PROBABILISTIC PREDICTION OF MULTIPLE FRACTURE UNDER SERVICE CONDITIONS*

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SUMMARY

The probability of fracture initiation, propagation, and arrest is one of the important problems facing designers, analysts, and operators of modern structures including nuclear reactors. The question of fracture requires special considerations which include the random nature of the loading and the statistical nature of the material's response to fracture under the imposed service loads. In most structural applications it is essential to know not only if and when fracture will occur under normal and off-normal operating conditions, but also to have some knowledge of the fracture propagation configurations and their resulting influence on the integrity of the structure.

In this presentation we address the problem of multiple fracture propagation configurations in structures under service conditions. This is accomplished by introducing a generalized energy criterion for multiple brittle fracture in nonhomogeneous and anisotropic materials.

The fracture criterion is expressed in terms of the so-called Hartz function, H , which measures the difference between the total instantaneous strain energy release rate, G_T , and the rate at which energy is required for the formation of new fracture surfaces, R_T . Strain energy release rates are computed for a variety of symmetric as well as asymmetric fracture propagation configurations from finite element solutions of incrementally related boundary value problems. These solutions yield a deterministic influence parameter, α , which is used to relate the applied loading to the probabilistic expression for the strain energy release rate. A similar treatment is given to the function, R_T , for which the influence parameter, β , must be determined experimentally. The parameters, α and β , depend upon the relative locations, sizes and orientations of the primary and secondary cracks as well as flaws, and secondary imperfections present in the material. In addition, the parameter, β depends upon the relative values of the specific surface energy associated with the possible primary and secondary fracture paths. This is important for anisotropic materials such as ceramics mixed-oxide fuels and concrete, and for materials experiencing stress-corrosion fracture, where the energies associated with intergranular and transgranular fracture, for example, may differ significantly.

From the probabilistic expressions for G_T and R_T , the Hartz function is determined as a random variable, h , which describes the multiple fracture process.

The formulations developed in this presentation are applied to a single edge notched panel experiencing multiple fracture under assumed random loadings. Some interesting symmetric as well as nonsymmetric fracture configurations are studied and the results are related to reported experimental observations.

The approach presented here may have applications in the areas of fatigue crack propagation, stress corrosion cracking and fail-safe design optimization. Current studies are aimed at simulating the influence of grain structure anisotropy, intergranular corrosive attack and creep deformation on multiple fracture in polycrystalline materials.

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INTRODUCTION

The probability of fracture initiation, propagation, and arrest is one of the important problems facing designers, analysts, and operators of modern structures including nuclear reactors. With the development of new materials with higher ultimate strengths, and sophisticated analytical techniques for computing stresses and strains, designers are tempted to specify higher allowable loads with lower margins of safety. This may be questionable from a safety viewpoint since structural integrity may not be ensured by the use of conventional safety factors. This is true particularly for a structure whose integrity is determined by its ability to withstand fracture. Increased static strength is not always accompanied by a corresponding increase in the ability of the structure to withstand fracture under service loads. The question of fracture requires special considerations which include the random nature of the loading and the statistical nature of the material's response to fracture under the imposed service loads.

In most structural applications it is essential to know not only if and when fracture will occur under normal and off-normal operating conditions, but also to have some knowledge of the fracture propagation configurations and their resulting influence on the integrity of the structure. In this presentation we address the problem of multiple fracture propagation configurations in structures under service conditions. This is accomplished by introducing a generalized energy criterion presented earlier by Chopra and Hartz [1,2,3] for multiple brittle fracture in nonhomogeneous and anisotropic materials.

The fracture criterion is expressed in terms of the Hartz function, H , which measures the difference between the total instantaneous strain energy release rate, G_T , and the rate at which energy is required for the formation of new fracture surfaces, R_T . Strain energy release rates are computed for a variety of symmetric as well as asymmetric fracture propagation configurations from finite element solutions of incrementally related boundary value problems [1,4]. These solutions yield a deterministic influence parameter, α , which following Chopra and Wang [5], is used to relate the applied loading to the probabilistic expression for the strain energy release rate. A similar treatment is given to the function, R_T , for which the influence parameter, β , must be determined experimentally. The parameters, α and β , depend upon the relative locations, sizes and orientations of the primary and secondary cracks as well as flaws, and secondary imperfections present in the material. In addition, the parameter, β depends upon the relative values of the specific surface energy associated with the possible primary and secondary fracture paths. This is important for anisotropic materials such as ceramics mixed-oxide fuels and concrete, and for materials experiencing stress-corrosion fracture, where the energies associated with intergranular and transgranular fracture, for example, may differ significantly.

