THREE DIMENSIONAL RUPTURE ANALYSIS OF A PRESTRESSED CONCRETE PRESSURE VESSEL INCLUDING CREEP EFFECTS

B. SAUGY, Th. ZIMMERMANN, M. HUSSAIN KHAN

*Institut de production d'énergie, Département du génie civil,
École Polytechnique fédérale de Lausanne, CH-1006 Lausanne, Switzerland*

SUMMARY

This paper describes briefly the results obtained since September 1971 by the research work carried out in our Institute in the field of non-linear analysis up to rupture of a PCRV taking into account of creep effects.

In the earlier work done in this field (and reported in paper H3/5, Vol. 4, 231-251 presented at the 1st SMIRT Conference) the influence of redistribution of stresses was restricted to short-time effects in the evaluation of rupture pressure of a PCRV. The research carried out since then aimed firstly to make a synthesis of constitutive laws describing the behavior of concrete under short-time or sustained loads and temperature gradients and of its rupture strength under tri-axial state of stresses which allowed to bring into evidence the principal parameters governing the behavior of concrete and to take them into account in the analysis. Secondly, it aimed to examine the influence of redistribution of stresses due to the rheological behavior of concrete on the rupture pressure of a PCRV.

First the method of non-linear analysis for creep and rupture is described briefly. The mathematical model, of a general application, is based on the Finite Element Method, utilising the isoparametric elements, non-linearities are introduced by use of iterative techniques. It allows to predict under satisfactory limit the behavior of massive prestressed concrete structures loaded up to destruction. The use of classical parameters for definition of the physical characteristics of materials for formulating the constitutive laws makes the model particularly interesting for practical applications.

Then the analysis of the PCRV for a Gas-Cooled Fast Reactor developed by the Swiss Federal Institute for Reactor Research is presented. This PCRV has large cavities in its walls to house direct cycle gas turbines and other mechanical equipments. First the creep analysis is done taking account of the envisaged construction schedule and the loading conditions of internal pressure and thermal gradients created during the testing period and finally during the normal exploitation of the PCRV. Proceeding from the state of stress obtained as such the rupture analysis is carried out for a sudden increase of internal pressure and of temperature gradient due to an hypothetical accident.

1. Introduction

The work described in this paper is carried out under the direction of Professor Dr A. Gardel at the Swiss Federal Institute of Technology in Lausanne and financed by the Swiss National Fund for Scientific Research. This theoretical research aims to develop deformation models predicting the non-linear behavior of massive structures. In a first phase, a deformation model to predict short time load effects was developed and a computer code for three dimensional analysis based on the Finite Element Method was established. This model gave satisfactory results for analysis of structures such as end slabs of a cylindrical PCRV, tunnels in rock massifs and foundations. The following phase aims to define a conventional model of creep which integrates suitably in the first model and which will allow to analyse the behavior of the structure upto rupture under increasing loads taking into account the evolution of material properties and the state of stress and strain in function of time. This model is being tested to analyse structures submitted to laboratory tests. Besides these tests it appeared interesting to apply it to a real structure for which such an analysis seems to be necessary. The PCRV for a Gas-Cooled Fast Reactor of 1000 MWe developed by the Swiss Federal Institute for Reactor Research (EIR) has been retained for such an analysis. We are thankful to EIR and in particular to Dr Sarlos for furnishing us their report on the assessment studies of this reactor.

2. Description of the PCRV and of the Loads

The integrated variant of the Helium-Cooled Fast Reactor of 1000 MWe capacity retained by the EIR is a three loop design. Beside the reactor core, three helium driven turbosets, each composed of two compressors and two turbines capable of generating 335 MWe, and six heat recuperators (two per turboset) are housed in the PCRV. Three electrical generators are vertically arranged under the PCRV.

The PCRV is a cylindrical type vessel of 29,5 m. external diameter (Fig. 1). The reactor core is housed in a central cylindrical cavity of 6,7 m. diameter and 13,3 m. height. Nine large vertical cylindrical cavities, each of 3,9 m. diameter and of 33,9 m. height are provided in the vessel wall and arranged in a symmetrical manner around the reactor cavity. The turbosets and the recuperators are housed in them. The PCRV has a reduced height of 27,7 m. in the central part of 9,5 m. diameter and a height of 36,9 m. for the rest of the structure. Three penetrations of different diameters, varying from 81 cm. to 157 cm., connect each of the recuperator cavities with the turboset and reactor core cavities. Each turboset cavity is connected to the reactor cavity by a horizontal penetration of 73 cm. diameter.

