SOME APPLICATIONS OF BEAM, COLUMN AND PLATE THEORY TO PROBLEMS OF REACTOR FUEL ELEMENT DEFORMATIONS

R. G. ANDERSON

*United Kingdom Atomic Energy Authority, The Reactor Group,
Reactor Fuel Element Laboratories, Springfields, Salwick, Preston PR4 0RR, United Kingdom*

SUMMARY

Nuclear reactor fuel designers have always accepted that the design fuel life will not be met by every fuel element. Most early discharges of nuclear reactor fuel elements have been associated with exposure of the fuel to the coolant. Such failures have dominated the design and development of gas and water-cooled thermal reactor fuel. It is less well-known that a number of early discharges of fuel have taken place in thermal reactors because of fuel element deformations which have occurred without loss of clad integrity.

One example is the discharge of certain Uranium-Magnox fuel elements because of the unexpected development of extensive waviness on the heat transfer surfaces. A second example is the discharge of some early Prototype SGHWR fuel because unacceptable lateral deformations might occur after the initial axial clearances had been closed up by unexpectedly large fuel pin length increases. The importance of fuel element deformations increases for fast reactors. It is considered, for example, that the voidage-induced bowing of fast reactor fuel element wrappers rather than pin failure could limit the fuel life.

It is, therefore, timely to review the development of the physical understanding of fuel element deformations during life. The important features of these deformations are established in this paper using approximations leading to a simple theoretical analysis of fuel deforming in the presence of creep. This theoretical treatment is applied illustratively to the following problem areas:

- (1) the change of thermal bow as the bending moments induced by lateral restraints relax due to creep,
- (2) the causes of waviness of the heat transfer surfaces of certain Uranium-Magnox fuel elements,
- (3) the estimation of lateral deflections in lengthening fuel pins which are restrained axially,
- (4) the restraint of fast reactor sub-assembly bowing.

It is concluded from these examples that many of the characteristics of fuel element deformations during life can be obtained using quite simple ideas and mathematical analyses. The methods thus provide a useful tool at the design stage and complement the much more expensive computer solutions of structural problems.

1. Introduction

The design of nuclear reactor fuel, cores and circuits has always been dominated by fuel pin failure modes in which fuel is exposed to the coolant. Information on released fission products is then used to decide the discharge time for the fuel. It is less well known that there have been discharges of fuel caused by fuel element deformations which have not involved loss of clad integrity. Anderson [1,2] has discussed two examples that have occurred in the United Kingdom: Magnox fuel discharged because of waving of the heat transfer fins and Prototype SGHWR fuel discharged because of uncertain lateral deformations of the fuel pins, after impending closure of axial gaps caused by unexpectedly large length increases. The normal design process for thermal reactors has, of course, aimed to exclude such failure modes. The high power densities of fast reactors and the neutron induced swelling of non-fissile core metals both mean that deformations in fast reactors merit more detailed attention than in thermal reactors. Indeed US authors [3] concluded after an assessment of the implications of metal swelling that:

"Core lifetime in future high-flux fast reactors will probably be limited by something other than the intrinsic burn-up capability of high-quality fuel elements of contemporary design".

The failure mode of concern in [3] was the lateral deformation of the wrapper in the neutron flux gradient.

The increased interest in fuel deformations makes it an appropriate time to review some relevant results obtained by the present author during investigations made over a number of years. These investigations have been aimed at understanding the mechanics of certain deformation modes related to fuel behaviour. It has been found that good descriptions of these modes can be obtained using quite elementary structural models. These models are of significant importance to the projects with which they are associated and will no doubt prove to have a wider Structural importance.

2. The Reference Stress and the I-beam Approximation

Over the last 10 years a number of workers - mostly in the United Kingdom - have pursued the concept that, in view of the uncertainties in creep data and its interpretation, many problems in the creep of structures can be adequately solved using approximate methods. Early observations indicating the possible power of such methods were made by Calladine and Drucker [4], Anderson, Gardner and Hodgkins [5] and Marriott and Leckie [6]. The main observation stemming from these authors' work is that the creep deformation of certain structures and loading systems can be related to a single creep test carried out on the material at a particular stress history determined by the structure geometry, the loading system and loading history. The stress of this single test is the Reference Stress.