From the probabilistic expressions for G_T and R_T , the Hartz function is determined as a random variable, h , which describes the multiple fracture process. The formulations developed in this presentation are applied to a single edge notched panel experiencing multiple fracture under assumed random loadings. Some interesting symmetric as well as nonsymmetric fracture configurations are studied and the results are related to reported experimental observations. Examples of multiple fracture configurations observed in fast breeder reactor mixed oxide fuel element cladding are shown in Figures 1 and 2.

MULTIPLE FRACTURE CRITERION

A fracture configuration can be described in terms of a primary crack and secondary cracks which may be either connected to (branch cracks) or remote from the primary crack. An earlier review of fracture studies related to multiple fracture by Chopra [1] indicated that analytical investigations [6,7,8] suffer from a dependence on the stress and strain fields associated with a single Griffith-type crack. It has been shown by Chopra [1] and Hartz and Chopra [2,3,4] that during multiple fracture, the true stress and strain fields and the corresponding values of the total strain energy release rates are a function of the primary and secondary cracks, their lengths, orientations and locations, in addition to their dynamic behavior. Considering observed fractures in polycrystalline materials which exhibit varying degrees of roughness along the fracture surfaces, and secondary micro and/or macro fractures, and using the extended potential energy principle of Rice and Drucker [9], the Griffith criterion [10] was extended to a hypothesis for a generalized energy criterion for multiple branching and secondary cracking in brittle materials including nonhomogeneous and anisotropic materials [1-4].

The generalized energy criterion for multiple fracture can be written as:

$$H(a_T) = G_T(a_T) - R_T(a_T) \geq 0 \quad (1)$$

where $G_T(a_T)$ is the instantaneous total strain energy release rate for crack formation as a function of the fracture incremental area, Δa_T ; and $R_T(a_T)$ is the instantaneous rate of conversion of available energy into the energy required for crack formation.

The strain energy release rates, $G_T(a_T)$ can be obtained from finite element solutions of successive boundary value problems, corresponding to progressive fracture configurations, which yield the total strain energy stored in the body, $U_T(a_T)$, as fracture proceeds.

$$G_T(a_T) = (\pm) \frac{\partial U(a_T)}{\partial a_T} \quad (2)$$

The method of computing $U_T(a_T)$ should account for all cracks as well as flaws present in the body. A simple expression for the instantaneous rate of

* + for constant stress boundary conditions
- for fixed displacement boundary conditions

conversion of available energy into the energy required for crack formation $R_T(a_T)$, during quasi-static brittle fracture, has been proposed in the form [1]:

$$R_T(a_T) = \frac{(T_p[a_p]\Delta a_p + \sum_{k=1}^m T_{sk}[a_{sk}]\Delta a_{sk})}{\Delta a_T} \quad (3)$$

where $T_p(a_p)$ is the instantaneous specific surface energy of the primary crack extended in area by Δa_p ; $T_{sk}(a_{sk})$ is the instantaneous specific surface energy of the k th, of m secondary cracks, extended in area by Δa_{sk} ; and Δa_T is the total increment in the fracture surface. For the case of a single crack propagating in a homogeneous and isotropic body, the expressions (1) and (2) are identical to Griffith's criterion for rupture [10].

Based upon considerations of fracture propagation behaviors, it has been suggested [1,3] that "minimization of the total potential energy for any form of quasi-static crack growth is enhanced by the attainment of a state for which H is maximized." This leads to the hypothesis that in an elastic solid body experiencing fracture under the action of specified surface tractions, the relative locations, size and orientation of crack initiation and propagation configurations are determined by the corresponding values of the Hartz function, H .

The Hartz function has a unique value, H_{qij} , corresponding to the specified fracture configuration, F_i , the specified surface tractions, L_j , and the specific surface energy on strength of the body, S_q . The range of values of the function, H_{qij} , may be expressed in the form of a three-dimensional fracture matrix (H_{qij}) as shown below.

