SEISMIC ANALYSIS OF THIN SHELL STRUCTURES

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SUMMARY

We consider in this paper a vertically erected, axisymmetric shell, resting on a horizontal foundation. The only contact between the shell and the foundation is at the circular edge of the shell which touches the foundation. Such a structure can represent the containment shell, which houses a nuclear reactor, or a cooling tower, which is used to discharge the thermal effluents of a nuclear power plant into surrounding atmosphere.

The foundation is subjected to a time-dependent motion in both the vertical and horizontal directions. The motion may be produced by an event such as an earthquake or explosion. It is assumed that a sufficient number of time histories of the motion of the foundation, caused by such events, is known. The object of this paper is to obtain an estimate of the response of the shell to an event that is going to occur in the future.

It is assumed in this paper that the foundation is being translated as a rigid body; no rotation (rocking) of the foundation is considered, although it could have been included easily in the analysis. It is also assumed that the base of the shell is imbedded rigidly in the foundation, so that the circular edge of the shell, which touches the foundation, remains circular during the motion.

An estimate of the response of the shell to such excitation can be obtained from the solution of the boundary value problem of an axisymmetric shell subjected to time-dependent boundary conditions. The solution to such a problem for shells of arbitrary shape has been given by A. Kalnins (*Nuclear Engineering and Design*, vol. 20, 1972, pp. 131-147) in terms of normal modes of free vibration of the shell. This solution is adapted here for an analysis with the average velocity spectra of earthquakes, which has been pioneered by M.A. Biot and G.W. Housner. The modes of free vibration are calculated by the multisegment, direct numerical integration method developed by A. Kalnins (*JASA*, vol. 36, 1964, pp. 1355-1365).

Using the proposed approach, an actual design case of the containment vessel of a nuclear power plant built in the U.S.A. is presented. An estimate of the dynamic response of the shell to an earthquake is obtained for all the relevant variables, such as stresses and displacements. The results are evaluated within the framework of the current specifications for seismic design of nuclear power plants.

1. INTRODUCTION

The dynamic response of structures to earthquakes has received renewed attention in the last decade, partly because of its importance in the seismic design of nuclear power generating stations. The concern for the safety of nuclear power plants has produced significant analytical and technological advances in the capabilities to understand and design for future seismic events. An excellent, comprehensive account of the latest capabilities is contained in a recent book by Newmark and Rosenblueth [1].

We consider in this paper the general problem of an axisymmetric shell erected vertically on a horizontal foundation. Only horizontal and vertical motion of the foundation is considered, and it is assumed that the shell is built in rigidly into the foundation.

The main interest of this paper is limited to axisymmetric shell structures which are built at the site of a nuclear power generating plant. Typical examples include the free-standing, steel containment vessel that houses the primary cooling system, the concrete reactor building that serves as a missile and biological shield for the containment vessel, and a cooling tower.

The seismic analysis of thin shell structures carried out in this paper is based on the response spectrum concept, that has been pioneered by Biot [2] and Housner, et al [3], and the classical approach of normal mode superposition. The contribution of this paper is that the parameters entering into the normal mode superposition are defined and calculated with the use of shell theory rather than from equivalent lumped-mass or beam models. It is regarded that a lumped-mass or beam model will often introduce inadmissible errors in the solution. When the axisymmetric shells are conical or hyperboloidal, such errors are especially significant, but even in a cylindrical shell topped with a spherical dome, which is a typical geometry of a containment vessel, the errors become appreciable unless the height-to-diameter ratio is very large. Also, in calculating the vertical modes, the dome of the containment shell controls the lower resonant frequencies which cannot be modeled with a simple lumped-mass or beam model. Inasmuch as methods for the free vibration analysis of axisymmetric shells have advanced to a state where they have reduced the problem to a routine calculation, there is no reason at this time to employ a less accurate model than that of shell theory.

