TEXTURE AND ANISOTROPY OF ZIRCONIUM IN RELATION TO PLASTIC DEFORMATION

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ABSTRACT

Possible modes of plastic deformation of tubular materials are analyzed in a general, yet illustrative way. Based on symmetry considerations applied on known textures of zirconium wire and strip, textures of zirconium tubes are predicted as resulting from a wide range of deformation modes. This is made possible by focusing on the main feature of texture, i.e. the orientation distribution of basal poles in large. A relation between texture and the mechanical anisotropy of zirconium is developed, and rules are suggested for softening and hardening caused by texture change. Experimentally, Zircaloy tubes were deformed by various techniques, in each case to a series of deformation degrees. The resulting texture was determined by X-ray diffraction and the mechanical anisotropy by Knoop-hardness measurement. Each determination is condensed into two representative figures, one describing the type of texture or anisotropy and the other describing the sharpness of texture or the degree of deviation from isotropy.

The experiments verify the predicted relation between the type of texture and mechanical anisotropy, and also indicate how the degree of anisotropy depends on the sharpness of texture. Changes in texture are investigated in relation to the plastic deformation, and further refined rules are derived for the direction and rate of these changes. The initially predicted textures are concluded to be stable, i.e. they are reached only after a certain degree of deformation. Zircaloy tubes of three separate textures were subjected to tensile testing, closed-end burst testing, and open-end burst testing. The observed texture-softening phenomenon corroborates the rules suggested and is concluded to occur commonly, as opposed to texture hardening, which requires special combinations of texture and testing method.

1. INTRODUCTION

The hexagonal close-packed metal zirconium has a crystal lattice with a unique direction, the basal plane normal or basal pole [0001], implying a pronounced anisotropy. This is manifested in plastic deformation, which occurs either by slip or twinning. The slip systems are of the type $\{10\bar{1}0\} \langle 1\bar{2}10 \rangle$ with all slip directions perpendicular to the [0001]. Thus tension or compression along [0001] can not be accommodated by slip. Instead, twinning occurs and this requires higher resolved shear stress, making [0001] a hard direction of the crystal. The anisotropy of zirconium crystals is transferred to the polycrystalline state of the metal, which normally is heavily textured and has anisotropic properties. This is of special importance as regards the mechanical properties of Zircaloy canning tubes in their use as fuel sheathing in nuclear reactors, table I. In Zircaloy the hcp alpha-phase is dominant, giving Zircaloy the same characteristics as regards texture and anisotropy as pure zirconium. In the following, the word zirconium should be interpreted as including Zircaloy as well as other dilute alloys where the alpha-phase is dominant.

Zirconium textures in wire and strip have been recorded and reported by Burgers et al [1], McGeary et al [2], and also thoroughly reviewed by Dillamore et al [3]. The anisotropy of mechanical properties has been illustrated in a number of investigations by Picklesimer and Rittenhouse [4] - [7], Cheadle et al [8], and Steward et al [9], where the influence was revealed of the basal pole orientation on the yield strength, strain ratio, and ductility in tensile and burst testing. An excellent study of how the texture of tubes is related to the preceding deformation and heat-treatment has been reported by Cheadle et al [10], who concluded that the basal poles have a tendency to align themselves parallel to the direction of maximum compressive strain. Hobson [11] and Tenckhoff et al [12] found the texture of Zircaloy tubes to be a function of the Q-value of the deformation, i.e. the ratio between the reduction of wall thickness and the reduction of diameter. However, these studies do not answer the important question of what degree of deformation is required to bring about a change in texture. Also, the use of Q-value has restricted the range of deformation to reduction in both wall and diameter. It is here felt that much would be gained by a more general approach, where the main features of texture and anisotropy are related to both the mode and degree of deformation.

Focusing on the main features of texture is very important, because these should of course be mapped out before it is worthwhile considering details. It is also most often only the texture in large that influences the measurable anisotropy of the material. The main features may be regarded as a first order effect and are a direct consequence of the basal pole orientation, whereas the rotation of crystals around the basal pole contributes to effects of higher order.

For texture and anisotropy studies, material in tubular form allows for one more degree of freedom in selecting the mode of deformation, because arbitrary changes in both diameter and wall thickness are possible. This is as opposed to wire drawing or strip rolling, which imply fixed modes of deformation. In this work the following limitation is used: The principal strain and stress directions during deformation are to be parallel with the principal directions of the tube, i.e. the radial, the tangential, and the axial direction, denoted r , θ , z . This parallelity makes all anisotropic properties, including the texture, reflexion symmetric with respect to the three principal planes of the tube, i.e. the three planes perpendicular to the radial, tangential and axial directions, respectively.

1.1 Texture of zirconium

The textures from wire drawing, compression of cylinders and strip rolling are characterized in a simple way in table II, both directly after deformation and after subsequent recrystallization, see Burgers et al [1], McGeary et al [2] and Dillamore et al [3]. Pole figures provide a more detailed description of textures, and since only the basal pole orientation is of interest here, only basal pole figures are relevant. In fig. 1 typical basal pole figures are given for a drawing texture with the drawing axis indexed 3, for a compression texture with compression axis 1, and for a rolling texture with rolling plane normal 1 and rolling direction 3. The rolling deformation process implies elongation and thinning with most often constant width. This might be regarded as the sum of a longitudinal drawing and a compression in the normal direction, so that the narrowing from the drawing is compensated by the broadening from the compression. In accordance, the rolling texture is, at least approximately, the sum or the average of a drawing texture and a perpendicular compression texture.

During recrystallization the crystals rotate 30° around $[0001]$, which means the basal pole texture is essentially unaffected, Dillamore et al [3]. Thus, the basal pole figures in figure 1 are valid for recrystallization textures as well as deformation textures, and the following discussions of texture and anisotropy are independent of whether recrystallization has occurred or not. The duplex appearance of the rolling texture in fig. 1 is characteristic for highly reduced zirconium strip. However, in order to consider only the main features of texture in the following discussions, the textures will be regarded as not so pronounced or more approximate, as shown in fig. 2.

2. SYMMETRY CONSIDERATIONS

2.1 Plastic deformation characterization

The precondition of parallelity between the principal strain directions and the principal tube directions is valid if an entire tube is looked upon

only before and after a deformation, provided there has been no torsion. During the deformation the tube is conical. This implies shear strains and loss of parallelity, but these strains are relatively small and disappear when the tube resumes its cylindrical shape. Also, the precondition of parallelity need not be fulfilled by the local deformation in a small element of the tube wall. In spite of these exceptions, it is justified to assume parallelity as generally prevailing, because the approximation made is normally insignificant, and because much is gained in clarity.