PROBABILISTIC EXPRESSION OF HARTZ FUNCTION

The generalized formulation of the Hartz function for multiple fracture can be expressed in a probabilistic form in order to describe fracture in real materials under service conditions. This is done simply by assuming that the sets of surface tractions, l_j , and the material strength functions, s_q , are both random and by establishing a relationship between these random functions and the probabilistic expression of the Hartz function, h .

It is postulated that the total instantaneous strain energy release rate, g_{Tqij} , is a random function of the loading, l_j ; and the instantaneous rate of conversion of available energy into the energy required for crack formation, r_{Tqi} , is a random function of the strength of the material, s_q . It follows that:

$$g_{Tqij} = g_{Tqij}(\alpha_{qij}, l_j^2) \quad (4)$$

$$r_{Tqi} = r_{Tqi}(\beta_{qi}, s_q^2) \quad (5)$$

where α_{qij} and β_{qi} are deterministic functions of the i -th fracture configuration, determined by the location, size, and orientation of cracks as well as secondary imperfections and the corresponding values of the surface tractions and the relative values of the specific surface energy along the fracture surfaces. l_j and s_q are the applied loading and the strength of the material, respectively; both regarded as random variables.

Equations (4) and (5) may be expressed more explicitly by postulating that:

$$g_{Tqij} = \alpha_{qij} l_j^2 \quad (6)$$

and

$$r_{Tqi} = \beta_{qi} s_q^2 \quad (7)$$

where the functions α_{qij} and β_{qi} are influence coefficients obtained from equations (2) and (3), respectively.

Based upon the relationships (6) and (7), the distribution functions and density functions of g_{Tij} and r_{Tiq} can be expressed in terms of the known distribution functions l_j and s_q as follows:

$$F'_{g_{Tqij}}(x) = F_{l_j}\left(\sqrt{\frac{x}{\alpha_{qij}}}\right) \quad (8)$$

$$F'_{r_{Tqi}}(x) = F_{s_q}\left(\sqrt{\frac{x}{\beta_{qi}}}\right) \quad (9)$$

$$f_{g_{Tqij}}(x) = \frac{1}{2\sqrt{\alpha_{qij}x}} f_{g_j} \left(\sqrt{\frac{x}{\alpha_{qij}}} \right) \quad (10)$$

$$f_{r_{Tqi}}(x) = \frac{1}{2\sqrt{\beta_{qi}x}} f_{s_q} \left(\sqrt{\frac{x}{\beta_{qi}}} \right) \quad (11)$$

where $F_X(x)$ and $f_X(x)$ are the distribution functions and the density functions of the random variable, X , respectively.

The generalized energy criterion for multiple fracture; equation (1), now can be expressed in a probabilistic form as:

$$h_{qij} = P \left\{ (g_{Tqij} - r_{Tqi}) \geq 0 \right\} \quad (12)$$

or

$$h_{qij} = \iint_D f_{g_{Tqij}}(x) f_{r_{Tqi}}(y) dx dy \quad (13)$$

where D is the domain of integration for $(g_{Tqij} - r_{Tqi}) \geq 0$. h_{qij} is the probability of occurrence of the i -th fracture configuration corresponding to the j -th set of surface tractions and q -th set of material properties.

If the number of possible fracture configurations in a given structure is known or assumed to be n , and the occurrences of different fracture configurations are mutually exclusive, the probability that the structure will experience fracture is:

$$P_{f_{qj}} = \sum_{i=1}^n h_{qij} \leq 1 \quad (14)$$

Simplified expressions for h_{qij} and $P_{f_{qj}}$ can be obtained if the structure is subjected to a single random set of surface tractions, l , and the material properties are defined by a single random strength function, s . Thus:

$$h_i = \iint_D f_{g_{Ti}}(x) f_{r_{Ti}}(y) dx dy \quad (15)$$

and

$$P_f = \sum_{i=1}^n h_i \leq 1 \quad (16)$$

If it is assumed further that the structure is subjected to a set of deterministic surface tractions, $l = L$, we obtain, G_{Ti} as a discrete function of L and the fracture configuration, i . Thus,