The actual response spectrum followed in the evaluation of the results is the straight-line design spectrum proposed by Newmark [4]. The latter is constructed on the basis of estimates of maximum ground acceleration, velocity and displacement at the plant site. The modes of free vibration of the axisymmetric steel containment shell and the participation factors of each mode in the response are calculated with the multisegment, direct numerical integration approach developed by Kalnins [5]. Results are given for the design of a steel containment vessel presently under construction in the U.S.A.

2. STATEMENT OF PROBLEM

Consider a shell structure which is symmetric about a straight axis, and let its axis be oriented vertically with respect to a horizontal foundation. The foundation is capable of moving horizontally and vertically. The object is to find the response of the shell to given motions of the foundation. Mathematically, the problem consists of finding the solution to the following boundary-value problem. Given a shell of revolution and a mathematical model for its deformation, find the stresses and displacements of the shell when it is subjected to time-dependent boundary conditions at one end, the rest of the shell being free of any constraints (Figure 1).

Consider now an axisymmetric shell with an arbitrary contour. We shall assume that a continuous axisymmetric reference surface of the shell has been chosen and that a thickness has been assigned at every point of the reference surface. The location of the material points within the thickness is given by the two coordinates, ϕ and θ , on the reference surface, and by the distance ζ , which is measured from the reference surface along its positive normal (Figure 1).

A general mathematical model for the deformation of a thin shell is based on the hypothesis that the displacement field $\underline{v}$ is linear in ζ in the form

$$\underline{v}(\phi, \theta, \zeta) = \underline{u}(\phi, \theta) + \zeta \underline{\beta}(\phi, \theta) \quad (1)$$

Since the reference surface is defined by $\zeta = 0$, then $\underline{u}$ is the displacement vector of the material points lying on the reference surface. We resolve

$$\underline{u} = u_\phi \underline{t}_\phi + u_\theta \underline{t}_\theta + w \underline{t}_\zeta \quad (2)$$

and

$$\underline{\beta} = \beta_\phi \underline{t}_\phi + \beta_\theta \underline{t}_\theta + \beta_\zeta \underline{t}_\zeta \quad (3)$$

where $\underline{t}_\phi$, $\underline{t}_\theta$, and $\underline{t}_\zeta$ is a triad of unit vectors, tangent to the ϕ , θ , and ζ coordinate curves, respectively. For infinitesimal deformations, β_ϕ and β_θ are the angles of rotation of the normal in the directions of $\underline{t}_\phi$ and $\underline{t}_\theta$, respectively, while β_ζ times the thickness gives the change in thickness of the shell. We obtain next the conditions that a foundation imposes on the six shell displacements, u_ϕ , u_θ , w , β_ϕ , β_θ , and β_ζ , at the points where the shell touches the foundation.

Let the shell be built in rigidly into a rigid foundation (Figure 1), which is subjected to time-dependent displacements given by

$$\begin{aligned} \tilde{U}_x &= U_x f_x(t) \\ \tilde{U}_z &= U_z f_z(t) \end{aligned} \quad (4)$$

where U_x and U_z are some amplitude factors and the f 's are the time variations.

The displacements of the foundation are transferred to the shell by the following conditions at its base

$$\begin{aligned} u_r &= U_x \cos \theta f_x(t) \\ u_\theta &= -U_x \sin \theta f_x(t) \\ u_z &= U_z f_z(t) \\ \beta_\phi &= \beta_\theta = \beta_\zeta = 0 \end{aligned} \quad (5)$$

where u_r is the displacement perpendicular to the axis of symmetry and u_z is parallel to the axis on the circle of contact between the reference surface and the foundation. In order to express the displacements of the foundation in terms of the shell displacements, we must use the transformation

$$\begin{aligned} u_\phi &= -u_z \sin \phi + u_r \cos \phi \\ w &= u_z \cos \phi + u_r \sin \phi \end{aligned} \quad (6)$$

Finally, the boundary value problem can be stated as follows: find the response of the shell when the displacements

$$\begin{aligned} u_\phi &= -U_z f_z(t) \sin \phi + U_x f_x(t) \cos \phi \cos \theta \\ u_\theta &= -U_x f_x(t) \sin \theta \\ w &= U_z f_z(t) \cos \phi + U_x f_x(t) \sin \phi \cos \theta \end{aligned} \quad (7)$$

and

$$\beta_\phi = \beta_\theta = \beta_\zeta = 0 \quad (8)$$

are prescribed at the circle of contact between the reference surface of the shell and the foundation, the rest of the shell being free of any constraint.