When a rectangular parallelepiped with the dimensions t_0 , w_0 , l_0 is deformed plastically with the principal strain directions parallel to the edges, it will stay a rectangular parallelepiped after the deformation but with the new dimensions t , w , l . Constant volume implies

$$\ln \frac{t}{t_0} + \ln \frac{w}{w_0} + \ln \frac{l}{l_0} = 0 \quad (1)$$

The edges, t , w , l are identified with the wall thickness, the circumference, and the length of a thin-walled tube, and the logarithmic strains ϵ_r , ϵ_θ , ϵ_z are introduced

$$\epsilon_r = \ln \frac{t}{t_0}, \quad \epsilon_\theta = \ln \frac{\pi d}{\pi d_0} = \ln \frac{d}{d_0}, \quad \epsilon_z = \ln \frac{l}{l_0} \quad (2)$$

Here d stands for the mean diameter. Substitution into (1) gives

$$\epsilon_r + \epsilon_\theta + \epsilon_z = 0 \quad (3)$$

This means a deformation is defined by only two independent variables, and a function of two variables can be illustrated in a plane diagram. To emphasize the equivalence of the principal directions a triaxial plane 120° diagram is chosen, and in the following referred to as the strain diagram, fig. 3.

Any change in the dimensions of a tube is represented by a vector in the strain diagram. The size and angle of the strain vector correspond directly to the degree and to the mode of deformation. The effective strain ϵ_e and the strain ratio α are introduced, see Hill [13] and Johnson et al [14].

$$\epsilon_e = \sqrt{\frac{2}{3} (\epsilon_r^2 + \epsilon_\theta^2 + \epsilon_z^2)} \quad (4)$$

$$\alpha = \arctan [(\epsilon_\theta - \epsilon_r)/\epsilon_z \sqrt{3}] \quad (5)$$

The strain ratio is expressed as an angle α counted clockwise from the positive ϵ_z -axis to the direction of the strain vector, fig. 3. Defining a ratio by an angle is made in order to underline the equivalence of all α -values from -180° to $+180^\circ$. The generality of the definition of α is illustrated by the fact that, as the earlier mentioned Q-value goes from 0 via 1 to ∞ , α goes from -30° via 0° to $+30^\circ$.

A strain vector can be translated parallelly in the strain diagram

without changing its significance, and vectors can be summed up to represent a series of deformation steps. Also, a deformation can be divided into a series of very small vectors, which together define a curve in the strain diagram. Such a curve will be referred to as a strain path. Strictly according to the theory of plasticity, the above formulas should be based on infinitesimal strain elements de_i , the effective strain should be an integral and a strain ratio should be valid only at one single stage during the deformation, see Hill [13] and Johnson et al [14]. However, for practical purposes the given finite formulas are justified if used with care. To further illustrate the use of the strain ratio α , the deformation of tubes with $\alpha = n \cdot 30^\circ$ is described in words in table III.

2.2 The texture rose

The drawing texture is typical of wire, which can only be deformed through elongation with rotational symmetric contraction. The corresponding deformation of tubes is elongation with equal wall thinning and diameter contraction, which gives the strain ratio $\alpha = 0^\circ$. The compression texture is a result of shortening of a cylinder with rotational symmetric expansion. One corresponding deformation of tubes is wall thinning with equal diameter expansion and elongation, which gives the strain ratio $\alpha = 60^\circ$. The two rotational symmetric pole figures from fig. 2 is accordingly inserted in a strain diagram at the direction of strain ratio 0 and 60° , fig. 4. The rolling texture is obtained from elongation with constant width. The corresponding deformation of tubes is elongation with constant diameter, which is the strain ratio $\alpha = 30^\circ$. The rolling texture pole figure from fig. 2 is inserted at the strain ratio 30° in fig. 4.

Rotational symmetric deformation is of course equivalent either the symmetry axis is radial, tangential or axial, as is also reflected by the treefold strain diagram. Thus, it is possible to introduce fibre textures at all the axes of the strain diagram both in their positive and negative directions, by permutating the texture axis of the pole figures, fig. 4. Like the rolling texture at $\alpha = 30^\circ$ is the average of the two neighbouring fibre textures, there must be average textures between all the fibre textures in fig. 4. Thus, the strain diagram is completed with twelve pole figures, but should in fact be regarded as representing a continuous ring of textures which are weighted averages of the surrounding fibre textures. The strain diagram completed with pole figures is referred to as the texture rose and expresses a general relationship between the mode of deformation of zirconium tubes and the resulting texture.

However, the relation between deformation and texture as given by the texture rose is expected to be valid only for stable textures, which requires either a texture-free starting material or such a high degree of deformation that stability is reached. To be able to handle not stable textures, the

strain diagram in the center of the rose is taken away and a new parameter introduced: The texture ratio is an angle, denoted η , counted clockwise from the rising vertical line through the center of the texture rose, to the position of any recorded texture in the rose. Thus for stable textures $\alpha = \eta$, while for not stable textures $\alpha \neq \eta$.

2.3 Yield locus and mechanical anisotropy

Yield loci are presented as projected on the deviator stress plane, containing the deviator stresses σ'_r , σ'_θ , σ'_z , see Hill [13] and Johnson et al [14]. This maintains the triaxial 120° symmetry also for the stress diagram, and makes the von Mises yield locus of an isotropic material circular. On considering the main features of anisotropy, the yield locus in its first order deviation from the circular form should be regarded as elliptical. A fibre texture must have a yield locus symmetric to the same axis as the texture. In fig. 5a a compression texture with radial axis is shown. Radial basal poles make the radial direction hard, and the corresponding yield locus consequently has its major axis along the σ'_r -axis. In fig. 5b a drawing texture with axial axis is shown. Here the axial direction is soft, because the basal poles are perpendicular to this direction, and the corresponding yield locus has its minor axis along the σ'_z -axis. The excentricity of the ellipse in fig. 5a has been made greater than that in fig. 5b in order to point out that basal poles parallel to a direction imply a greater anisotropy than do basal poles oriented perpendicular to a direction. As the rolling texture is an average of the drawing and compression textures, the rolling texture yield locus could be assumed to be an average of the drawing and compression texture yield loci, fig. 5c.

Thus, it is possible to replace every pole figure in the texture rose with a schematic yield locus, by applying a simple symmetry consideration, fig. 6. If the direction of the major axis of the yield locus is expressed with the angle β , as defined in fig. 6, the following formula relates the strain ratio α to the orientation of the yield locus

$$\alpha = 2(90 - \beta) \quad (6)$$

Again, this relation is valid only for stable textures and yield loci. To be able to handle not stable yield loci it is convenient to introduce the new parameter anisotropy ratio ν , where $\nu = 2(90 - \beta)$ is a mere function of the type of mechanical anisotropy. As a consequence of these definitions ν is expected to be equal to η for all textures, while $\nu = \alpha$ only for stable textures. These statements are to be verified in the experiments.

