotes the length of the bars, and Δu the displacement accumulated over a cycle.

When Δt is very large, the instantaneous solution becomes the steady state solution given by

$$\dot{v}_1 = \dot{v}_2$$

$$\text{i.e.} \quad \sigma_1^n \exp(\gamma(\theta_1 - \theta_0)) = \sigma_2^n \exp(\gamma(\theta_2 - \theta_0))$$

The accumulated displacement over a cycle is given by,

$$\Delta v = \frac{\Delta u}{l} = \int_0^{\Delta t} \dot{v}_1 dt = \int_0^{\Delta t} \dot{v}_2 dt.$$

It may easily be shown that the ratio $U = \Delta U^U / \Delta U^L$ is dependent upon three parameters, σ^P / σ^T , n and

$$\beta = \frac{2 \gamma \Delta \theta}{n} \quad (5)$$

The physical meaning of β arises from the relationship between the creep rates which occur at $\theta_0 - \Delta\theta$ and $\theta_0 + \Delta\theta$. Consider tests conducted at these two temperatures. Raising the temperature from $\theta_0 - \Delta\theta$ to $\theta_0 + \Delta\theta$ will increase the creep rate. Suppose the same increase in creep rate is caused by maintaining the temperature at $\theta_0 - \Delta\theta$ but increasing the stress to $\lambda\sigma$. Then

$$\dot{\gamma} = k' (\lambda\sigma)^n \exp(-\gamma\Delta\theta) = k' \sigma^n \exp(\gamma\Delta\theta).$$

Hence $\beta = \lambda n \lambda$. In a graph of $\ln \dot{\gamma}$ against $\ln \sigma$ at constant θ then β becomes the distance between the lines corresponding to the two temperatures, as shown in Fig. 2.

In Fig. 3 contours of constant $U = \Delta U^U / \Delta U^L$ are exhibited for $n = 3$ and a range of values of σ^P / σ^T and β . It can be seen that the contours exhibit four distinct regions which we will discuss in turn.

Region 1. $U > 1$. In this region, which corresponds to smaller values of β , the most severe increase in displacement rate is shown for small cycle times. Consider the extreme case when $\beta = 0$, i.e. the creep rate independent of temperature. The rapid cycling solution is self-evident and is shown in Fig. 4(a). Effectively all the deformation occurs when the stress is largest and equal to $\sigma_p + \sigma_t$ and $\rho = 0$. If we completely ignore the creep strain which occurs when the stresses have their lower values and evaluate U we obtain the dashed line shown in Fig. 3. It clearly can be seen that these lines closely approximate the exact solution throughout this region. Hence the deformation is effectively equal to that of the structure subject to a constant load $P = 2A(\sigma^P + \sigma^T)$ with half the cycle time and temperature $\theta_1 = \theta_0 - \Delta\theta$, $\theta_2 = \theta_0$, followed by a lower or zero applied load over the remainder of the cycle. This approximation is least accurate near the boundary between Region 1 and Region 3.

Region 2. $U \approx 1$. This region corresponds approximately to $\beta > 2$ and $\sigma_p / \sigma_t > 1$. As U is near unity then the slow and rapid cycling solutions are nearly identical in their prediction of displacement rate. As β is large then the creep rates during the first part of the cycle when the highest temperature occurs provides the major contribution to the displacement. In bar 1, although the temperature remains constant the stress reduces during $\Delta t/2 < t < \Delta t$. In bar 2 the stress increases but the temperature reduces and the large value of β ensures that the creep rates in $0 < t < \Delta t/2$ dominates. In fact, in the rapid cycling solution the stresses are virtually identical to those of the slow cycling solution during $0 < t < \Delta t/2$ and the contribution from the second half of the cycle is negligible, resulting in a value of U close to unity. In this region the deformation may be assumed to be equal to one half of that which would occur if the condition of the first part of the cycle remained constant in time.

Region 3. $U < 1$. This region corresponds to $\sigma_p / \sigma_t < 1$ and lines of constant U become independent of β for large β . If we consider the case when β is very large the behaviour may easily be understood. During $0 < t < \Delta t/2$, σ_1 becomes very small ($\rho \approx \sigma_p + \sigma_t$) and either negative or positive, and during $\Delta t/2 < t < \Delta t$ although $\sigma_1 = 2\sigma_t$ as $\theta = \theta_0 - \Delta\theta$

the creep rate is small. Hence the deformation is governed by σ_2 which changes from $2\sigma_p$ to $2(\sigma_p - \sigma_t)$. As $\theta = \theta_0$ in this bar the displacement rate is independent of β . When $\sigma_p/\sigma_t = 0.5$ the net accumulation of strain is zero. Hence for $1 < \sigma_p/\sigma_t < 0.5$, $1 > U > 0$ and for $0.5 < \sigma_p/\sigma_t < 0$ then $U < 0$. In this latter case the displacement rate in the rapid cycle solution is of opposite sign to the applied load σ_p , and a load of $\sigma_p = 0.5\sigma_t$ is required to maintain zero displacement rate.