$$h_i = F_{r_{Ti}}(G_{Ti}) \quad (17)$$

or

$$h_i = F_{r_{Ti}}(\alpha_i L^2) \quad (18)$$

Similarly, if the strength of the material is assumed to be represented by a deterministic function, $s = S$, then the rate at which energy is required for crack formation, R_{Ti} is obtained as a discrete function of S and the fracture configuration, i ; whence:

$$h_i = 1 - F_{g_{Ti}}(R_{Ti}) \quad (19)$$

or

$$h_i = 1 - F_{g_{Ti}}(\beta_i S^2) \quad (20)$$

FRACTURE PREDICTIONS UNDER RANDOM LOADING

The application of equations (13) and (14) to the general case of fracture in a structure, represented by random material properties and subjected to random loading is complex, and will be dealt with in a subsequent communication. In the current investigation, the probability of fracture occurrence, p_f and the probability of occurrence of the i -th fracture configuration, h_i , in a structure, possessing specified material properties, subject to random service loads, are obtained with the use of equations (16) and (19).

It is assumed that the structure is composed of a homogeneous and isotropic material for which R_{Ti} is equal to a constant value, R_T . The value, R_T , therefore may be referred to as the average fracture toughness of the material. If it is assumed further that the structure is subjected to random loading, l , which follows a Gaussian distribution with a mean μ_l , and a standard deviations, σ_l ,

$$h_i = 1 - F_{g_{Ti}}(R_T) = 1 - F_l\left(\sqrt{\frac{R_T}{\alpha_i}}\right) \quad (21)$$

or

$$h_i = 1 - \Phi\left[\left(\sqrt{\frac{R_T}{\alpha_i}} - \mu_l\right) / \sigma_l\right] \quad (22)$$

where Φ is the standardized normal distribution function.

Equation (22) can be used to determine the relative probabilities of occurrence of n selected fracture propagation configurations in a structure subjected to a random set of surface tractions which follows a Gaussian distribution. If the loading assumes a log-normal distribution, equation (21) becomes:

$$h_i = 1 - \Phi\left[\frac{\ln\left(\sqrt{\frac{R_T}{\alpha_i}} \frac{(V_l^2 + 1)}{\mu_l}\right)}{\sqrt{\ln(V_l^2 + 1)}}\right] \quad (23)$$

where $V_l = \sigma_l / \mu_l$

RESULTS AND DISCUSSION

Application of the probabilistic multiple fracture propagation hypothesis is illustrated with an edge-cracked panel shown in Figure 3, subjected to a random uniaxial tensile stress with a mean value of 1,000 psi. The panel is assumed to be perfectly homogeneous and isotropic with a constant value of R_T , a modulus of elasticity, $E = 5 \times 10^5$ psi, and a Poisson's ratio $\nu = 0.35$ representing Holomite 100 material.

Values of the loading influence coefficient, α_1 , are obtained from previous finite element investigations of multiple fracture paths by Chopra [1] for various fracture propagation configurations. These computed values of the loading influence coefficient, α_1 , in conjunction with known or assumed constant values of R_T in equations (22) and (23), yield the relative probabilities of occurrence of the analyzed fracture configurations.

In the present application, four parametric studies are selected which appear to resemble fracture profiles obtained from aerospace and nuclear structural components. In applying these finite element results, the general limitations of the numerical method of solution must be noted. It should be recognized, in particular, that solutions are obtained only for those fracture configurations which are selected and imposed by the analyst. Consequently, unless the fracture configurations to be studied are selected judiciously, based upon an understanding of the parameters which are expected to influence fracture behavior, there may remain other configurations with higher fracture probabilities than those obtained for the investigated profiles.

When a primary crack tip approaches a grain boundary flaw or a material defect, the relative probabilities of the crack deflecting or branching may be of interest. Figure 4 shows the relative probabilities for a primary crack to form a single non-symmetric branch or two symmetric branches. As is expected, from numerous fracture profiles reported in the literature, the predicted probability of crack deflection is significantly higher than that for symmetrical crack branching for a homogeneous and isotropic material. This result is true for loading with a normal distribution or a log-normal distribution. This prediction offers an explanation for the generally observed zig-zag pattern of single cracks in polycrystalline materials as shown in Figure 5.

