FRACTURE OF CYLINDRICAL AND SPHERICAL SHELLS CONTAINING A CRACK

F. ERDOGAN, M. RATWANI,

*Department of Mechanical Engineering and Mechanics,
Lehigh University, Bethlehem, Pennsylvania, U.S.A.*

Within the past decade the so-called linear fracture mechanics has established itself as a very satisfactory theoretical working tool in studying the phenomena of fatigue crack propagation and brittle fracture in structural solids. The fracture theories which are based on the use of the stress intensity factor or, its equivalent, the crack extension force have been most effective in cases for which generally the size of the plastically deformed region around the fracture front is small compared to the length parameter characterizing the fracture area. However, in the presence of moderately large plastic deformations around the crack front these fracture models need some re-interpretation and modification. Generally, such a modification is accomplished either by the notion of "plasticity correction" or by introducing the concept of "crack opening displacement". The former requires the evaluation of the plastic zone size ahead of the crack periphery, and in the latter the crack surface displacement in the plastically deformed medium has to be calculated.

In the application of these theories to shell structures such as pressure vessels, pipes, fuselage, rocket motor casings, etc., one encounters two types of problems. The first is analytical and concerns the evaluation of the critical variables, namely the stress intensity factor, the plastic zone size, and the crack opening displacement. The second refers to the proper modification of the fracture models to take into account the effect of the combined stress state which, because of the coupled nature of the differential equations for shells, always occurs around the crack front even if the external loads are membrane loads having a certain symmetry with respect to the plane of the crack.

In this paper we will first present the analytical results for a cracked spherical shell, a cylindrical shell with an axial crack and a cylindrical shell with a circumferential crack. The results will be given for both symmetric and anti-symmetric loading conditions. We will then introduce the plastic strip model to calculate the plastic zone size and the crack opening displacement.

The objective of the analytical work presented in this paper is to provide the theoretical information which can be used in predicting the fatigue crack propagation and fracture behavior of shells from that of flat plates, which may easily be obtained by relatively simple laboratory experiments. To demonstrate this the results are applied to a set of existing experimental fatigue crack propagation and fracture data obtained in our laboratory and by NASA. In the analysis of the fatigue data the stress intensity factor is used as the correlation parameter. In studying the fracture data we use a) the stress intensity factor K , b) K with plasticity correction, and c) the crack opening displacement.

* Published in *Nuclear Engineering and Design* 20 (1972) 265-286.