3. RESPONSE OF SHELL

The response of a shell subjected to general time-dependent boundary conditions can be constructed by means of the normal mode superposition approach [6]. The object of this section is to specialize the general response for the application considered in this paper.

As given in [6], the displacement response to time-dependent boundary conditions is constructed in the form

$$\begin{aligned} u(\phi, \theta, t) &= \underline{u}^s(\phi, \theta) + \sum_i \underline{u}^i(\phi, \theta) m_i F_i(t) \\ g(\phi, \theta, t) &= \underline{g}^s(\phi, \theta) + \sum_i \underline{g}^i(\phi, \theta) m_i F_i(t) \end{aligned} \quad (9)$$

The terms with the superscript s identify variables of a static solution for

the shell when it is subjected to the boundary conditions given by eqs. (7) and (8) with the time functions omitted; the index i designates variables belonging to the i -th mode of free vibration of the shell, corresponding to a real resonant frequency ω_i in radians per second. Thus, u^i and β^i give the displacement field in the i -th mode of free vibration; m_i is a mode participation factor, given by

$$m_i = J_i / (\omega_i M_i) \quad (10)$$

where

$$J_i = \int_S [\rho_0 u^s \cdot u^i + \rho_1 (\beta^s \cdot u^i + u^s \cdot \beta^i) + \rho_2 \beta^s \cdot \beta^i] dS \quad (11)$$

$$M_i = \int_S [\rho_0 u^i \cdot u^i + 2\rho_1 \beta^i \cdot u^i + \rho_2 \beta^i \cdot \beta^i] dS \quad (12)$$

and $F_i(t)$ denotes the Duhamel integral

$$F_i(t) = \int_0^t a(\tau) \sin \omega_i(t-\tau) d\tau \quad (13)$$

where $a(\tau)$ is the ground acceleration, either in the horizontal or vertical direction. Thus, we can have either

$$a(t) = \ddot{x}(t) \quad (14)$$

or

$$a(t) = \ddot{z}(t) \quad (15)$$

where the dots denote time derivatives. The integrals J_i and M_i can be interpreted as the generalized force and mass, respectively, in the i -th mode. They must be evaluated over the whole reference surface S of the shell, with the density measures given by

$$\rho_n = \int_{\zeta_1}^{\zeta_2} \rho(\phi, \theta) \zeta^n d\zeta \quad (16)$$

for $n=0,1,2$, where ζ_1 and ζ_2 denote the distances of the bounding surfaces of the shell from the reference surface, and ρ denotes the mass density of the shell material.

For a vertical, axisymmetric shell, excited through the foundation, the static solution (with the superscript s) is represented by a rigid-body movement of the whole shell through a distance U_x horizontally and U_z vertically. With reference to eqs. (7) and (8), such a rigid-body solution throughout the shell is given by

$$\begin{aligned} u^s = & (-U_z \sin \phi + U_x \cos \phi \cos \theta) \underline{t}_\phi \\ & - U_x \sin \theta \underline{t}_\theta + (U_z \cos \phi + U_x \sin \phi \cos \theta) \underline{t}_\zeta \end{aligned} \quad (17)$$

and

$$\beta^S = 0 \quad (18)$$

while all other (stress) variables are identically zero. Using eq. (18) from the static solution, and expressing for a shell of revolution the surface integral for J_i as an iterated integral, J_i from eq. (11) can be written as

$$J_i = \int_{s_1}^{s_2} \int_0^{2\pi} u^S \cdot (\rho_0 u^i + \rho_1 \beta^i) r d\theta ds \quad (19)$$

where s is the arc length along the meridian (curve of intersection of reference surface and a plane through axis of symmetry); s_1 and s_2 are the end-points of the meridian; θ is the circumferential angular coordinate; and r is the distance of a point on the reference surface from the axis (Figure 1).