2.4 Texture softening and hardening

So far the plastic deformation has mainly been forced upon the material by the tools in a forming process. In mechanical testing, on the other hand, the material is loaded under a given stress ratio. Under the precondition of

parallelity between the principal stress directions and the principal tube directions, a state of stress in the deviator stress diagram is given by a point, or by a stress vector from the origin to that point. The size of the stress vector is proportional to the effective stress σ_e and its angle corresponds to the stress ratio γ , see Hill [13] and Johnson et al [14].

$$\sigma_e = \sqrt{\frac{1}{2} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} \quad (7)$$

$$\gamma = \arctan [(\sigma_2 - \sigma_1) \sqrt{3} / (2\sigma_3 - \sigma_1 - \sigma_2)] \quad (8)$$

Like the strain ratio α , the stress ratio γ is expressed as an angle counted clockwise from the positive σ'_2 -axis in the deviator stress diagram. Strain ratios of specific interest are $\gamma = 0^\circ$ in tensile testing, $\gamma = 90^\circ$ in closed-end burst testing, and $\gamma = 120^\circ$ in open-end burst testing.

On loading a tube with a given stress ratio until plastic deformation occurs, the strain ratio will be a function of both the stress ratio and the type of anisotropy. A rule is that the strain vector is perpendicular to the yield locus in the point where the yield locus is reached by the stress vector, if a combined strain-stress diagram is used, Johnson et al [14]. For an isotropic material the rule is quite natural, but it is also said by Hill [13] to be valid in an anisotropic material. The pure geometric relation between the angle γ of the radius to a point on an ellipse and the angle α of the normal to the periphery at the same point is displayed as a periodic curve in fig. 7. By changing the orientation β of the major axis of the ellipse the curve is translated parallel to the $\alpha = \gamma$ -line. The periodic curve of fig. 7 can be assumed to illustrate the relation between the stress and strain ratios during mechanical testing. The periodicity 180° implies that

$$\alpha = \gamma \quad \text{when} \quad \alpha = \beta + n \cdot 90 \quad (9)$$

where n stands for any positive or negative integer. The question here is if the strain during testing will bring about a change in texture and anisotropy or if the texture is stable for the strain. The relation between the strain ratio, the stable anisotropy ratio, and the orientation of major axis of yield locus, is as previously stated $\alpha = \nu = 2(90 - \beta)$, eq. 6. Replacing α in eq. 9 gives

$$\nu = \gamma \quad \text{when} \quad 2(90 - \beta) = \beta + n \cdot 90 \quad (10)$$

or after transformation

$$\nu = \gamma \quad \text{when} \quad \nu = n \cdot 60 \quad (11)$$

Thus the relation between the stable anisotropy ratio and the stress ratio must be as shown in fig. 8 which is similar to fig. 7, but with the smaller periodicity 120° .

If the anisotropy ratio is not stable for deformation under a given

stress ratio, the texture will change so as to bring the anisotropy ratio towards its stable value. In this process either a softening or a hardening due to the texture change is anticipated, of course, superimposed on the normal strain hardening. Texture softening is foreseen when the angle between the stress vector and the minor axis of the yield locus is greater at start, than when stable anisotropy is reached. Texture hardening, on the other hand, is foreseen when the angle between the stress vector and the major axis of the yield locus is greater at start, than when stable anisotropy is reached. To illustrate this, parallel dashed lines with the slope -2 are drawn in fig. 8, indicating those anisotropy ratios for which the major and the minor axes of the yield locus are parallel to the stress vector. Examples for $\gamma = 30^\circ$ and 90° are given in fig. 8, where the range for hardening (H) is defined by the two distances marked a being equal, and where the range for softening (S) is defined by the two distances marked b being equal. In this way it is possible to divide the entire anisotropy ratio - stress ratio diagram into regions alternately giving texture softening (S) and texture hardening (H), fig. 9. The region boundary curves are approximately straight lines with a slope of -5.

3. EXPERIMENTAL

3.1 Deformation experiments and mechanical testing

Tubes of Zircaloy-2 were deformed with six widely separate techniques, simply referred to as numbered I through VI. The ranges of strain ratios obtained with the six techniques are summarized in table IV. In each experiment specimens were taken from the starting material and from a series of stages during the deformation. The specimens were numbered in sequence as shown in table IV. The starting material was normally recrystallized, but in one case, specimen 40, it was stress relieved. A more detailed display of the result of the deformation experiments is given as strain paths in the strain diagram. For easier access, the triaxial strain diagram have been further developed by introducing logarithmic scales for wall thickness and mean diameter on the ϵ_r - and ϵ_θ -axes, respectively, see fig. 10 and 11, where the strain paths are obtained directly by plotting wall thickness versus mean diameter.

Tubes of Zircaloy-4 were selected for mechanical testing. The tubes were of three widely separate textures obtained through the deformation techniques I, III, and VI, in this case applied so that the final dimension of the tubes were the same, outer diameter 10.7 and wall thickness 0.7 mm. The tubes I, III, and VI were subjected to tensile testing, closed-end burst testing, and open-end burst testing. The tensile testing was made in a conventional electronic-mechanical machine, using a strain rate of 0.02 per minute. In burst testing the specimens were connected to a high pressure oil supply capable of giving a steady rate of volume increase inside the specimens, corresponding to the same strain rate. The pressure was recorded with an

electronic pressure gauge. During testing the length and diameter of the specimens were continuously recorded. For the diameter measurement a special extensometer was constructed based on three differential-transformer strain gauges separated 120° but in perpendicular contact with the outer periphery of the tube specimen. The strain recordings had to be interrupted when necking or growth of a bubble started. To follow the strain all the way to fracture, a square photogrid was applied on the specimens. Measuring the squares before and after testing defined the effective strain and the strain ratio in each point of the specimen after failure. The strain in the uniform part of the broken specimens was in acceptable agreement with the extensometer recordings.

3.2 Texture characterization

The textures of the tube specimens were determined by X-ray diffraction, using the inverse pole figure technique. X-ray diffraction specimens were prepared with chemically polished surfaces perpendicular to the radial, tangential, and axial directions, composite specimens being necessary for the latter two. The recordings were made with CoK α radiation in a rotating specimens holder by scanning from 2θ 36° to 90° to cover reflections from ten crystallographic planes, see table in Appendix. The texture coefficient T_q for the q:th reflection was calculated with the formula

$$T_q = 10 \cdot \frac{I_q/I_q^0}{\sum_{i=1}^{10} (I_i/I_i^0)} \quad (12)$$

where I_q and I_i are experimentally determined, integrated intensities evaluated as the area under the respective peak, and where I_q^0 and I_i^0 are intensities calculated for a texture-free specimen, see table in Appendix, and also Klug et al [15]. The calculated intensities were checked to agree acceptably with recordings from a randomized specimen of filings. The procedure and formula used are in accordance with Harris [16].

