STOCHASTIC PREDICTION OF MAXIMUM SEISMIC RESPONSE OF LIGHT SECONDARY SYSTEMS

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SUMMARY

A stochastic model is presented for predicting the elastic response of light multi-degree-of-freedom secondary systems to strong motion earthquakes. Secondary systems may include light mechanical or electrical equipment, piping, or other light systems attached at one or at several points to walls or floors of the supporting or primary structures. The critical functions of these secondary systems in nuclear power plants make the accurate prediction of their maximum responses important. The response of such secondary structures may be obtained by a direct time-history analysis, or more approximately, by the response spectrum method. The time-history solutions is, of course, expensive; moreover, there is no single representative earthquake and thus a number of possible earthquake ground motions have to be considered. On the other hand, the response spectrum method applied to secondary systems can lead to unreliable results.

Within the framework of the normal mode method, a decoupled stationary random vibration model is developed based on the assumption of Gaussian response process and Poisson barrier crossings. The accuracy of the proposed model is verified by comparing the calculated responses, at the 10 percent and 50 percent probability of exceedance level, with the second highest and average of the time-history responses from eight normalized accelerograms. The influence of decoupling, i.e., ignoring the dynamic interaction between the primary and the secondary systems, on the response is examined.

The influence of nonstationarity is also evaluated. It is observed that nonstationarity is unimportant for earthquakes of relatively long duration, and that for a given damping most of the error can be accounted for by a simple scaling. It is also shown that one aspect of the proposed method constitutes the basis for some of the approximations in the response spectrum method; however, the proposed method yields results that are consistently more reliable than the response spectrum method. Moreover, results obtained with the proposed method represent maximum response statistics from an ensemble of earthquakes rather than a single earthquake.

1. Introduction

The response of light multi-degree of freedom secondary systems to strong-motion earthquakes is considered. Secondary systems may include light electrical or mechanical equipment, piping or other light system attached at one or at several points to wall or floors of the primary supporting structure. In most instances, a secondary system will be subjected to a greater or more harmful input than that coming directly from the ground. The critical functions of these secondary systems in nuclear power plants make the accurate prediction of their responses important. The response of such a secondary system may be obtained by a direct time-history analysis, or more approximately, by the response spectrum method. The time-history solution is expensive -- there is no single representative earthquake and thus a number of possible earthquake ground motions have to be generated. On the other hand, the available response spectrum method applied to secondary systems does not always lead to reliable results.

An alternative random vibration approach to the response prediction of such secondary systems is developed and described.

2. Formulation of Random Vibration Method

The problem of determining the response of a structure for a given probability of exceedance is closely related to the first passage problem in the theory of probability. No exact solution exists for the dynamic response of elastic structures; however, several approximate solutions are available (see Lin [1]). Figure 1 shows one record of a process $r(t)$ with barrier levels $\pm b$. Assuming that the up-crossing of $+b$ and the down-crossing of $-b$ are independent events and the crossing of level $+b$ from below and $-b$ from above constitute a Poisson process, it can be shown (see e.g., Singh [2]) that the condition for safe performance during the interval $(0, t_d)$, corresponding to no crossing of either $+b$ or $-b$, is

$$p_e(b, t_d) = 1 - \exp[-2 \int_0^{t_d} v_b(s) ds] \quad (1)$$

where $v_b(t)$ is the crossing rate of either $+b$ or $-b$ given by

$$v_b(t) = \int_0^\infty \dot{r} f_{r\dot{r}}(r, \dot{r}, t) d\dot{r} \quad (2)$$

where $f_{r\dot{r}}(r, \dot{r}, t)$ is the joint density function of the random processes $r(t)$ and $\dot{r}(t)$ at time t . The assumption that $r(t)$ and $\dot{r}(t)$ are jointly Gaussian with zero means gives

$$f_{r\dot{r}}(r, \dot{r}, t) = \frac{1}{2\pi\sigma_r\sigma_{\dot{r}}\sqrt{1-\rho_{r\dot{r}}^2}} \exp\left\{-\frac{1}{2(1-\rho_{r\dot{r}}^2)}\left[\left(\frac{r}{\sigma_r}\right)^2 - 2\rho_{r\dot{r}}\left(\frac{r}{\sigma_r}\right)\left(\frac{\dot{r}}{\sigma_{\dot{r}}}\right) + \left(\frac{\dot{r}}{\sigma_{\dot{r}}}\right)^2\right]\right\} \quad (3)$$