The free vibration solutions (with superscript i) are separable in ϕ and θ coordinates in the form

$$u_\phi(\phi, \theta) = u_{\phi n}(\phi) \cos n\theta \quad (20)$$

with similar expressions for all other variables. There is an infinite number of resonant modes of the shell for each given value of the circumferential wave number n , and all the modes, for all wave numbers, must be used in the integrand of eq. (19). However, when the terms of the scalar products of eq. (19) are integrated with respect to θ , only the modes for $n=0$ and $n=1$ survive. This can be seen by considering the first term of the first scalar product in the integrand of eq. (19) in the form

$$J_i = \int_{s_1}^{s_2} \int_0^{2\pi} [\rho_0 (-U_z \sin \phi + U_x \cos \phi \cos \theta) \times (\sum_n u_{\phi n}^i \cos n\theta) + \dots] r d\theta ds \quad (21)$$

Noting that

$$\int_0^{2\pi} \sum_n u_{\phi n}^i \cos n\theta d\theta = 2\pi u_{\phi 0}^i \quad (22)$$

and

$$\int_0^{2\pi} \cos \theta \sum_n u_{\phi n}^i \cos n\theta = \pi u_{\phi 1}^i \quad (23)$$

it is clear that out of all the modes of free vibration, as given by eq. (20) with $n=0,1,2,\dots$, only those with $n=0$ and $n=1$ participate in the general solution of our problem.

Thus, for our problem, J_i can be simplified to

$$J_i = U_z j_{zi} + U_x j_{xi} \quad (24)$$

where

$$\begin{aligned}
 j_{zi} &= 2\pi \int_{s_1}^{s_2} [-(\rho_0 u_{\phi 0}^i + \rho_1 \beta_{\phi 0}^i) \sin \phi + (\rho_0 w_{\phi 0}^i + \rho_1 \beta_{\phi 0}^i) \cos \phi] r ds \\
 j_{xi} &= \pi \int_{s_1}^{s_2} [(\rho_0 u_{\phi 1}^i + \rho_1 \beta_{\phi 1}^i) \cos \phi - (\rho_0 u_{\theta 1}^i + \rho_1 \beta_{\theta 1}^i) \\
 &\quad + (\rho_0 w_{\phi 1}^i + \rho_1 \beta_{\phi 1}^i) \sin \phi] r ds
 \end{aligned} \tag{25}$$

Thus, it is clear that the participation factor, j_{zi} , for the vertical motion involves only axisymmetric modes, with modal displacements $u_{\phi 0}^i$, $w_{\phi 0}^i$, $\beta_{\phi 0}^i$, $\beta_{\phi 0}^i$, while that for the horizontal motion, j_{xi} , involves only the $n=1$ modes, with modal displacements $u_{\phi 1}^i$, $u_{\theta 1}^i$, $w_{\phi 1}^i$, $\beta_{\phi 1}^i$, $\beta_{\theta 1}^i$, $\beta_{\phi 1}^i$. No modes with wave numbers other than $n=0,1$ enter in our general solution.