Texture coefficients of all 30 reflections from each triple specimen are normally presented in three 30° sectors of a stereographic projection. However, such a lot of data would here be difficult to survey, and would also make a temptation to consider details of the textures. Instead the data are to be condensed into two single representative parameters.

The texture of any polycrystalline material is completely described by a distribution function $w(\varphi, \omega, \psi)$, where the angles φ , ω , ψ define the orientation of a single grain, fig. 12. The function implies that $w(\varphi_1, \omega_1, \psi_1) \sin\varphi_1 d\varphi_1 d\omega_1 d\psi_1$ is the volume fraction of grains with crystal orientation within a small angular range ($d\varphi$, $d\omega$, $d\psi$) around the orientation $(\varphi_1, \omega_1, \psi_1)$. A function of this type may be expanded into a series of generalized spherical harmonics

$$w(\varphi, \omega, \psi) = \sum_{l=0}^{\infty} \sum_{m=-l}^l \sum_{n=-l}^l W_{lmn} Z_{lmn}(\cos\varphi) e^{-jm\omega} e^{-jn\psi} \quad (13)$$

Here W_{lmn} are the coefficients of the terms in the series, and $Z_{lmn}(\cos\varphi)$ are generalized Legendre functions. The significance and formalism for the use of spherical harmonics in describing texture have been developed by Roe [17], from whom some of the present notations are borrowed, and have recently been thoroughly worked over by Bunge [18].

Reflexion symmetry of texture with respect to the three principal tube planes is, as previously stated, a consequence of the parallelity between principal strain directions and the principal tube directions. Also, inverse pole figures from only the three principal directions must be interpreted assuming such a reflexion symmetry. Together with the inherent symmetry of the hcp crystal, this reduces the number of W_{lmn} that are independent and non-zero to the lmn combinations, 000, 200, 220, 400, 420, 440, etc.

The coefficients W_{lmn} for the first few terms in the series are of interest because they describe the main feature of the texture, W_{000} corresponds to the average distribution. Since the summation of all volume fractions must be 1, the W_{000} has a constant value for both textured and texture-free materials. The next coefficients W_{200} and W_{220} express the main feature of the deviation from randomness or isotropy. The W_{lmn} of higher order describe the deviation from isotropy more in detail. A formal evidence that only the basal pole orientation is included in the first order effect, is that n is zero in W_{200} and W_{220} , i.e. the rotation around [0001] as expressed by the angle ψ is irrelevant.

The coefficients W_{lmn} can be expressed explicitly as integrals of the distribution function w . After disregarding some normalizing constants and eliminating the angle ψ , the hence modified W'_{200} and W'_{220} are

$$W'_{200} = \int_0^{\pi} \int_0^{\pi} w(\varphi, \omega) (3\cos^2\varphi - 1) \sin\varphi d\varphi d\omega \quad (14)$$

$$W'_{220} = \sqrt{3} \int_0^{\pi} \int_0^{\pi} w(\varphi, \omega) \sin^3\varphi \cos 2\omega d\varphi d\omega \quad (15)$$

These formulas cannot be used for the experimental determination of W_{200} and W_{220} because $w(\varphi, \omega)$ is not known. However, the formulas are applicable in illustrating the significance of W_{200} and W_{220} . This is made in table V by considering some special cases of basal pole distribution, for which $w(\varphi, \omega)$ may be realized directly.

For graphical representation of textures a Cartesian co-ordinate system of W'_{200} and W'_{220} is chosen with the axes oriented as in fig. 13. On plotting the six very extreme textures, these outline an equilateral triangle, fig. 13. The three more moderate textures are plotted as schematic pole figures in

relation to the equilateral triangle in fig. 14. They fall on the vertical line through the center of the triangle. The center implies randomness, which is marked out with a pole figure indicating fully even distribution. For reasons of symmetry the triangle is completed with another six moderate fibre textures, which makes fig. 14 very similar to the texture rose. Like the texture rose, it contains information on the type of texture, but in addition also on the sharpness of texture. The position of a plotted texture can be defined by a texture vector $\bar{W}$, fig. 13. The angle of this vector is identified as the texture ratio η and the magnitude is called the texture sharpness W

$$\eta = \arctan (-W'_{220}/W'_{200}) \quad (16)$$

$$W = \sqrt{W'_{200}{}^2 + W'_{220}{}^2} \quad (17)$$

On disregarding some proportional constants but keeping the factor $\sqrt{3}$ in the above expression for W'_{200} and W'_{220} , the intentions were to obtain the triangular equilaterality, which emphasizes the equivalence of the three principal directions, and to have the maximum texture sharpness equal to two, $W = 2$, for the following reasons: Textures on the middle of the triangle sides have the basal pole distributed perpendicularly to one direction. This implies the distribution has lost one degree of freedom and hence the sharpness should have the value one. Textures in the triangle corners have all basal poles parallel to one direction. This implies two degrees of freedom are lost, and the sharpness should have the value two.

The formalism for evaluation of W'_{200} and W'_{220} from the total 30 texture coefficients determined for each triple specimen is presented in the Appendix. The final result is two explicit expressions suitable for routine computing. Repeated texture determinations from a series of specimens taken from the same tube, indicate that the total error from specimen preparation, X-ray diffraction, and evaluation is ± 0.04 in W'_{200} and W'_{220} . When represented graphically in relation to the equilateral triangle, a texture is plotted as a circle with the radius 0.04, to indicate the range within which the true texture probably falls. The error in the texture sharpness is also ± 0.04 , while the error in the texture ratio is smaller the greater the texture sharpness, as is realized from the graphical representation.

3.3 Mechanical anisotropy characterization

The Wheeler and Ireland [19] technique for determination of the Knoop-hardness yield locus was used as a rapid method of defining the main features of mechanical anisotropy. The technique might lack a solid theoretical basis, but have given yield loci in good agreement with conventional ones, see Lee et al [20]. The Knoop hardness is measured with six separate orientations of the indenter, and the hardness values are plotted in a deviator stress diagram for six separate stress ratios. The yield locus may be fitted by

hand to the six points, but here computer fitting of an ellipse was made standard procedure. The general equation of an ellipse contains only five constants, which motivated a least square method to utilize all six experimental points. The elliptical yield locus is characterized by the orientation β of the major axis, by the excentricity, and by the geometric mean of half the major and minor axes. Based on these parameters are defined the anisotropy ratio ν as equal to $2(90-\beta)$, the anisotropy degree A as equal to the excentricity, and the mean hardness H as equal to the geometric mean.