As a simple example, a rectangular beam of length l , thickness $2h$ under a constant bending moment M /unit length has a curvature after time t of

$$K = \epsilon_R / [h (1 + 1/2n)^{-n}] \quad (1)$$

and a centre deflection

$$y = (l^2/8h)(\epsilon_R)(1 + 1/2n)^n \quad (2)$$

where $t^2/8h$ is a geometrical term,

ϵ_R is the creep due to a stress M/h^2

and n is the creep stress index.

The Reference Stress is M/h^2 and eq. 2 gives estimates of the deflection to within $\pm 5\%$ when there is no knowledge of the value of the creep stress index n . Whilst it is hardly surprising that the deformation can be determined by a range of creep data covering a small but important range of stresses in the structure it is remarkable that a single creep test should be sufficient to describe the deformation so accurately.

This result was extended to more complex beam systems e.g. simply-supported beams, cantilevers etc in [5] and single Reference Stresses could be identified. The existence of single Reference Stresses over a range of beam loadings and support systems, suggests that it may be possible to solve creep deformation problems of beams more easily than by considering the full geometry with the concomitant integrations over the beam cross-section. There are many possible approaches to such an exploitation - the one adopted here derives from the so-called Skeletal Point concept of Marriott and Leckie.[6] These authors noted that in a beam with constant bending moment M and the dimensions discussed previously there is a point in the beam at which the stress depends almost entirely on M and on the dimensions but not on the creep stress index. This result may be extracted succinctly from eq. (1) where the curvature is related to a strain ϵ_R at a point distant $h(1 + 1/2n)^{-n}$ from the neutral axis. The value of $(1 + 1/2n)^{-n}$ varies from 0.667 at $n = 1$ to 0.606 at $n = \infty$ so that a Skeletal Point exists at about $2h/3$ from the neutral axis. The constant bending moment beam may thus be thought of as an I-beam with flanges situated at a distance of $4h/3$ apart.

Since the position of the Skeletal Point is dependent only on the thickness of the cross-section it is assumed that a wide class of beam problems with non-linear creep material laws may be treated as though an equivalent I-beam existed and all the stresses concentrated at the Skeletal Point.

3. Applications of the I-beam approximation to fuel element problems

The first application is to the calculation of the deformations of AGR pins during the creep relaxation of bending moments generated when the free thermal bow of the pin at start up is restrained laterally by a centre grid. The complete numerical solution of such a problem is time-consuming both for the theoretician and the computer.

The consequences of treating the fuel element can (a thin cylinder) by means of an equivalent I-beam have been worked out by Anderson [7] who showed that the completely relaxed shapes in a number of simple but important cases could be evaluated simply. It was apparent that a wide range of relaxation shapes could occur depending on the creep stress index (n) and the axial distribution of temperature. High values of n concentrate the relaxation near bending moment maxima i.e. grids and the effect is to increase the initial bow. Alternatively, a value of $n = 1$ produces no change of shape so long as no axial temperature gradient was present. High local temperatures also concentrate the relaxation and if they occur near a mid-span position could cause a reversal of the initial bow. Complex relaxation bows are produced by combination of non-linear creep laws and axial temperature gradients.

The success of the I-beam approximation for a thin cylindrical can (some results had been checked numerically) led to a simple approach to analysing the behaviour of wrappers in fully restrained Fast Reactor Cores. This problem arises if the design solution to excessive lateral movement of wrappers subjected to neutron induced voidage gradients requires the core to be constrained radially at a number of axial stations (typically two or three). To design such a restraint system it is necessary to have estimates of the deflections of the restrained system and of the loads necessary to restrain the wrappers. At the design stage where many uncertain material parameters (e.g. irradiation creep) have to be examined it is useful to have simple methods available. The full problem becomes particularly difficult analytically if non-linear creep laws have to be incorporated to take account of thermal creep over the coolant exit half of the wrapper.