Similarly, the generalized mass for the $n=0$ modes is given by

$$\begin{aligned}
 M_{i0} &= 2\pi \int_{s_1}^{s_2} \{ \rho_0 [(u_{\phi 0}^i)^2 + (w_{\phi 0}^i)^2] \\
 &\quad + 2\rho_1 [u_{\phi 0}^i \beta_{\phi 0}^i + w_{\phi 0}^i \beta_{\phi 0}^i] \\
 &\quad + \rho_2 [(\beta_{\phi 0}^i)^2 + (\beta_{\phi 0}^i)^2] \} r ds
 \end{aligned} \tag{26}$$

and for the $n=1$ modes by

$$\begin{aligned}
 M_{i1} &= \pi \int_{s_1}^{s_2} \{ \rho_0 [(u_{\phi 1}^i)^2 + (u_{\theta 1}^i)^2 + (w_{\phi 1}^i)^2] \\
 &\quad + 2\rho_1 [u_{\phi 1}^i \beta_{\phi 1}^i + u_{\theta 1}^i \beta_{\theta 1}^i + w_{\phi 1}^i \beta_{\phi 1}^i] \\
 &\quad + \rho_2 [(\beta_{\phi 1}^i)^2 + (\beta_{\theta 1}^i)^2 + (\beta_{\phi 1}^i)^2] \} r ds
 \end{aligned} \tag{27}$$

The static solution, represented by u^s and β^s in eq. (9), is given by eqs. (17) and (18). Since the u^s terms represent a rigid motion of the shell that is the same as that of the foundation, then the u^s term in the response can be omitted if it is agreed that the displacements u_{ϕ} , u_{θ} , and w are measured with respect to the foundation. No change is needed for β^s , because, according to eq. (18), β^s is zero.

Finally, we can write from eq. (9) the total response of the shell as follows:

$$A = U_z \sum_i A_i m_{zi} F_{zi}(t) + U_x \sum_i A_i m_{xi} F_{xi}(t) \tag{28}$$

where the symbol A represents any desired variable of the solution (such as a displacement, stress, or resultant), and A_i represents the same variable taken from the solution of the i -th mode of free vibration. The first term on the right-hand side of eq. (28) represents vertical motion, and the second term

represents horizontal motion. The mode participation factors for each motion are now given by

$$\begin{aligned} m_{zi} &= j_{zi}/(\omega_i M_{i0}) \\ m_{xi} &= j_{xi}/(\omega_i M_{i1}) \end{aligned} \quad (29)$$

The Duhamel integral in eq. (28), $F_{zi}(t)$, contains the acceleration of the vertical motion, and $F_{xi}(t)$ contains the horizontal motion [see eq. (13)]. It must be remembered that the components of the displacement vector $\underline{u}$, when represented by A , are measured relative to the foundation. The summation in eq. (28) is carried out over a number of normal modes which are expected to ensure convergence.

When the free vibration problem of an axisymmetric shell is solved numerically, as for example with the multisegment direct, numerical integration technique [5], then it is a routine matter to evaluate the integrals of J_i and M_i , and finally the mode participation factors m_i , together with all the variables of the solution for each mode of free vibration. A number of axisymmetric modes ($n=0$) are needed for the vertical response and a number of $n=1$ modes for the horizontal response. We can regard, then, that we have at our disposal the means to evaluate $\underline{u}^i$, $\underline{\beta}^i$, and m_i for any desired number of $n=0$ and $n=1$ modes of free vibration for the shell, so that the complete response of the shell, as given by eq. (9), is determined, once the time variation, $F_i(t)$, is known. The following section is concerned with the finding of the peak value of $F_i(t)$ from the response spectrum.

4. RESPONSE SPECTRUM

If a deterministic response to a known ground motion is desired, the Duhamel integral, given in eq. (13), must be evaluated at various times t , which then provides the time dependence of the response in the solution, eq. (28). However, seismic design is concerned with a ground motion that is expected to occur in the future, so that its deterministic time record cannot be anticipated. The use of an acceleration record of a previous earthquake in eq. (13) is possible, but, as far as the design of an unbuilt shell structure is concerned, that is not very meaningful. A logical procedure to follow for seismic design is the response spectrum approach. The object of this section is to show how the response spectrum concept can be applied to the vertically erected shell structure.