The gas pressures and temperatures for the working conditions of the power station are as described in table I.

For more details on flow and temperature-entropy diagrams of the direct Helium-Turbine cycle the reader may refer to the EIR report [1].

The loading conditions taken into account in the present analysis are shown in the figure 2. The origine of time in the diagrams shown in this figure is taken as the date of realization of 50 % of the concreting of the PCRV. After one year from this date

100 % of the prestressing loads are applied. At the age of 3 years, the loads corresponding to the normal working conditions of the power station are applied. These loads are admitted to be the following :

- internal pressure of 90,9 bars in the reactor cavity,
- average internal pressure of 35,7 bars in the peripheral cavities,
- internal temperature of 50°C in the concrete adjacent to the liner of all the cavities and temperature of 20°C on all the external walls of the PCRV.

The maximum hypothetical accident is supposed to occur after 2 years of normal servicing of the power station. At this moment the internal pressure in the peripheral cavities suddenly increases to 90,9 bars and then the pressure in all the cavities continues to increase simultaneously till the rupture of the pressure vessel occurs. Unsymmetrical loading conditions as well as proof test, start up, shut down and faulty operational condition of the liner cooling water system have not been examined in the present study.

3. Design criteria

The design criteria can be grouped under four following headings.

- a) Limiting allowable state of stress for construction and normal operating conditions with sustained loads (long time behavior) including start up and shut down loadings.
- b) Limiting allowable state of deformation for normal operating conditions with sustained loads.
- c) Limiting allowable state of stress for short-time exceptional operating and faulty functioning conditions.
- d) Minimum ultimate load capacity.

If β_w is the cube strength of concrete at 28 days, the group a) criteria limits the maximum stresses to 0,33 to 0,4 β_w in compression and 0 to 10 kg/cm² in tension. In the zones where there are stress concentrations in general, around penetrations in particular, these limiting values are increased upto 1,5 to 2 times as the penetration liner and the reinforcement around it are not taken into account in the analysis.

The group b) criteria limits the maximum values of deformations and displacements which are imposed by the liner designer so that the amount of strain cycling to which liner is submitted remains in acceptable limits and by the equipment designer, in particular of the control-rod assembly. The deformations are limited to 0,5 to 0,8 o/oo and the displacements to 2 to 3 mm.

The group c) criteria limits the stress to 0,45 to 0,5 β_w in compression and to 10 to 30 kg/cm² in tension when there is a 20 % rise in internal pressure and the temperature gradient is increased by 50 % due to a faulty functioning of the liner cooling water system.

Finally the minimum ultimate load capacity for a sudden rise in pressure simultaneously in all the cavities is required to be of the order of 3 times the pressure in the reactor cavity during normal operating condition.

4. Constitutive laws

4.1 Stress-strain law

An isotropic non-homogeneous model with characteristics varying in function of the state of stress, of the temperature and of the loading age of the material has been used. The constitutive laws are expressed in Lame's tensorial form for the initial elastic model.

$$\begin{aligned} S &= 2\mu_e \\ \sigma_m &= 3K\epsilon_m \end{aligned} \quad (1)$$

where S = deviatoric stress

e = distortion

μ = shear modulus

K = bulk modulus

σ_m = average stress

ϵ_m = average strain

The non-linear behavior of concrete under static short-time loads is idealised in the following manner.

$$\begin{aligned} K &= \text{constant} \\ \mu &= \mu_e - \alpha \log_e (I_2/I_{2e}) \quad \text{for } I_2 > I_{2e} \\ \mu &= \mu_e \quad \text{for } I_2 \leq I_{2e} \end{aligned} \quad (2)$$

where $I_2 = 1,5 \Sigma S_i^2$ is the second invariant of the stress tensor and the index e indicates elastic behavior range. These laws have been established on the basis of experimental results published by several authors and commented in an earlier paper [2]. They have already been successfully tested and compared with published experimental studies.

The non-linear model presented above is very practical as far as application is concerned as it uses exclusively current material characteristics. The proposed rheological model which follows does not require also any further special experiments and adapts naturally to the non-linear one. This rheological model which defines the secant shear and bulk moduli in function of time is of the form :

$$\begin{aligned} \mu_{(t-\tau)} &= \mu_i / (1 + a(t-\tau)^n) \\ K_{(t-\tau)} &= K_i / (1 + a(t-\tau)^n) \end{aligned} \quad (3)$$

where τ = loading age

t = time

a and n are creep factors

and index i designates the value at loading age.