Symmetric crack branches have been known to occur in some cases [11,12,13]. However, the predicted probability of the continued propagation of a single branch is greater than that for the simultaneous propagation of both branches as shown in Figure 6 for both normal and log-normal loadings. This result is supported directly by numerous fracture profiles obtained by Bradley [13] where, immediately after symmetrical crack branching, one branch crack arrested and the other continued to propagate.

There are several reported cases [11,13] where symmetrical multiple fracture patterns have been observed. The angles at which these branch cracks initiate and propagate have been the object of some earlier investigations [6,7]. Figure 7 shows the relative probabilities of various branch crack propagation angles. The results indicate that branch cracks will tend to propagate at angles ranging from 0° to 60° to the primary crack axis with a tendency to turn parallel to the primary crack axis. The relative probabilities for the branches to turn inwards towards one another ($-30^\circ < \theta < 0^\circ$) or to flare outwards ($60^\circ < \theta < 90^\circ$) is low.

Crack branching has been observed by Carter [11] to occur more often at angles ranging from 15° to 45° . An example of the secondary cracking patterns obtained by Carter is shown in Figure 8. These observations do not support Yoffe's [15] and William's [16] predictions based upon crack tip stress analyses for a single primary crack, that branching should occur at an angle of 60° to the crack axis. The probabilistic predictions shown in Figure 7, which follow from the deterministic solutions presented earlier by Chopra and Hartz [1-4], indicate that material properties play an important role in determining branch crack propagation angles. This is evident from the small difference in the predicted probabilities for branch crack propagation angles, particularly in the range of 0° to 60° .

The direction of propagation of a single branch crack is examined in Figure 9. The initial branch crack is inclined to the primary crack axis at an angle of 45° . The results indicate that the probability of continued propagation at angles between -30° and 30° is considerably higher than that for the remaining angles investigated. However the highest probability is obtained for propagation at angles between 0° and -30° . This indicates that contrary to the case of two symmetrical branch cracks, a single non-symmetric inclined branch crack has a tendency to turn inwards towards the primary crack axis. Subsequent propagation may result in a change in the propagation angle and can be investigated subsequently.

An interesting comparison may be made between the results shown in Figure 9 and the experimental results of Iida and Kobayashi [14] for crack propagation from an initially inclined central crack in a plate subjected to uniaxial tension. The crack paths were found to turn inward from the initially inclined direction until they become orthogonal to the applied loading. Iida and Kobayashi concluded that under bimodal loading a crack rotates immediately on propagation and propagates in the direction where the stress intensity factor, K , is maximum. The results in Figure 9 suggest that crack rotation is determined by the maximum value of the probabilistic expression for the Hartz function, h .

CONCLUSIONS

The approach presented here may be employed to simulate some multiple brittle fracture behaviors in anisotropic and nonhomogeneous structures and materials while recognizing that fracture is a discontinuous and random process. The probability of fracture occurrence and the relative probabilities of occurrence of various possible fracture paths may also be determined by the method described. It appears

that probabilistic expression of the Hartz function may have application in the areas of fatigue crack propagation, stress corrosion cracking and fail-safe design optimization, as has been indicated earlier by Chopra [1,15] and Hartz et al. [16]. Current studies are directed towards evaluating the influence of grain structure anisotropy, intergranular corrosive attack and creep deformations on multiple fracture in polycrystalline materials. These effects may be important for applications to fast breeder reactor fuel element cladding.

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Figure 1: Transverse section of a fast breeder reactor, type 304L S.S., fuel element cladding showing multiple fracture path.

Figure 2: Inter-granular multiple fractures on the inner surface of a fast breeder reactor, type 304L S.S., fuel element cladding.

Figure 3: Edge-cracked panel for multiple fracture investigations.

Figure 4: Fracture probabilities for symmetrical and non-symmetrical branching (loading assumed normal/log-normal).

Figure 5: Zig-zag fracture propagation in a fast breeder reactor fuel element cladding of type 304L S.S.

Figure 6: Fracture probabilities for symmetrical and non-symmetrical branch crack propagation (loading assumed normal/log-normal).

Figure 7: Fracture probabilities for symmetrical propagation (loading assumed normal).

Figure 8: Stress-corrosion secondary cracking in a high strength steel [11]

Figure 9: Fracture probabilities for non-symmetrical propagation (loading assumed normal).