Let us first write out the general solution for horizontal motion from eq. (28), term by term, for some variable A , in the form

$$A = [A_1 m_{x1} F_{x1}(t) + A_2 m_{x2} F_{x2}(t) + \dots] \quad (30)$$

where the amplitude U_x has been absorbed by $F_{xi}(t)$. As argued in the foregoing, it makes little sense for seismic design to evaluate the functions

$F_{xi}(t)$ from some ground acceleration record of a past earthquake in order to produce an exact time-dependent response for all the shell variables represented by A . The goal is to produce an estimate for A which is based on many records of past earthquakes and which can then be used for design purposes for the protection from future earthquakes.

One procedure that leads toward this goal is described by Housner [7], where it is suggested to consider in eq. (30) the maximum value of each multiplier of each of the product terms of the modal series and then add them in some agreed-upon manner. The maximum values of the variables A_i and of m_{xi} are simply their magnitudes, without an algebraic sign. The maximum value of $F_{xi}(t)$ for each mode is obtained from the response spectrum. The remainder of this section will be devoted to the construction of a response spectrum for design purposes.

If we assume that horizontal ground acceleration records of some past earthquake are available (i.e., in eq. (13) $a(t)$ is known for a sufficient range of t), then we can calculate the function

$$F_x(t) = \int_0^t a_x(\tau) \sin \omega(t-\tau) d\tau \quad (31)$$

for a given value of ω and record only the maximum value of F_x , denoted by

$$V_x = \max_t [F_x(t)] \quad (32)$$

over all times t . It does not matter at what time the maximum occurs. Repeating this calculation for a range of ω 's, within which the shell structure is expected to have its resonant frequencies, a plot can be produced between V_x and ω , for a single earthquake. Such a plot is called the velocity response spectrum of an earthquake. One example of such a response spectrum, which has been obtained from the ground acceleration recorded at Taft, California, July 21, 1952, is shown in Fig. 50-10 in [7]. Since there is no reason to prefer the response spectrum of one earthquake to that of another, Housner [7] has proposed further to average such spectra of many earthquakes in order to produce a representative response spectrum which can be used for design purposes. A response spectrum, averaged over four earthquakes, is given in Fig. 50-11 of Housner's article [7] and reproduced in our Figure 2. Such maximum values of F_x can then be read off Figure 2 for a known resonant frequency and used to calculate each term of the series of eq. (30). Since Figure 2 has an arbitrary scale for V_x , multiplication factors are required to set the actual amplitudes of V_x that are to be used for seismic design at a given building site. Some factors are identified on p. 50-16 of [7].

Another method for the construction of the response spectrum between V_x and ω for design purposes has been advanced by Newmark [4]. After plotting the response spectrum V_x vs. ω from the ground acceleration data recorded at El Centro, California, May 18, 1940, on a log-log scale, Newmark observed

that below $\omega = 0.3$ cycles per second (cps) the quantity

$$D = V_x / (2\pi\omega) \quad (33)$$

is approximately constant with respect to ω ; between $\omega = 0.3$ and $\omega = 2.0$ cps, V_x is about constant; and above $\omega = 2.0$ cps, the quantity

$$A = 2\pi\omega V_x \quad (34)$$

appears constant. If V_x were the peak value of a harmonic ground velocity at some ω , then D would have been the peak displacement and A the peak acceleration of the ground.

On the basis of this observation, Newmark suggested in [4] to construct a design response spectrum as follows. From the seismological and geological characteristics of the building site, estimate maximum values of ground acceleration, $A_{\max}$, velocity, $V_{\max}$, and displacement, $D_{\max}$. Using eqs. (33) and (34), plot the straight lines

$$V_x = 2\pi\omega D_{\max} \quad (35a)$$

$$V_x = V_{\max} \quad (35b)$$

$$V_x = A_{\max} / (2\pi\omega) \quad (35c)$$

on a log-log scale of a V_x vs. ω graph. Use amplification factors given in Table 3 of [4] for acceleration, velocity, and displacement, for a given amount of damping, to produce the corresponding three straight lines for the response relation between V_x and ω . Finally, using the rules given on pp. 18-19 of [4], connect the response lines with maximum ground displacement for low frequencies and with the maximum ground acceleration for high frequencies.