Using the experience of the relaxation work, Collinge [8] assumed an equivalent I-beam in analysing the behaviour of thin hexagonal wrappers. It is worth noting that a similar conclusion has been drawn by the authors of [3] after an extensive analysis of the wrapper using a finite element approach. The conclusion appears to have been precipitated by high running times of the finite element programs [3].

The elementary analysis of Collinge [8] has enabled a useful understanding of restrained wrapper behaviour to be obtained. A number of observations on important design decisions can be made. For example, it may be important to decide whether a third restraint point should be imposed on a wrapper above or below the core region. A decision would depend on the expected loads and deflections. Irradiation creep in the core leads to only fairly minor differences between the two positions. If thermal creep of the wrapper is important the positioning of the central restraint can significantly affect the deflections in the core. If the central restraint is above the core the non-linear thermal creep can be large enough to concentrate the relaxations and deformations to a hinge-like motion near this restraint. This would allow movements of the wrapper towards the core centre. If the central restraint is below the core, the thermal creep is most effective in the core length where it is ideally situated to reduce wrapper movements towards the centre due to core bowing. If substantial thermal creep is expected therefore it may be appropriate to site the central restraint point below the core.

These two examples illustrate the use of the I-beam approximation. In the next Section an elementary analysis of restrained structures made of creeping material is used to assist in understanding two more important fuel element deformation problems.

4. Approximate analyses for simple expanding, creeping, restrained structures

The theory of elastic thermal buckling has received substantial attention. A review is given, for example, by Boley and Wiener [9]. Although the normal treatment has been to solve the problem at the moment of buckling - the Euler problem - there has also been interest in the post-buckling, large deflection problem. The design of bimetalllic temperature controllers which snap from one slightly bent, stable configuration to another has, for example, received attention.

The equivalent problems for a creeping material have been discussed by Anderson [1,2, 10] and a number of important results obtained. These will now be outlined.

The crucial result of elastic thermal buckling theory is that at a certain expansion of the structure against its restraints, an Euler buckling load system is attained which induces an uncertain deflection behaviour of the structure which may be large and will

normally be avoided by designers. This expansion may be called "one Euler strain". Anderson [1,2,10] has tackled two important, simple questions:

- (1) Can an expansion of "one Euler strain" be exceeded if a material is creeping?
- (2) Does repeated thermal cycling of a creeping structure produce any continuing development of an initial deflection?

The answers to these two questions were developed with reference to a simple structure obeying a linear creep law. The structure is a beam-column with its ends fixed against axial movement and its deformations assumed to develop as a sine curve. With these assumptions, Anderson found that an expansion of "one Euler strain" produces an increase of the initial deflection by a factor e ($= 2.718$) for a material deforming by creep alone. Much larger increases (perhaps by factors of 10 or more) occur for elastic structures so that the answer to Question (1) is that creep extends the allowable expansion. The physical reason for this is simply that creep reduces the build up of the elastic energy which causes the Euler buckle. Indeed, if there is sufficient creep the sudden buckle is avoided altogether although large deflections may still occur at larger expansions.

The answer to Question (2) is equally simple and important because it is entirely different from the elastic solution. Thermal cycling of a slightly deformed structure in an environment allowing stress relaxation does produce a continuing development of the initial deflection until the initial deflection reaches a value which is of the order of the thickness of the structure. The explanation of this continuing deformation for small deflections lies in the fact that a deflection increases rather more under a compressive load than it reduces under an equal tensile load. When the deflections reach the order of the thickness this difference is substantially reduced because the thermal expansions and contractions are substantially accommodated by length changes.

5. The application of analyses for expanding, restrained structures to fuel element problems

The results just described were derived for simple structural systems which are not normally mirrored exactly in fuel element configurations. Nevertheless the results were the nub of arguments of crucial importance in two project applications.