yielding

$$v_b(t) = \frac{\sigma_{\dot{r}}\sqrt{1-\rho_{r\dot{r}}^2}}{2\pi\sigma_r} \left\{1 + \sqrt{2\pi}\eta_{br} \cdot \Phi(\eta_{br}) \cdot \exp\left[\frac{1}{2}\eta_{br}^2\right] \cdot \exp\left[-\frac{1}{2}\lambda_{br}^2\right]\right\} \quad (4)$$

where σ_r and $\sigma_{\dot{r}}$ are the respective standard deviations, and $\rho_{r\dot{r}}$ is the correlation coefficient, at time t , of the random variables $r(t)$ and $\dot{r}(t)$. Also, $\lambda_{br} = b/\{\sigma_r\sqrt{1-\rho_{r\dot{r}}^2}\}$, $\eta_{br} = \rho_{r\dot{r}}\lambda_{br}$ and $\Phi(x)$ is the standard normal distribution function.

For a non-stationary response, numerical integration of eqs. (1) and (2) is inevitably necessary. However, when it is appropriate to assume a stationary response, eqs. (1) through (4) yield

$$\bar{b}(p_e) = \sigma_r [2 \ln \{ \frac{\sigma_f^2 t_d}{\pi \sigma_r^2 \ln(\frac{1}{1-p_e})} \}]^{1/2} \quad (5)$$

where b is the response corresponding to a probability of exceedance p_e .

Figure 2 shows a shear-beam secondary system mounted on a shear-beam primary system. A direct or coupled approach would be to consider such a coupled system as an $(n_s + n_p)$ degree-of-freedom system, where n_p is the number of primary masses and n_s is the number of secondary masses. A decoupled (and preferred) approach is to treat the primary and the secondary systems as two separate systems and use the response of the primary system as input to the secondary system. Thus, it is assumed that the secondary system does not dynamically affect the response of the primary system; this will be approximately the case when the secondary system is light relative to the primary system. The decoupled approach is preferred as it is usually impractical to include equipments and piping systems located in buildings in the dynamic model representing the building; and if a coupled approach is used, the results may be unreliable because of the excessive number of degrees of freedom and the large difference between the masses of the primary and secondary systems.

To calculate the response for a given probability of exceedance using eq. (5), the standard deviation σ_r and σ_f are required. Expressions for σ_r and σ_f are derived for the spring distortions of the system in Fig. 2 for the decoupled model. The development, however, can be readily modified (Singh [2]) to consider other types of systems. Based on a normal-mode approach, the decoupled deterministic analysis yields

$$u_{sk}(t) = - \sum_{m=1}^{n_s} \gamma_{sm} \psi_{sm}(k) \int_0^t h_{sm}(t-s) \ddot{x}_1(s) ds \quad (6)$$

where $\ddot{x}_1$ is the acceleration of primary mass 1; u_{sk} is the relative displacement of the k -th secondary mass with respect to the first primary mass; γ_{sm} , $\psi_{sm}(k)$ and $h_{sm}(t)$ are the participation factor in mode m , the k -th element of the m -th eigenvector, and the impulse response function for the m -th mode for the secondary system, respectively. The spring distortion is

$$z_{sk} = - \sum_{m=1}^{n_s} \{ \gamma_{sm} [\psi_{sm}(k) - \psi_{sm}(k-1)] \int_0^t h_{sm}(t-s) \ddot{x}_1(s) ds \}$$

$$= - \sum_{m=1}^{n_s} \sum_{j=1}^{n_p} \{ A_{mj}(k) \int_0^t h_{sm}(t-s) q_{pj}(s) ds \} \quad (7)$$

where $q_{pj}(t)$ is the response of the decoupled primary system in mode j and

$$A_{mj}(k) = \gamma_{sm} [\psi_{sm}(k) - \psi_{sm}(k-1)] \gamma_{pj} \psi_{pj}(1) \omega_{pj}^2 \quad (8)$$