A slightly modified technique of Newmark's method for the construction of a design spectrum is based on the specification of a single ground motion parameter and its amplification factor. This technique requires that the constant portion of V_x , for a desired damping ratio, lies between some fixed frequency limits, say $\omega = 0.3$ and 2.0 cps. Then the specification of one maximum ground motion parameter determines the other two by means of eqs. (35) at the points $\omega = 0.3$ and 2.0 cps, respectively. The usual single parameter chosen is the maximum ground acceleration.

A design spectrum based on the modified technique is shown in Figure 3. The maximum ground acceleration for the particular plant site was selected as $0.12g$ ($g = 385.9 \text{ in/sec}^2$) and the amplification factor as 3.5 , which gives the constant acceleration line of the response at $0.42g$. Both of these lines are shown in Figure 3. At the point where the $0.42g$ line intersects $\omega = 2.0$ cps a horizontal line is drawn for the constant V_x portion of the curve. The constant value of V_x is calculated from eq. (35c) to be 12.9 in/sec . The horizontal line for $V_x = 12.9$ is continued until it intersects $\omega = 0.3$ where the constant displacement line, calculated from eq. (35a) as $D = 6.843$ inches, is drawn. Finally, the constant response acceleration line is connected to the

ground acceleration line at frequencies that are four times and ten times the frequency at the intersection of the $V_x = 12.9$ and $A = 0.42g$ lines. This means that on Figure 3 a line is drawn between the points $V_x = 3.22$, $\omega = 8$, and $V_x = 0.369$, $\omega = 20$. The values of V_x between $\omega = 8$ and $\omega = 20$ are determined either by interpolation on the log-log scale, or, in this particular case, by the equation

$$V_x = 443.1/\omega^{2.367}$$

For frequencies larger than 20 cps, the constant ground acceleration line of 0.12g determines the values of V_x from eq. (35c).

Once a design response spectrum is agreed upon, using either Housner's [7] or Newmark's [4] technique, the maximum values of all the terms in the modal series of eq. (30) can be obtained. They can be added for the solution in various ways. Since the peak values of each modal term will not occur at the same time, the responses will be overestimated at various degrees. Housner (p. 50-13 of [7]) states that if satisfactory convergence of the series is achieved in two modes, then simple addition is in order, but when the solution requires four or more modes for convergence, then the square root of the sum of squares of the modal terms is advisable. The later procedure is also advocated by Newmark (p. 20 of [4]).

The response to a vertical ground motion can be handled in the same way, if vertical acceleration records, $a_z(t)$, are available from which

$$V_z = \text{Max}_t \left[\int_0^t a_z(\tau) \sin \omega(t-\tau) d\tau \right]$$

can be computed. However, for seismic excitation, vertical acceleration records are scarce, and it is a common practice to use for the determination of V_z the same design spectrum as for horizontal motion, except that all amplitudes are multiplied by a common factor, such as two-thirds.

In summary, then, a design estimate of the response of a shell to an earthquake can be obtained from the summation

$$A = \sum_i |A_i| |m_{zi}| V_z + \sum_i |A_i| |m_{xi}| V_x \quad (37)$$

over a sufficient number of terms. The vertical lines mean that for all the variables listed among A_i , and for m_{xi} , their algebraic signs are to be dropped. The value of V_x can be read off either Figure 2 or 3 as a function of a given resonant frequency ω , and that of V_z from a similar curve.

It is clear that the summation in eq. (37) will overestimate the response of the shell for any time t , for any one of the past earthquakes that were used in the construction of the average spectrum. But it must be kept in mind that the estimate for the response variables is intended for a future earthquake rather than for any ones of the past. Eq. (37) gives a reasonable

distribution of the variables in A over the shell which might occur in earthquake excitation. The actual amplitude of the response is set by the amplitudes of the response spectra which must be selected for each building site.