The evolution of the values of μ and K in function of τ et t admitted in the present analysis is shown in the Figure 3. Firstly the instantaneous shear modulus

is determined in function of the second invariant of the stress tensor I_2 with the aid of Figure 3a). Then the effect of age is taken into account with the aid of Figure 3b) which allows to determine the value of μ_i at the time τ of the application of the

loads. The Figure 3c) gives the evolution of the value of the creep factor a in function of the loading age τ and the temperature T of the concrete. The value of a is independent of the state of stress. Even for a good degree of precision, the creep factor n can be assumed constant. Finally the Figure 3d) allows to determine the fictive secant modulus $\mu_{(t-\tau)}$ in function of the parameter $a \cdot (t-\tau)^n$ which itself is evaluated from the known values of the creep factors and $(t-\tau)$ the time under load. The value of $K_{(t-\tau)}$ is determined in a similar manner.

It may be remarked that the equation (3) applied to a one dimensional case corresponds to a creep law, defining the deformation, of the following type :

$$\epsilon = \epsilon_1 \{1 + a(t-\tau)^n\} \quad (4)$$

4.2 Rupture criterium

This criterium had been discussed in a more detailed form in an earlier paper [4]. We will just recall here its essential elements. The range of validity of the laws described in the previous paragraph is limited by a rupture surface ψ in the space of principal stresses of the type :

$$\psi = f(\sigma_1, \sigma_2, \sigma_3) \quad (5)$$

and defined in a specific manner in terms of the stress invariants by the following equations :

$$I_1 = \sum_{i=0}^3 \alpha_i (I_2^{0,5} - E)^i \quad (6)$$

$$\text{with } E = \alpha_4 - \alpha_4 / \{ (I_2^{0,5} - E)^2 T^2 / (\alpha_4 + \gamma/2) + (I_2^{0,5} - E) T + \alpha_4 \}$$

$$T = \{1 / \cos(\Sigma 1,5 (S_1 / I_2^{0,5})^3) - 1\}$$

where the I_1 are the stress invariants; the α_i are functions of β_c , m and γ . The uniaxial compressive strength on cyl. β_c and the coefficients m , the ratio of biaxial to uniaxial compressive strength, and γ , the ratio of uniaxial tensile to compressive strength define completely the surface of rupture. The five parameters α_0 to α_4 are determined by utilizing the five points corresponding to the following states of stresses :

Definition of the state of stress	Principal stresses		
	σ_1	σ_2	σ_3
uniaxial compression	β_c	0	0
biaxial compression	$m\beta_c$	$m\beta_c$	0
uniaxial tension	$\gamma\beta_c$	0	0
biaxial tension	$\gamma\beta_c$	$\gamma\beta_c$	0
triaxial tension	$\gamma\beta_c$	$\gamma\beta_c$	$\gamma\beta_c$

The parameters β_c , m and γ vary in function of the age of concrete. As such the general equation of the surface of rupture is of the type

$$\psi(\sigma) = \psi(\beta_c, m, \gamma, \tau) \quad (7)$$

The rupture defined by such a surface depends on the average pressure, the deviatoric stress, the ratio of principal stresses and the age of the concrete.

The tensile cracking of the concrete is taken into account by introducing a local orthotropy oriented along the plane of cracking and placed discontinuously in the model at the integration points of the finite elements. The stresses incompatible with the local orthotropy are then corrected and redistributed in the structure through fictive equivalent nodal forces.

5. Mathematical model

The basic numerical model utilizes the elastic isoparametric elements with 20 nodal points elaborated at the University of Swansea by the research group of Professor D.C. Zienkiewicz [5]. A frontal elimination process developed by B. Irons [6] is used in the computer code established to solve the set of linear equations.

The non linear behavior of the material is taken into account by a cycle of iterations of the pseudo-elastic model without modifying the stiffness matrix. At each iteration, fictive loads, corresponding to the difference between pseudo-elastic model and the constitutive laws idealizing the non-linear and rheological behavior of the material, are introduced as external forces at the nodal points. In fact these fictive forces are the unbalanced forces which the material can not take up in the zone considered. The solution obtained at the end of the cycle of iteration is the one which simultaneously satisfies with satisfactory precision the internal equilibrium and the constitutive laws at a given number of integration points of each element.

The computing time which tends to become excessive for such an iteration process has been fairly reduced by the introduction of accelerating processes based on the interpolation of values of the variables obtained during the preceding iterations and optimized in the structure by a least square process.