The Knoop measurements were made with 500 g load on chemically polished surfaces. The maximum error in the excentricity, i.e. the anisotropy degree, is estimated to ± 0.04 . The error in the anisotropy ratio $\Delta\nu$ is smaller the greater the anisotropy degree, $\Delta\nu = \pm 10, 6, 3, 2^\circ$ for $A = 0.5, 0.6, 0.7, 0.8$, respectively.

4. RESULTS AND DISCUSSION

4.1 Texture and mechanical anisotropy

It was predicted from symmetry consideration that the anisotropy ratio ν equals the texture ratio η . To check this prediction, $(\eta-\nu)$ is plotted as a function of η in fig. 15 for all specimens from the deformation experiments. Since the strain history should not affect this relation, results from a number of odd specimens from both tubular and strip material are included to strengthen the evidence. Values less accurate due to low W and A are plotted square-formed, while values with maximum error $\Delta\eta = \pm 5^\circ$ and $\Delta(\eta-\nu) = \pm 11^\circ$ are plotted circular. A deviation from the prediction $(\eta-\nu) = 0$ outside the error limits is apparent. There are very strong reasons that $\nu = \eta$ for $\eta = n \cdot 60^\circ$, i.e. for rotational symmetric textures. However, in between a periodic deviation seems reasonable, because of the two separate ways of defining and determining the η and ν . Least square fitting of a sinus function yields

$$\nu = \eta + 6\sin 3\eta \quad (18)$$

Most plotted values of $(\eta-\nu)$ are within the error limits from this relation. Those few values, plotted as circles in fig. 15, not within the error limits must be regarded as representing textures of a more complex type, e.g. originating from inhomogeneous deformation.

The anisotropy degree A is throughout smaller in strain hardened material than in recrystallized. This might be interpreted as if strain hardening counteracts slip deformation more than twinning. Among those specimens investigated both before and after recrystallization, seven had a texture sharpness change less than 0.1. Based on these seven specimens the following relation for the hardness sensitivity of the anisotropy degree is derived

$$\frac{1}{A} \cdot \frac{\partial A}{\partial H} = -(3.5 \pm 1.4) \cdot 10^{-3} \quad (19)$$

All values of A determined were corrected to A', as corresponding to the soft annealed mean hardness 185 KHN. The corrected values of A' are plotted in fig. 16 as function of W. In a texture-free isotropic material W = 0 and A' = 0, while for a single crystal with W = 2 A' can be assumed to be approximately 0.9. One equation fulfilling these boundary conditions is A' = 0.9(W/2)^a. Using the experimental data, the exponent a = 0.2 has been determined by a least square method. Thus, the following equation describes the relation between corrected anisotropy ratio and texture sharpness

$$A' = 0.9(W/2)^{0.2} \quad (20)$$

4.2 Strain path and texture change

The textures of specimens taken from the deformation experiments are represented in equilateral triangles, truncated to save space, fig. 17. Within each series the textures are joined by a curve, which constitutes the texture-change path. These paths which define the size and direction of the texture change, are to be correlated with the corresponding strain paths in fig. 10 and 11.

For the series 30-32, 40-44, 50-55, 60-68, and 70-72 the texture-change path and the strain path are approximately parallel. For series 10-15 and 80-85, there is no such parallelity, but the texture-change path is parallel to the upper side of the triangle. The reason for this is that the starting textures of specimen 10 and 80 already are so close to the upper side that there is no room for texture change parallel to the strain path. This is because the sides of the equilateral triangle are defined as impossible to surpass, and because it ought to be difficult to reach the more sharp texture, implied by an approach towards the side. The texture change instead occurs in the direction of the projection of the strain on the triangle side. The series 40-44 and 50-55, which differ only as regards the heat-treatment of the starting material, have parallel texture-change paths. A general rule for the texture-change path applicable for any starting texture is suggested:

A texture-change path is parallel to its corresponding strain path, until one of the triangle sides is approached, where the texture-change path deviates to become parallel to the projection of the strain path on the triangle side.

A texture is stable if the texture ratio has not changed during the last stage of deformation. However, in very few cases such constancy is completely obtained, and so, on judging the possible stability of a texture, considerations must be given to the degree of deformation, in some cases to the length of the projection of the strain path, and to the texture sharpness, this last because a certain texture change might alter the texture

ratio much more at low texture sharpness. Thus, stable textures should have been reached by specimen 15, 65-68, and 85. Specimens from some more tubes, deformed by technique I with strain ratio 25-27°, were also considered to have reached stable texture. The relation between these stable texture ratios η and the strain ratio α is displayed in fig. 18, where $(\eta - \alpha)$ is plotted as a function of α . For most textures $(\eta - \alpha) = 0$, and the only texture somewhat off is the one for specimen 15. Maybe number 15 was not truly stable after all. Together with the very strong symmetry reasons that $(\eta - \alpha) = 0$ for $\alpha = n.60$, figure 18 is the proof that η equals α for stable textures, thus making the texture rose in fig. 4 verified quantitatively.

Replacing η with α in eq. 18 gives the relation between stable anisotropy ratio and the strain ratio, $v = \alpha + 6\sin 3\alpha$. This is in fact a verification of the predicted relation illustrated in fig. 6. The sinus function implies only a minor modification: The yield locus for $\alpha = 30^\circ$ has its major axis angle $\beta = 72^\circ$ instead of 75° as was predicted. All yield loci for $\alpha = 30^\circ + n.60^\circ$ should be modified accordingly, while the symmetric yield loci for $\alpha = n.60^\circ$ are still true.

The rule for the texture-change path and the relation $\eta = \alpha$ make it possible to construct a number of texture-change paths from a starting texture (O) to stable textures (S), under the effect of any strain ratio. In a texture-free starting material these paths go straight from the center of the triangle towards the sides with η all the time equal to α . In an originally textured material some texture-change paths deviate as they approach the triangle sides, fig. 19. For some strain ratios the question arises whether the deviation occurs towards stable texture or in the direction of the projection of the strain path. Specimen 20-21 indicates the latter, fig. 17, because the texture is changed towards the right corner of the triangle. However, a definite proof cannot be extracted from the present data. The question is of great importance. If deviation occurs towards stable texture, only the prevailing strain ratio influences the resulting texture. If the deviation occurs in the direction of the projection, the starting texture may also influence the resulting texture. This would imply the material might remain an eternal memory of its previous strain history, that is an inheritance effect. An implication of this would be that the texture rose after all is not valid under all circumstances.