From eq. (7), the auto-correlation function of z_{sk} is

$$R_{z_{sk}}(t_2 - t_1) = E [z_{sk}(t_1) \cdot z_{sk}(t_2)]$$

$$= \sum_{m=1}^{n_s} \sum_{n=1}^{n_s} \sum_{j=1}^{n_p} \sum_{\ell=1}^{n_p} A_{mj}(k) A_{n\ell}(k) \int_0^{t_1} \int_0^{t_2} h_{sm}(t_1 - s_1) h_{sn}(t_2 - s_2) \cdot$$

$$\cdot E [q_{pj}(s_1) q(s_2)] ds_1 ds_2 \quad (9)$$

However,

$$E [q_{pj}(t_1) \cdot q_{p\ell}(t_2)] = \int_{-\infty}^{\infty} [H_{pj}(\omega) H_{p\ell}^*(\omega) G_{\ddot{y}}(\omega) e^{i\omega(t_1 - t_2)}] d\omega \quad (10)$$

where $H_{pj}(\omega)$ is the complex frequency response function for the primary mode j , and $G_{\ddot{y}}(\omega)$ is the spectral density of ground acceleration. Substitution of eq. (10) in eq. (9) and integration on s_1 and s_2 yields

$$R_{z_{sk}}(t_1 - t_2) = \sum_{m=1}^{n_s} \sum_{n=1}^{n_s} \sum_{j=1}^{n_p} \sum_{\ell=1}^{n_p} [A_{mj}(k) A_{n\ell}(k) \int_{-\infty}^{\infty} \{H_{sm}(\omega) H_{sn}^*(\omega) H_{pj}(\omega) H_{p\ell}^*(\omega) \cdot$$

$$\cdot G_{\ddot{y}}(\omega) e^{i\omega(t_1 - t_2)}\} d\omega \quad (11)$$

where $H_{sn}^*(\omega)$ is the complex conjugate of $H_{sn}(\omega)$. Thus

$$\sigma_{z_{sk}}^2 = R_{z_{sk}}(0) = \sum_{m=1}^{n_s} \sum_{n=1}^{n_s} \sum_{j=1}^{n_p} \sum_{\ell=1}^{n_p} [A_{mj}(k) A_{n\ell}(k) \sigma_{mnj\ell}^2] \quad (12)$$

and

$$\sigma_{z_{sk}}^2 = \left. \frac{\partial^2 R_z(t_1 - t_2)}{\partial t_1 \partial t_2} \right|_{t_1=t_2=t} = \sum_{m=1}^{n_s} \sum_{n=1}^{n_s} \sum_{j=1}^{n_p} \sum_{\ell=1}^{n_p} [A_{mj} A_{n\ell}(k) \sigma_{mnj\ell}^2] \quad (13)$$

where,

$$\sigma_{mnj\ell}^2 = \int_{-\infty}^{\infty} H_{sm}(\omega) H_{sn}^*(\omega) H_{pj}(\omega) H_{p\ell}^*(\omega) G_{\ddot{y}}(\omega) \cdot d\omega \quad (14)$$

$$\sigma_{mnj\ell}^2 = \int_{-\infty}^{\infty} \omega^2 H_{sm}(\omega) H_{sn}^*(\omega) H_{pj}(\omega) H_{p\ell}^*(\omega) G_{\ddot{y}}(\omega) d\omega \quad (15)$$

The above integrals can be evaluated by the method of residues (Churchill [3]). The Complex Arithmetic feature of FORTRAN IV language makes the numerical evaluation straight-forward, thus avoiding lengthy algebra.

The expressions for the variances when the system of Fig. 2 is considered as a coupled system would be (Gungor [4]),

$$\sigma_{z_k}^2 = \sum_{j=1}^n \sum_{l=1}^n \{ \gamma_j \gamma_l [\psi_j(k) - \psi_j(k-1)] [\psi_l(k) - \psi_l(k-1)] \sigma_{jl}^2 \} \quad (16)$$

and

$$\sigma_{z_k}^2 = \sum_{j=1}^n \sum_{l=1}^n \{ \gamma_j \gamma_l [\psi_j(k) - \psi_j(k-1)] [\psi_l(k) - \psi_l(k-1)] \sigma_{jl}^2 \} \quad (17)$$

where n , γ_j and $\psi_j(k)$ are the degrees of freedom, participation factor in mode j , and k -th element of the j -th eigenvector for the coupled system, respectively. Also,

$$\sigma_{jl}^2 = \int_{-\infty}^{\infty} H_j(\omega) H_l^*(\omega) G_{\ddot{y}}(\omega) d\omega \quad (18)$$

$$\sigma_{jl}^2 = \int_{-\infty}^{\infty} \omega^2 H_j(\omega) H_l^*(\omega) G_{\ddot{y}}(\omega) d\omega \quad (19)$$