It can clearly be seen that these simple solution regions are quite distinct and are separated by regions in which fairly rapid transition occurs. The most marked transition occurs near the origin of Fig. 3 where large changes in U occur for small changes in σ_p/σ_t . For larger values of n a similar picture emerges except that the regions become more distinct, the transitions occupying a smaller area of the diagram. These transitional regions are defined by the contours of U of values close to unity. In Fig. 5 contours of $U = 0.99$ and 1.01 are shown for $n = 3, 5$ and 7 , and they are seen to be very close to each other. For $U = 0.00$ the contours are indistinguishable. Contours for $U = 50$ are also included to demonstrate that within the regions (with the exception of Region 2) the value of U is dependent of n , but the stresses are governed by the calculation described above. We see therefore that within Regions 1 and 2 the rapid cycling solution may be co-related with the behaviour of the same structure subject to constant load and constant temperature. For Region 1 the relevant applied load is $P = 2A(\sigma_p + \sigma_t)$ with $\theta_1 = \theta_0 - \Delta\theta$ and $\theta_2 = \theta_0$. In Region 2, $P = 2A\sigma_p$ and $\theta_1 = \theta_0 + \Delta\theta$ and $\theta_2 = \theta_0$. These situations may themselves be co-related with a constant reference and temperature history, as described by Ponter and Walter [5]. In Region 3, a variable stress history, fluctuating between $2\sigma_p$ and $2(\sigma_p - \sigma_t)$ at temperature $\theta = \theta_0$, defines the displacement rate.

3. Strain Hardening

In this section we describe a similar analysis for a strain hardening material. Consider the uniaxial strain hardening model

$$\dot{v} = m B^{1/m} \sigma^{n/m} v^{(m-1)/m} \quad (6)$$

For constant stress commencing at $t=0$ when $v=0$

$$v(t) = B \sigma^m t^m, \quad B = B' \exp \gamma (\theta - \theta_0)$$

In accordance with experiment we take $m = 1/3$. The rapid cycling solution is given by [4]

$$\frac{\Delta v}{\Delta t} = \left\{ \frac{1}{\Delta t} \int_0^{\Delta t} B^{1/m} (\hat{\sigma} + \rho)^{n/m} \right\}^m m t^{m-1} \quad (7)$$

where $\hat{\sigma}$ and ρ have the same meaning as in the time hardening case. Again ρ becomes determinate from the condition that $\Delta v/\Delta t$ shall be equal in the two bars. In fact the calculation for ρ becomes identical to the time hardening case with n/m substituted for n and $\Delta H/m$ substituted for ΔH .

There exist no lower bound solution, but for purposes of providing a normalization comparable with the time hardening case, the solution was computed so that the integrand of (7) remains equal in each bar during $0 < t < \Delta t$. This solution corresponds to assuming that the cycle time is sufficiently long for redistribution to occur at each instant

within the cycle but that the average creep rate is calculated assuming that, from the point of view of the constitutive relationship, the cycling is rapid. This solution has no direct physical meaning but will reduce to the solution arising from equation (7) if the conditions of Region 2 occur.

The displacements predicted by these two methods are denoted by ΔU^U and ΔU^L and their ratio by U .

We find that the solutions so generated have a behaviour which is very similar to that of viscous material and corresponds to the conditions described above. The boundary lines of the regions are shown in Fig. 6, and are seen to be very similar to these in the viscous case Fig. 5. There are however differences in the values of U as the relationship (6) provides a higher value of the creep rate under varying stress, when compared with constant stress maintained at the maximum value than does the viscous relationship. The stress histories themselves are however very similar and are divided into the same sub-regions.

We conclude therefore that the behaviour of a viscous material and a strain hardening material are very similar when described in this way, and the same reference stress histories are relevant.

4. Bailey-Orowan Theory

This theory is described in references [6]. The state of the material is described by an internal flow stress s which increases due to strain hardening and decreases due to thermal softening,

$$\dot{s} = h(s) |\dot{\epsilon}| - r(s) \quad (8)$$

where $h(s)$ and $r(s)$ are coefficients of strain hardening and thermal softening respectively. The creep rate $\dot{\epsilon}$ is given by,

$$\dot{\epsilon} = \text{sign}(\sigma) f(|\sigma| - s) \quad (9)$$

where

$$f(|\sigma| - s) = 0, \quad |\sigma| < s$$

$$\geq 0, \quad |\sigma| = s.$$

Stationary state creep occurs when $|\sigma| = s$ and hence

$$\dot{\epsilon} = \text{sign}(\sigma) r(s) / h(s).$$

Assuming $r(s) = k_1 s^{n-\alpha}$ and $h(s) = 1/k_2 s^\alpha$ yields Norton's flow

$$\dot{\epsilon} = \text{sign}(\sigma) k_1 k_2 |\sigma|^n, \quad k_1 k_2 = k' \exp(\gamma(\theta - \theta_0)).$$