The parameter texture-change rate is introduced to express the texture change occurring per unit effective strain along the texture-change path, until the texture becomes stable also with respect to sharpness. If the distance between two subsequent textures in the equilateral triangle is ΔW , i.e. $|\Delta W|$, and if the effective strain causing the change is $\Delta \epsilon_e$, the texture-change rate is defined by $\Delta W / \Delta \epsilon_e$. The heavy deformation of series 10-15 gave a small texture change compared to most other series, which means the texture-change rate varies. It seems probable that the projection of the

strain path on the texture-change path is one of the factors governing the rate. Under otherwise constant conditions, $\Delta W/\Delta \epsilon_e$ should be proportional to $\cos(\alpha - \Delta \eta)$, where $\Delta \eta$ stands for the direction of the straight line between the two textures in their graphical representation, i.e. the argument of $\overline{\Delta W}$. Calculated values of $\Delta W/(\Delta \epsilon_e \cos(\alpha - \Delta \eta))$ are compiled in table VI for most of the deformation steps. The values range from 0.8 to 2.5, except for a higher value for step 40-42, and lower values for step 14-15 and 84-85. No reason for the higher value is evident, while the lower values are for texture changes close to stability. No other systematic variation in $\Delta W/(\Delta \epsilon_e \cos(\alpha - \Delta \eta))$ is apparent, and hence the conclusion is that this parameter has a constant value, except close to textures stable also with respect to sharpness. Average and standard deviation of the values between 0.8 and 2.8 are 1.7 ± 0.6 . Thus, the texture-change rate is given by

$$\Delta W/\Delta \epsilon_e = 1.7 \cos(\alpha - \Delta \eta) \quad (21)$$

This ultimate formula certainly has a low accuracy, since experimental errors have been accumulated, but it still is of importance in prescribing production routes to reach specific textures.

4.3 Yield locus and texture softening

The texture and mechanical anisotropy of the three mechanically tested tube materials I, III, and VI, are given in table VII. The true stress-effective strain curves recorded are shown in fig. 20, marked out with $\gamma = 0^\circ$ for tensile testing, $\gamma = 90^\circ$ for closed-end burst, and $\gamma = 120^\circ$ for open-end burst testing. The strain paths followed to fracture are illustrated in fig. 21. However, the maximum effective strain reached must not be used in comparing the separate textures, because especially the tube material III had variable dimensional tolerances, owing to the experimental state of the deformation technique. The strain paths have a tendency to bend towards $\alpha = 60^\circ$, because the stress ratio in a necking or bubble, respectively, changes towards $\gamma = 60^\circ$.

The mechanical testing offers an opportunity to check on the elliptical Knoop-hardness yield loci, fig. 22. The mechanical testing gives three points on the true yield locus both as regards the distance from the origin and the orientation of the periphery. Best agreement was obtained using the stress at $\epsilon_e = 0.012$ and the conversion factor $3.9 \text{ KHN}/(\text{kg}/\text{mm}^2)$, fig. 22. Of course, a yield locus at the strain $\epsilon_e = 0.012$ should not be characterized as virtually true, because this strain is enough to bring about a texture change as will soon be discussed.

Some of the stress-strain curves in fig. 21 display something like a yield-point phenomenon. This must be caused by the anisotropy, since it appears more or less pronounced dependent on the combination of tube material and testing method. Evidently the deformation brings about a texture

and anisotropy change, which alters the flow stress required. The stress increase between $\epsilon_e = 0.004$ and 0.1 varies from 10.6 to 21.2 kg/mm², mainly because of strain hardening, table VII. The greatest increase 21.2 is in tensile testing. Independent of the predicted regions for softening or hardening, tensile testing with $\gamma = 0^\circ$ can only result in texture softening, since the stable yield locus has its minor axis parallel to the stress vector. Thus pure strain hardening should increase σ_e somewhat more than 21.2. A reasonable assumption is 23 ± 2 kg/mm². Thus, the texture softening corresponds to the difference between 23 and the increases in σ_e given in table VII.

The anisotropy ratios of the materials tested are plotted in fig. 9 as function of the stress ratios applied. A number indicating the texture softening is given inside each plotted point. All points fall within predicted regions of texture softening. The softening is also in general greater the farther the plotted point is from the stable $\nu - \gamma$ relation. These are evidences in agreement with the predicted behaviour, even if they do not constitute the definite proof. Texture hardening apparently is not so common, since it requires unusual testing methods or extreme textures.

The texture softening occurs at the very beginning of the testing, and is more or less completed before $\epsilon_e = 0.1$. This is very rapid. With a texture-change rate as given by eq. 21, a strain of 0.1 should not change the properties noteworthy. The main difference is the free deformation under a given stress ratio in mechanical testing as compared to the forced-on strain in the deformation experiments. An explanation for the observed behaviour might be, that rapid texture change is obtained through abundant twinning, if the shear strain of the twins not necessarily must conform with a forced-on deformation.

5. CONCLUSIONS

- a. Experiments on Zircaloy tubes have verified the predicted texture rose as describing the relation between the type of stable texture and mode of plastic deformation. In addition, the texture-change path has been revealed and the texture-change rate evaluated as functions of plastic deformation.
- b. An important but unanswered question is whether the starting texture might influence the final texture and impose an inheritance effect. Thus, under certain circumstances the texture rose is perhaps not completely valid.
- c. The predicted relation between type of texture and type of mechanical anisotropy, as manifested by the Knoop-hardness yield locus, have been examined and found to be valid with a minor modification. In addition, the anisotropy degree has been found to correlate with the texture sharpness.

- d. As a consequence of conclusions above, the predicted relation between type of mechanical anisotropy and the fore-going plastic deformation has been verified.
- e. Mechanical testing of tubes of separate textures has resulted in texture softening, in agreement with the predicted behaviour of zirconium material. Although not a proof, this supports the visualized regions, alternatingly predicting softening and hardening.

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APPENDIX

Evaluation of W'_{200} and W'_{220} from inverse-pole-figure texture data

The angle between the normal to the crystallographic plane q and the basal pole is denoted ϕ_q , and compiled in table VIII for the hcp zirconium crystal with $c/a = 1.593$. Grains with the plane q normal oriented parallelly to a principal direction, indexed p , have possible basal pole orientations describing a circular line L_{pq} on the orientation sphere.

L_{zq} is defined by $\phi = \phi_q$ while $\omega = 0 \rightarrow \pi/2$, fig. 23a

L_{rq} is defined by $\phi' = \phi_q$ while $\omega' = 0 \rightarrow \pi/2$, where from spherical trigonometry applied on the triangle ABC in fig. 23b, $\cos\phi = \sin\phi' \sin\omega'$ and $\cos\omega = \cos\phi' / \sin\phi$.