The amount of allowable axial expansion of Prototype SGHWR elements after a few early pins had been found to be closing the axial gaps to the bottom support grid has been discussed in detail by Anderson [2]. An important factor determining the allowable expansion was the extent to which irradiation creep delayed the build up of the Euler load. This delay was worth a significant amount in allowable burn-up.

The observations of heat transfer fin waving on Uranium/Magnox fuel elements provide an important illustration of a discharge of fuel due entirely to deformation. The discovery of the causes of this deformation resembles a detective story.

The first suspect was that thermal cycling in the reactor caused the waving. Analysis [10] indicated that thermal cycling could increase deformation in beam-columns. Extension of the beam-column analysis to plates with a free edge revealed that thermal-cycling induced deformations would also occur in plates (though on a reduced scale compared with beams) and hence on heat transfer fins. The extension of the analysis to large deflections however produced a major difficulty for the thermal cycling hypothesis: the deflections could not develop much beyond the fin thickness. Since deflections up to

several times the fin thickness had been observed it appeared unlikely that thermal cycling was causing the larger deflections. When this conclusion was appreciated the search for alternative explanations ended when the large deflections were attributed to very thin surface corrosion layers forming in such a way as to stretch the heat transfer fins.

6. Concluding remarks

The Structural models discussed briefly here have been at the simplest level necessary to describe the Project phenomena. The models complement and augment the large computer programs now available. They are complementary because intelligent application of simple models enables the effort on large computer programs to be deployed most effectively. They augment the large computer programs because they reveal new phenomena which the large general computer programs may be ill equipped to cope with. The use of a general purpose finite element program to analyse the behaviour of a beam thermally cycled 10^6 times in a stress relaxing medium is improbable. Whether to commission a program to cope with such a problem once it is recognised is a separate decision on which the simple solution may throw light.

The objective of this COMPACT has been to outline the Structural background to the LECTURE and provide indications of the Project applications. The LECTURE will explore both aspects further.

REFERENCES

- [1] ANDERSON, R G. "Theoretical waving behaviour of Magnox fuel element fins due to coolant-gas induced corrosion and to thermal cycling". The Reactor Group Report 2121(S), HMSO.
- [2] ANDERSON, R G. "Behaviour of lengthening Prototype SGHWR fuel pins subjected to axial constraints during normal operation". Journal of the British Nuclear Energy Society, Vol. 12, No. 2, pp 193-197. (1973).
- [3] HUEBOTTER, P R, BUMP, T R, SHA, W T, EGGEN, D T and FULFORD, P J. "Design, Research and Development implications of metal swelling in fast reactors". ANL-7786.
- [4] CALLADINE, C R and DRUCKER, D C. "A bound method for creep analysis of structures: direct use of solutions in elasticity and plasticity". Journal of Mechanical Engineering Science, Vol. 4, No. 1, 1962, pp 1-11.
- [5] ANDERSON, R G, GARDNER, L R T and HODGKINS, W R. "Deformation of uniformly loaded beams obeying complex creep laws". Journal of Mechanical Engineering Science, Vol. 5, No. 3, 1963, pp 238-244.
- [6] MARRIOTT, D L and LECKIE, F A. "Some observations on the deflections of structures during creep". Paper 17, Conference on Thermal Loading and Creep in Structures and Components, 12-15 May, 1964.
- [7] ANDERSON, R G. "A simple model for estimating the bow of fuel elements after complete stress relaxation". The Reactor Group Report 1836(S), HMSO.
- [8] COLLINGE, I R. "The restraint of fast reactor sub-assemblies against voidage-induced bowing". Journal of the British Nuclear Energy Society, Vol. 11, No. 3, pp 251-258, (1972).
- [9] BOLEY, B A and WEINER, J H. "Theory of Thermal Stresses", Chapter 13, John Wiley, 1960.
- [10] ANDERSON, R G. "The Deformation of an Axially held Strut during Repeated Thermal Cycling at Creep Relaxation Temperature". Creep in Structures 1970, Springer Verlag, Berlin 1972, pp 269-272.