2.1 Earthquake Spectral Density

In the random vibration approach, the expression for the spectral density of ground excitations must be known. Based on Gungor [4], the following expression is used,

$$G_{\ddot{y}}(\omega) = S_0 \frac{1 + 4\beta_f^2 (\omega/\omega_f)^2}{(1 - \omega^2/\omega_f^2)^2 + 4\beta_f^2 (\omega/\omega_f)^2} \cdot \frac{a + 4\beta_g^2 (\omega/\omega_g)^2 + c (\omega/\omega_g)^4}{(1 - \omega^2/\omega_g^2)^2 + 4\beta_g^2 (\omega/\omega_g)^2} \quad (20)$$

Where $\beta_f = 0.64$, $\omega_f = 15.5/\text{sec.}$, $a = 0.46$, $\beta_g = 0.81$, $c = 1.25$, $\omega_g = 12.6/\text{sec.}$ and S_0 is the scaling factor. Constants a , β_g and c are found by minimizing the square of the error in the average velocity spectra. Figure 5 shows the average velocity spectra obtained by the random vibration method in relation to the points obtained by a time-history solution of the average of eight earthquakes. A value of 0.0036, 0.0024 and 0.0016 for S_0 was used for a system with 5%, 2% and 1% damping, respectively. The smaller values of S_0 for successively smaller values of damping, in part, signifies the non-stationarity of the earthquake motions and of the structural response. However, the non-stationarity to a large extent can be accounted for by a constant scaling factor depending on system damping. It is to be noted that the response varies as the square-root of S_0 , and S_0 itself varies within a narrow limit; thus, the error involved is small, i.e., if S_0 corresponding to a 2% damping is used for systems with a damping of 1% to 5%, the error would always be less than 20%, which is not large considering the errors in the recording and digitizing of earthquake records (Amin and Ang [5]). This suggests that any reasonable adjustment for S_0 would be adequate. When the damping in the secondary and the primary systems were assumed to be different, the following adjustment for S_0 was applied: (i) for the diagonal terms σ_{mmjj} -- linear variation between S_s and S_0 depending on the ratio ω_{sm}/ω_{pj} ; (ii) for the terms σ_{mmjl} , $j \neq l$ -- $S_0 = S_s$; and for the term σ_{mnjj} , $m \neq n$, -- $S_0 = S_p$ here S_p and S_s are the values of S_0 associated with the damping in the primary and secondary systems, respectively. The rule

for σ_{mnjl} is the same as the one described for σ_{mnjl} . The rule adopted is based on the behavior exhibited by σ_{mnjl} which is shown in Figs. 6 and 7.

3. Relation Between Random Vibration and Response Spectrum Method

In the response spectrum method for light secondary systems (e.g., Biggs and Rosset [6], and Kassawara [7]), an iterated response spectra is used to compute the modal responses for each primary and secondary modes. These modal responses are subsequently combined using the square-root of sum-of-squares rule; thus,

$$\max |u| = \sqrt{\sum_{m=1}^{n_s} \sum_{j=1}^{n_p} [u_{mj}^2]} \quad (21)$$

where u is the total response and u_{mj} is the maximum modal response for the m -th primary and j -th secondary modes. Using eq. (21), the spring distortion for the system of Figure 2 is

$$z_{sk} = \sqrt{\sum_{m=1}^{n_s} \sum_{j=1}^{n_p} [A_{mj}^2(k) \eta_{mj}^2]} \quad (22)$$

where η_{mj} is the ordinate of the iterated response spectra corresponding to the m -th secondary and j -th primary modes.