$L_{\theta q}$ is defined by $\phi' = \phi_q$ while $\omega' = 0 \rightarrow \pi/2$, where from spherical trigonometry applied on the triangle DEF in fig. 23c, $\cos\phi = \sin\phi' \cos\omega'$ and $\sin\omega = \cos\phi' / \sin\phi$.

Integration of the function w along the line L_{pq} and dividing through with the same integral of w_0 for texture-free material corresponds to the texture coefficient T_{pq} from inverse pole figure recordings of a surface perpendicular to the direction p .

$$T_{pq} = \frac{\int_{L_{pq}} w dL}{\int_{L_{pq}} w_0 dL} \quad (22)$$

Series expansion yields

$$T_{pq} = 1 + \sum_{l=2}^{\infty} \sum_{m=-l}^l \sum_{n=-l}^l W_{lmn} K_{lmn}^{pq} \quad (23)$$

After disregarding some proportional constants, the same for both K_{200}^{pq} and K_{220}^{pq}

$$K_{200}^{pq} = \int_{L_{pq}} (3\cos^2\varphi - 1) dL / \int_{L_{pq}} dL \quad (24)$$

$$K_{220}^{pq} = \int_{L_{pq}} \sin^2\varphi \cos 2\omega dL / \int_{L_{pq}} dL \quad (25)$$

By replacing the two variables φ and ω with the constant φ_q and the variable ω' and by integrating for $dL = d\omega'$, the expressions are condensed to

$$K_{200}^{rq'} = -k_q/2 \quad K_{200}^{\theta q'} = -k_q/2 \quad K_{200}^{z\sigma} = k_q \quad (26)$$

$$K_{220}^{rq'} = k_q/2 \quad K_{220}^{\theta q'} = -k_q/2 \quad K_{220}^{zq'} = 0 \quad (27)$$

where $k_q = 3\cos^2\varphi_q - 1$ is given in table VIII. To obtain the desired W'_{200} and W'_{220} , a least square procedure is applied in which the following summation is minimized.

$$\sum_p \sum_q [I_q^0 (T_{pq} - 1 - W_{200}^{pq} K_{200}^{pq} - W_{220}^{pq} K_{220}^{pq})^2] \quad (28)$$

Here I_q^0 , the X-ray intensity for a texture-free specimen, is introduced as a weighting factor, table VIII. The statistical error in an X-ray recording is proportional to $\sqrt{I_q^0}$, and hence I_q^0 is entered in the summation to give the more accurate T_{pq} more weight. The final expressions for W'_{200} and W'_{220} used routinely are

$$W'_{200} = \frac{1}{39} \sum_q [I_q^0 k_q (2T_{zq} - T_{rq} - T_{\theta q})] \quad (29)$$

$$W'_{220} = \frac{\sqrt{3}}{39} \sum_q [I_q^0 k_q (T_{rq} - T_{\theta q})] \quad (30)$$

The factors $1/39$ and $\sqrt{3}/39$ are introduced to give $W'_{200} = -1$ and $W'_{220} = \sqrt{3}$ for a single crystal with radial basal pole. The figure 39 is a somewhat odd assumption on the following basis: The greatest texture coefficient ever recorded among hundreds of specimens was 12.6 for $\{11\bar{2}0\}$. Probably this specimen surface was almost 100 percent $\{11\bar{2}0\}$, and a single crystal can be expected to give the texture coefficient 13, which requires the factors $1/39$ and $\sqrt{3}/39$. It should be emphasized that this estimation is dependent on the divergence of the X-ray beam, i.e. characteristic only for the specific diffraction set-up used. This uncertainty does not influence the texture ratio derived, but only the texture sharpness values with a constant factor.

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Table I. Alloying elements in Zircaloy-2 and Zircaloy-4, weight percent.

	Sn	Fe	Cr	Ni	O
Zircaloy-2	1.5	0.13	0.10	0.05	0.12
Zircaloy-4	1.5	0.20	0.10	-	0.12

Table II. Textures in zirconium.

	Drawing texture	Compression texture	Rolling texture
Deformation texture	$\langle 10\bar{1}0 \rangle$	$[0001]$	$(0001)\langle 10\bar{1}0 \rangle$
Recrystallization texture	$\langle 11\bar{2}0 \rangle$	$[0001]$	$(0001)\langle 11\bar{2}0 \rangle$

Table III. Types of deformation corresponding to the specific strain ratios $\alpha = n \cdot 30^\circ$

Strain ratio	Deformation of tubes as defined by changes in wall thickness, diameter, and length
$\alpha = 0^\circ$	Elongation with equal wall thinning and diameter contraction
$\alpha = 30^\circ$	Elongation with constant diameter
$\alpha = 60^\circ$	Wall thinning with equal expansion and elongation
$\alpha = 90^\circ$	Expansion with constant length
$\alpha = 120^\circ$	Expansion with equal shortening and wall thinning
$\alpha = 150^\circ$	Expansion with constant wall thickness
$\alpha = 180^\circ$	Shortening with equal wall thickening and expansion
$\alpha = -150^\circ$	Shortening with constant diameter
$\alpha = -120^\circ$	Wall thickening with equal diameter contraction and shortening
$\alpha = -90^\circ$	Diameter contraction with constant length
$\alpha = -60^\circ$	Diameter contraction with equal elongation and wall thickening
$\alpha = -30^\circ$	Elongation with constant wall thickness

Table IV. Deformation experiments with Zircaloy tubes.

Deformation technique	I	II	III	IV	V	VI
Range of strain ratio	20-27°	115-137°	120-145°	-145° - -139°	-84 - -56°	-35 - -31°
Specimen series no.	10-15	20-22	30-32 40-44 50-55	60-68	70-72	80-85

Table V. The coefficients W'_{200} and W'_{220} for some special basal pole distributions.

Basal pole orientation		W'_{200}	W'_{220}
All basal poles exactly parallel to the	radial direction	-1	$\sqrt{3}$
	tangential direction	-1	$-\sqrt{3}$
	axial direction	2	0
All basal poles evenly distributed but exactly perpendicular to the	radial direction	1/2	$-\sqrt{3}/2$
	tangential direction	1/2	$\sqrt{3}/2$
	axial direction	-1	0
All basal poles evenly distributed within limits defined by the angle φ from the axial direction	$60^\circ < \varphi < 120^\circ$	-0.75	0
	$0 < \varphi < 60^\circ, 120^\circ < \varphi < 180^\circ$	0.75	0
	$0 < \varphi < 30^\circ, 150^\circ < \varphi < 180^\circ$	1.37	0

Table VI. Evaluation of texture-change rate between specimens from deformation experiments.