The principle of this iterative process is shown in the Figure 4 which is valid for symbolic tensorial stress created firstly by an increasing or a decreasing loading condition realized at constant velocity and secondly by creep effects. The non-linear constitutive law is satisfied by defining the triangle ABC in which σ_j corresponds to the unbalanced stress which is equal to the difference between the elastic model and the non-linear one. When the equilibrium is obtained with a sufficient precision at each point, the real deformation is equal to ϵ_e plus ϵ_{nl} . The evolution in function of time is then searched by a new cycle of iterations in which the stress σ_c , equivalent to the creep deformation defined by the corresponding constitutive law, is evaluated at each step and introduced as a fictive stress. If creep leads to an acceleration of the failure process, a new non-linear equilibrium is searched out according to the first type of iterative process.

6. Analysis

The introduction of rheological model in the computer code is actually at the testing stage. We therefore choose a rather simple finite element mesh for the present analy-

sis to verify the algorithms and to avoid prohibitive computer time.

The PCRV has a symmetry of the order of 18 as it has 9 cylindrical peripheral cavities. The Figure 6 shows the finite element mesh for 1/18th part of the PCRV. It comprises of 84 isoparametric elements and 805 nodal points.

For normal operation conditions the prestressing tendons are not introduced in the mesh and uniform equivalent pressures representative of the prestressing forces are applied. The state of stress and deformation is determined for internal pressures in the cavity, thermal gradient and the prestressing forces and it is checked that the stress and deformation limits mentioned earlier (paragraph 3) are satisfied and that there is reversibility of deformations at all parts of the structure.

While examining the behavior of the structure under increasing pressure loads upto rupture, one dimensional steel elements are introduced in the finite element mesh to simulate the prestressing tendons so that a better picture of the failure mechanism is obtained.

First a creep analysis is done taking into account of the history of loading, then the pressure in the cavities is increased by steps till failure of the structure is obtained. Till now an analysis with complete loading and unloading cycle has not been attempted on the present model as it will be time consuming and expensive.

The introduction of steel elements in the finite element mesh for the analysis of composite structures like the PCRV allows to take into account of the redistribution of stresses, in a more realistic manner, between the concrete and the steel. This redistribution influences the ultimate load capacity of the structure to a degree depending upon the quantity and the location of the steel members.

Actually only test analysis have been carried out by now to check the algorithms of the mathematical model. Final results have not been obtained and therefore can not be given now. Moreover, the influence of the variation of some principal parameters of the constitutive laws on the behavior of the PCRV is also under study actually. All these results will be presented at the 2nd SMIRT conference and published later.

References

- [1] EIR, "1000 MWe Helium Cooled Fast Reactor, Assessment studies", Swiss Federal Institute for Reactor Research, Report no TM-St-260, Würenlingen, July 1972.
- [2] SAUGY, B., "Contribution à l'étude théorique du comportement des structures massives en béton armé sous charges rapides", Bulletin technique de la Suisse romande, no 22, 1969.
- [3] FREUDENTHAL, A.M., ROLL, F., "Creep and Creep Recovery of Concrete under High Compressive Stress", Journal of the American Concrete Institute, no 12, June 1958.
- [4] HUSSAIN, M., SAUGY, B., ZIMMERMANN, Th., "Analyse tridimensionnelle du comportement non linéaire d'un caisson de réacteur nucléaire en béton précontraint", Proceedings of the 1st SMIRT Conference, Berlin 1971.
- [5] ERGATOUDIS, J., "Isoparametric Finite Element in two and three Dimensional Stress Analysis ", Ph. D. Thesis, University of Wales, Swansea 1968.
- [6] IRONS, B., "A Frontal solution Program for Finite Element Analysis", Int. J. for Numerical Methods in Engineering, no 1, Vol. 2, 1970.

Table I. Normal operational condition pressure
and temperature characteristics

Cavity	Gas	
	Pressure Bars	Temperature °C
Reactor	90.9	120.7
Upper part : Intercooler	46.5	18.0
L.P. Compressor		
H.P. Compressor	91.0	121.0
Turboset : Middle part : H.P. Turbine	84.3	662.0
L.P. Turbine (inlet)	44.7	447.0
Lower part : L.P. Turbine (outlet)	28.1	365.0
Recuperator	28.1	365.0

1. PCR of a 1000 MWe helium cooled fast breeder with direct cycle gas turbines.

2. Variation of loads and temperature gradient in function of time.

3. Constitutive laws.

$$\psi(\sigma_1, \sigma_2, \sigma_3) = \psi(\beta_c, m, n, \tau)$$

4. Iterative technique.

5. Rupture criterium.

84 Elements
805 Nodes

6. Finite element mesh.