The random vibration method yields

$$\max |z_{sk}| = \sigma_{z_{sk}} \cdot c \quad (23)$$

where

$$c = \left[2\ell n \left\{ \frac{\sigma_{z_{sk}}^2 t_d}{\sigma_{z_{sk}}^2 \pi \ell n(2)} \right\} \right]^{1/2} \quad (24)$$

and

$$\sigma_{z_{sk}}^2 = \sum_{m=1}^{n_s} \sum_{j=1}^{n_p} [A_{mj}^2(k) \sigma_{mmjj}^2] \quad (25)$$

$$\sigma_z^2 = \sum_{m=1}^{n_s} \sum_{j=1}^{n_p} [A_{mj}^2(k) \sigma_{mmjj}^2] \quad (26)$$

Equations (25) and (26) are obtained by neglecting the off-diagonal terms of eqs. (12) and (13). By definition, let

$$c_{mj} = \left[2\ell n \left\{ \frac{\sigma_{mmjj}^2 t_d}{\pi \sigma_{mmjj}^2 \ell n(2)} \right\} \right]^{1/2} \quad (27)$$

It can be shown (see Singh [2]) that for a given system $(c_{mj})_{\min} \leq c \leq (c_{mj})_{\max}$ and that c_{mj} varies within a narrow limit. Thus, replacing c by c_{mj} , eq. (23) becomes

$$\max |z_{sk}| = \sqrt{\sum_{m=1}^{n_s} \sum_{j=1}^{n_p} [c_{mj}^2 A_{mj}^2(k) \sigma_{mmjj}^2]} \quad (28)$$

However, as η_{mj} is the iterated response, $\eta_{mj} = c_{mj} \sigma_{mmjj}$. Thus, eqs. (22) and (28) are equivalent. It follows that the response spectrum and random vibration methods are approximately the same.

4. Accuracy of Random Vibration Method

The accuracy of the random vibration method is evaluated by comparing the response obtained at 50% and 10% exceedance probability levels with the average and second highest among the time-history solutions for eight earthquake records. The two horizontal components of the following four earthquakes were used in the time-history solutions: El Centro (12/30/1934) NE, EW; Taft (7/21/1952) N 21° E, S 69° W; Olympia (4/13/1949) S 80° W, S 10° E; and El Centro (5/18/1940) NS, EW. The records were adjusted for base line correction and normalized by equating the area under the undamped velocity spectra. The time-history solutions were obtained by numerically integrating the equations of motion using the Beta method of Newmark [8]. Where possible, a coupled system was used for the time-history solution. For purposes of comparison, the average maximum responses obtained by the response spectrum method are also presented.

The mechanical systems considered are shown in Figs. 2 and 3. The solution method for both of these systems is similar; however, the analysis of a flexural system is more involved and the support continuity has to be taken into account. A mass ratio (the mass ratio is defined to be the ratio of the total mass of the secondary system to the mass of the supporting primary mass) of 0.01 is assumed. The results are shown in Table 1 relative to those of the corresponding time-history solutions.

These results indicate that except for resonant systems, the random vibration approach leads to results which are generally within 10% of the time-history solutions. The random vibration results are superior to those obtained by the response spectrum method. For resonant systems, much of the error in the random vibration solution can be attributed to the influence of decoupling (Singh [10]); this should not be a major drawback for the decoupled model, since in most practical problems resonant systems are avoided.

5. Influence of Non-Stationarity on Response

Earthquake response of structures are generally non-stationary; however, for many practical situations, a stationary model is often adequate. Here the degree of conservatism introduced by a stationary assumption is examined. Generally a deterministic envelope function together with a stationary random process is used to model actual earthquake motions (Amin and Ang [5], and Jennings, et al [9]). Here a stationary Gaussian white noise together with a deterministic envelope $en_B(t)$ (Jennings, et al [9]) is used to study the influence of non-stationarity on response. Modeling of earthquakes as white noise should not cause any significant error, as actual earthquakes are broad band processes with a fairly constant frequency content for the frequency range and damping considered. The auto-correlation function of the assumed excitation is

$$R_Y(t_1, t_2) = en_B^2(t_1) \delta(t_1 - t_2) \quad (29)$$

where $\delta(t)$ is the Dirac's delta function. Based on eq. (29), the variances of various responses can be obtained in closed form. Equations (1) and (4) are used to evaluate the response for a given probability of exceedance.