Step between specimen no.	Deformation		Texture change		$\frac{\Delta W}{\Delta \epsilon_e \cos(\alpha - \Delta \eta)}$
	$\alpha(^{\circ})$	$\Delta \epsilon_e$	$\Delta \eta(^{\circ})$	ΔW	
12-13	23	0.42	96	0.10	0.8
13-14	24	0.28	87	0.15	1.2
14-15	27	0.61	87	0.14	0.5
20-21	115	0.28	66	0.46	2.5
30-31	144	0.22	101	0.23	1.4
31-32	145	0.13	165	0.20	1.6
40-43	129	0.16	124	0.81	5.1
43-44	120	0.33	150	0.28	1.0
50-53	133	0.15	130	0.24	1.6
53-55	122	0.33	142	0.60	1.9
60-61	-145	0.12	-175	0.21	2.0
61-62	-144	0.11	-109	0.19	2.1
62-65	-144	0.25	-143	0.42	1.7
65-68	-140	0.20	-123	0.22	1.2
70-71	-56	0.25	-92	0.51	2.5
71-72	-84	0.27	-70	0.41	1.6
80-83	-35	0.23	-73	0.28	1.6
83-84	-33	0.23	-104	0.21	2.8
84-85	-31	0.50	-69	0.16	0.4

Table VII. Characterization of tube materials mechanically tested. Deformation hardening reduced by texture softening between 0.4 and 10 percent strain.

Tube	Texture		Mechanical anisotropy		Mean hardness (KHN)	Increase in σ_e between ϵ_e 0.004 and 0.1 (kg/mm ²)		
	$\eta(^{\circ})$	W	$\nu(^{\circ})$	A		$\gamma=0^{\circ}$	$\gamma=90^{\circ}$	$\gamma=120^{\circ}$
I	28	0.54	48	0.80	169	21.2	14.7	10.6
III	76	0.48	82	0.63	177	12.9	19.2*	19.2
VI	-23	0.56	-34	0.63	180	18.2	12.1	11.8

*extrapolated

Table VIII. Planes indexed q or i in hcp zirconium, X-ray intensities I° of texture-free specimen, angles ϕ_q between plane normal and basal pole, and $k_q = 3\cos^2\phi_q - 1$.

Plane	Index q or i	I_q° or I_i°	ϕ_q	k_q
10 $\bar{1}$ 0	1	0.91	90 $^{\circ}$	-1
0002	2	0.91	0	2
10 $\bar{1}$ 1	3	3.51	61.47 $^{\circ}$	-0.316
10 $\bar{1}$ 2	4	0.53	42.60 $^{\circ}$	0.626
11 $\bar{2}$ 0	5	0.57	90 $^{\circ}$	-1
10 $\bar{1}$ 3	6	0.62	31.52 $^{\circ}$	1.180
20 $\bar{2}$ 0	7	0.09	90 $^{\circ}$	-1
11 $\bar{2}$ 2	8	0.67	57.88 $^{\circ}$	-0.152
20 $\bar{2}$ 1	9	0.47	74.79 $^{\circ}$	-0.793
0004	10	0.09	0	2

Figure 1. Typical basal pole figures of drawing texture, compression texture, and rolling texture.

Figure 2. Basal pole figures of less pronounced or more approximate textures.

Figure 3. Plane triaxial strain diagram with strain vector, defined either by its strain components or by the effective strain ϵ_e and the strain ratio α .

Figure 4. The texture rose relating the strain ratio to the resulting stable texture.

Figure 5. Yield loci corresponding to compression, drawing, and rolling textures.

Figure 6. Relationship between the strain ratio and the resulting stable anisotropy as displayed by yield loci.

Figure 7. Relationship between the radius and the normal in a point on an ellipse.

Figure 8. Periodic relationship between the stress ratio and the stable anisotropy ratio. Principle of construction of softening and hardening regions.

Figure 9. Anisotropy ratio - stress ratio diagram divided into regions alternatingly implying texture softening (S) and hardening (H).

Figure 10. Double-logarithmic strain diagram with strain paths obtained with deformation techniques I, II, and III.

Figure 11. Strain paths obtained with deformation techniques IV, V, and VI.

Figure 12. The orientation of a crystal in relation to the principal directions of a tube.

Figure 13. Graphical representation of textures in a Cartesian and a polar co-ordinate system. Equilateral triangle outlined by extreme textures.

Figure 14. Positions of fibre textures in relation to the equilateral triangle.

Figure 15. The relation between anisotropy ratio and texture ratio illustrated by $(\eta - \nu)$ as a function of η . The error limits $\pm 11^\circ$ are indicated by dashed curves.

Figure 16. The relation between corrected anisotropy degree and texture sharpness.

Figure 17. Texture-change paths obtained in deformation experiments.

Figure 18. The relation between stable texture ratio and strain ratio illustrated by $(\eta-\alpha)$ as a function of α .

Figure 19. Constructed texture-change paths from starting texture (O) to stable textures (S) under the influence of separate strain ratios.

Figure 20. True stress-effective plastic strain curves for tensile, closed-end burst and open-end burst testing of tube material I, III, and VI.

Figure 21. Strain paths followed during mechanical testing.

Figure 22. Elliptical Knoop-hardness yield loci together with parts of the true yield loci.

Figure 23. Circular line L_{pq} defining orientations contributing to the texture coefficient T_{pq} .

DISCUSSION

W. G. NILSON, U. S. A.

C

This paper has made a contribution to zircaloy clad fuel rod design in clarifying the interface between deformation anisotropy and texture. The "texture rose" provides the basis for a texture catalog in the sense that designers may balance the economics of the cost of obtaining a particular texture against what advantages that texture may have in overcoming various life-limiting factors. In particular, observations reported by others of axial cladding cracks over radial pellet cracks suggest a possible advantage in using material with greater circumferential ductility. While this may be sought classically through annealing, it can alternatively be obtained and additionally enhanced through texture softening. While current zircaloy tubing for cladding typically possesses a strong radial basal pole texture with a component in the tangential (circumferential) direction, such texture softening could be achieved with material produced to have the non-radial component either eliminated, or perhaps more practically, rotated towards the axial direction. This latter alternative would especially contribute towards reducing axial stress-free irradiation growth while not affecting hydride circumferentiality.

J. H. GITTUS, U. K.

Q

Can you calculate the anisotropy coefficients from the texture ?

K. KÄLLSTRÖM, Sweden

A

Yes, it is possible to calculate approximate values. In my work there is an experimental correlation between the type of anisotropy and the type of texture, and also an experimental correlation between the degree of anisotropy and the sharpness of texture. The type and degree of anisotropy is represented by the major axis orientation and the eccentricity of the elliptical approximation of the yield locus. The strain ratio on mechanically testing the material is defined by the incremental strain vector being perpendicular to the yield locus, i. e., there is a unique correspondence between the anisotropy coefficients P and R and the orientation and eccentricity of the elliptical yield locus.