5. EXAMPLE - STEEL CONTAINMENT VESSEL

We consider as an example the seismic analysis of a steel containment vessel for a nuclear power plant. Its general shape consists of a cylindrical shell, topped with a hemispherical dome. The cylindrical shell is 1.25 inches thick from the base to the elevation of 389 inches, then 1.0 inch thick to the elevation of 869 inches, and 0.75 inches thick to the spring line. The dome thickness is 0.75 inches. (See Figure 4.)

For this example, we consider only the horizontal motion, so that we must calculate only the $n=1$ modes of free vibration. We employ the classical shell theory in which the transverse shear and normal strains are zero, so that β_z used in the preceding sections is zero, and relations exist between β_ϕ , β_θ and u_ϕ , u_θ , w . The specific system of equations solved for this shell is listed as eqs. (52) in [6]. The boundary conditions are prescribed as

$$w = N_\phi = M_\phi = N_{\phi\theta} = 0$$

at the crown of the dome and

$$w = u_\phi = u_\theta = \beta_\phi = 0$$

at the base. N_ϕ and $N_{\phi\theta}$ are the meridional and circumferential membrane resultants, and M_ϕ is the meridional bending moment.

We employ the numerical integration technique and the computer program described in [5] to solve for the first three modes of free vibration. The computer program gives the resonant frequencies ω , in cps, and mode participation factors, m_x , which are listed in Table 1. From the response spectrum of Figure 3, we can obtain the values of V_x for each of the modes. Since interpolation on log-log graphs may be tedious, we can also obtain the values of V_x directly from the equations for the spectrum, eqs. (35) and (36). The lowest mode is at 9.288 cps, so that we must use eq. (36). The second and third modes are governed by eq. (35c). The design values for V_x for each of the modes are also shown in Table 1.

The computer program gives also all the variables of the solution for each mode throughout the shell. From all the variables, we choose to present the axial stress at the outer surface of the cylindrical shell, and we denote this stress by σ_x . Any other variable could have been presented in a similar manner.

In order to obtain the estimate of the response of the first mode for σ_x at some elevation, we must record from the solution the value of σ_x at that

point. For example, at the base $\sigma_x = 14,440$ psi. Then, as shown in eq. (37),
the modal response of the first mode for σ_x is given by

$$S_1 = 14,440 \times 0.01857 \times 2.265 = 607.4 \text{ psi}$$

where the last two numbers in the product come from Table 1. Repeating such a calculation at other elevations, and also for the other two modes, the modal values for σ_x of each mode, S_1, S_2, S_3 , listed in Table 2, are obtained. Table 2 gives also the direct addition of the three modal values as well as the square root of the sum of squares. It is noted that for all practical purposes the first mode determines the response.

6. REFERENCES

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Table 1. Modal data for $n=1$ modes

Mode No.	ω	m_x	V_x
1	9.288	1.857 E-2	2.265
2	25.70	4.443 E-3	0.287
3	37.02	7.836 E-4	0.199

Table 2. Response of axial stress

Elevation	S_1	S_2	S_3	$S_1+S_2+S_3$	$(S_1^2+S_2^2+S_3^2)^{1/2}$
0	607.4	4.7	2.8	614.7	607.4
130	342.0	3.9	1.3	347.2	342.0
260	306.4	7.6	1.2	315.2	306.5
389	275.1	10.6	1.1	286.8	275.3
629	264.1	18.0	1.3	283.4	264.7
869	196.0	19.3	1.3	216.6	197.0
1105	175.7	22.4	1.9	200.0	177.1
1341	100.9	15.6	1.6	118.1	102.1

Fig. 2. Response spectrum from Housner [7].

Fig. 1. Axisymmetric shell on moving foundation.

Fig. 3. Response spectrum following Newmark [4].

Fig. 4. Containment vessel considered in example.