This model differs from both the viscous relationship, equation (1) and strain hardening, equation (6), by possessing thermal softening. When $|\sigma| < s$, then $\dot{s} = -r(s)$ and s decreases in time. If the stress is now suddenly increased, $\dot{s} = \dot{\sigma}$ and plastic strains occur according to

$$\dot{\sigma} = h(s) \dot{\epsilon}$$

For a cycle of stress, both plastic and creep strains occur. But in common with the viscous

relationship, an upper bound on the energy dissipation in a body of the material corresponds to assuming that the cycle time is very short compared with characteristic material times [6]. The stress history is given by $\sigma = \hat{\sigma} + \rho$, and the accumulation of strain over a cycle is given by

$$\frac{\Delta v}{\Delta t} = \text{sign}(\sigma(t_0)) |\sigma(t_0)|^n \frac{1}{\Delta t} \int_0^{\Delta t} b' \exp(\gamma(\theta - \theta_0)) dt.$$

where t_0 is the instant during the cycle when $|\sigma|$ achieves its maximum value. Hence the average strain rate is the same as if $\sigma = \sigma(t_0)$ throughout the cycle. The model predicts the same creep rate as the viscous relationship (1) for constant stress and a greater creep rate for any other stress history, the increase being due to the presence of recovery in the model.

The lower work bound is identical to that of viscous relationship, equation (1), described in Section (2), and the ratio of the displacements predicted by these two bounding solutions, U , may therefore be directly compared with the results in Section (2).

It can easily be shown that the upper bound average displacement rate of the two bar model is the same as if a constant applied load $P = 2A(\sigma_p + \sigma_t)$ were applied throughout the cycle, with one bar maintained at $\theta_2 = \theta_0$ and the other at $\theta_1 = \bar{\theta}$ where

$$\exp(\gamma(\bar{\theta} - \theta_0)) = \frac{1}{2} (\exp(\gamma \Delta \theta) + \exp(-\gamma \Delta \theta))$$

In Fig. 7 contours of constant U are shown for $n = 3$. This solution is appropriate in Region 1, but in Region 2 $|\sigma_1|$ achieves its maximum value when $\sigma_1 = \pm \sigma_t$ and $\rho = -\sigma_p$. The average creep rate in bar 1 becomes indeterminate and becomes determined by the stress history σ_2 . In bar 2, the stress fluctuates between $2\sigma_p + \sigma_t$ and $2\sigma_p - \sigma_t$ and hence the displacement rate is the same as if a constant applied load of $P = 2A(\sigma_p + \sigma_t)$ were maintained and both bars were at temperature $\theta = \theta_0$.

The behaviour shown in Fig. 7 is entirely different to that shown in either Figs. 3, 5 or 6. No reverse creep occurs and the ratio U remains at high values for large β , as recovery increases. Effectively the Region 1 of Fig. 3 now dominates a larger area of the diagram.

These calculations demonstrate a central problem of this type of calculation. When temperatures vary rapidly between two limits, the stress history, in the cyclic state, fluctuates between a higher stress at a lower temperature and a lower stress at a higher temperature. The rate of deformation is strongly governed by the amount of recovery which occurs during the high temperature period. To the authors knowledge no experiments have been conducted under these conditions.

In the full paper, experiments on a simulated two bar structure are described which allows some assessment of the behaviour of aluminium under these conditions. The temperature histories differ in detail from those assumed in Section 2 and the experiments were conducted in Region 1 of both Fig. 3 and Fig. 7. These experiments indicate the following:

- 1) The stress histories of the upper bound solution are achieved within a few cycles of 24 hours duration over a range of average strain rates.
- 2) Recovery does occur, but it appears to be somewhat less than that predicted by the Bailey-Orowan model.

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Fig 1.

Fig 4.b) Solution for \$\beta\$ large, Region 3 (\$P = -\sigma_p + \sigma_1\$)

Fig 2

Fig 4.a) Solution for \$\beta = 0\$, Region 1 (\$P = 0\$)

Fig 3. Contours of constant, \$U = \Delta u^0 / \Delta u^L\$ for a viscous material with \$n = 3\$.

Fig. 5 Contours of constant U for $n=3, 5$ and 7 , for viscous material.

Fig. 6 Contours of constant U for $n=3, 5$ and 7 , for strain hardening material.

Fig. 7 Contours of constant U for the Bailey-Crowan model and $n=3$.