The two degree of freedom system shown in Fig. 4 is considered. The primary and secondary systems are both single-degree-of-freedom systems with a mass ratio of 0.01. The system was analyzed as a coupled system; however, only the results for the distortion of the secondary spring is presented. Figures 8 and 9 show the influence of the secondary system period T_s , on the non-stationary response (measured by the ratio of non-stationary to stationary response, $b_{.10}/\bar{b}_{.10}$) for the primary system period, T_p of 1.0 and 1.5 seconds, respectively. The dotted lines represent the results for non-stationarity due to zero initial conditions only, whereas the solid lines are for cases when non-stationarity due to zero initial conditions and non-stationary excitation defined by envelope $e_B(t)$ is also considered. An earthquake duration of thirty seconds is used. The results are presented for a 2% and a 5% damping. From the figures, it is concluded that: (i) for strong-motion earthquakes of relatively long duration ($t \approx 30$ sec.) a stationary approximation is adequate; e.g., for a 2% damping the influence of non-stationarity is generally less than 20%. For higher damping, the influence is even less. Also, for a given damping, the error varies within 10% and thus a constant scale factor adjustment would reduce the error; (ii) for systems where $T_p > T_s$ the influence of non-stationarity depends primarily on the primary period, and when $T_p < T_s$ the influence of non-stationarity is primarily dependent on the secondary period.

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TABLE 1 COMPARISON OF TIME HISTORY, RANDOM VIBRATION,
AND RESPONSE SPECTRUM SOLUTIONS

T_s (1)	TIME HISTORY SOLUTION		$\frac{b}{.50}$ RV	$\frac{b}{.10}$ RV	$\frac{b}{RS}$
T_p (1)	AVERAGE	2ND HIGH	AVERAGE	2ND HIGH	AVERAGE
(1)	(2)	(3)	(4)	(5)	(6)
*STRUCTURE OF FIGURE 2, MASS RATIO = 0.01, $\beta_p = 0.05$ AND $\beta_s = 0.01$					
ACCELERATION OF MASS 4					
0.50	5.65	6.39	1.10	1.13	0.98
0.75	7.41	9.66	0.99	0.88	0.86
1.00	11.52	12.07	1.36	1.52	0.93
1.25	8.07	9.59	0.98	0.96	0.85
1.50	4.89	5.85	0.98	0.95	0.90
2.00	2.67	3.00	0.96	1.01	0.91
3.00	4.67	5.84	1.24	1.18	0.85
DISTORTION IN SPRING 5					
0.50	0.030	0.035	1.10	1.08	0.90
0.75	0.027	0.029	1.13	1.23	1.33
1.00	0.165	0.198	1.61	1.60	0.96
1.25	0.125	0.162	0.98	0.91	0.70
1.50	0.103	0.130	1.05	1.01	0.68
2.00	0.119	0.151	0.95	0.91	0.66
3.00	0.186	0.223	0.95	0.97	0.61
*STRUCTURE OF FIGURE 3, MASS RATIO = 0.01, $\beta_p = 0.02$ AND $\beta_s = 0.02$					
RELATIVE DISPLACEMENT OF MASS 5					
0.50	0.052	0.059	0.94	0.96	1.35
0.75	0.062	0.074	0.99	0.97	1.85
1.00	0.251	0.299	1.59	1.59	1.08
1.25	0.335	0.427	0.93	0.87	0.95
2.00	0.370	0.413	1.01	1.09	0.86
3.00	0.297	0.347	0.96	1.00	0.73
RELATIVE DISPLACEMENT OF MASS 6					
0.50	0.044	0.056	1.03	0.95	0.97
0.75	0.051	0.059	0.95	0.97	1.26
1.00	0.174	0.209	1.61	1.59	0.98
1.25	0.193	0.252	0.92	0.84	0.77
2.00	0.163	0.188	0.99	1.02	0.91
3.00	0.124	0.150	0.99	1.00	0.82

* Primary structure frequencies -- 6.28 16.0 23.3 rad/sec.
Secondary structure frequencies: System of Fig. 2 -- ω , 2.79ω , 4.04ω
System of Fig. 3 -- ω , 1.54ω , 4.88ω , 5.94ω , 8.96ω , 10.2ω

Figure 1 Random Process $r(t)$ and First Excursion

Figure 2 Singly-Supported
Secondary System

Figure 3 Multiply-Supported
Secondary System

Figure 4 System Considered for Nonstationary Response

Figure 5 Comparison of Median Pseudo Velocity Spectra

Figure 6

Effect of Primary and Secondary System Damping on σ_{mnjj}

Figure 7

Effect of Primary and Secondary System Damping on σ_{mnjj}

Figure 8 Influence of Nonstationarity on Response

Figure 9 Influence of Nonstationarity on Response