Stress Intensity Factors for Complete Internal and External Cracks in Spherical Shells

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INTRODUCTION

Cracks or flaws found in the nuclear structure must be fully evaluated to assure the safety of the plant. The weight function method presented by Bueckner (1970) and Rice (1972) has been widely used in the determination of stress intensity factors for cracks under stress gradient, e.g., for thermal shock loading. The unique feature of the weight function method is that once the weight function for a particular cracked geometry is determined the stress intensity factors at the crack tip for any loading applied to the flawed structure can be calculated by a simple integration.

In this paper the stress intensity factors of the complete, part-through internal and external cracks in a spherical shell are determined. The finite element method was used to develop the weight functions for the flawed geometry. The approximate crack surface profile proposed by Petroski and Achenbach (1978) was used to derive the weight functions. The stress intensity factors associated with the cracks in spherical shells under internal pressure are determined by both the weight functions and the direct finite element method.

THE WEIGHT FUNCTION METHOD

For a plane or axisymmetric problem the basic equation of the weight function method is, referring to the coordinate system in fig.1,

$$K = \int_0^a \sigma(x) h(a, x) dx \quad (1)$$

where K is the stress intensity factor under the stress distribution $\sigma(x)$ perpendicular to the crack surface in the uncracked component and $h(a, x)$ is the weight function for the geometry. The weight function $h(a, x)$ is independent of the stress distribution and can be obtained from the reference stress intensity factors $K_r(a)$ and the crack opening displacement $u_r(a, x)$ of a reference load case. For the axisymmetric case $h(a, x)$ has the form

$$h(a, x) = \frac{H}{K_r} \frac{\partial u_r(a, x)}{\partial a} \left(\frac{R+x}{R+a} \right) \quad (2)$$

where $H = E/(1-\nu^2)$, E is Young's modulus, and ν is Poisson's ratio. For the current study K_r in eqn.(2) can be determined from the finite element method. The $u_r(a, x)$ in its functional form, however, is not readily available from the finite element output. A few approximations have been proposed and proved to be very good and effective methods (Mattheck et al, 1984). In this paper the $u_r(a, x)$ proposed by Petroski and Achenbach (1978) is used to develop the weight functions.

FINITE ELEMENT ANALYSES AND DEVELOPMENT OF WEIGHT FUNCTIONS

In the paper by Petroski and Achenbach (1978), $u_r(x,a)$ is provided as :

$$u_r(a,x) = \frac{\sigma_0}{H \sqrt{2}} \left[4F(a/t) \sqrt{a(a-x)} + G(a/t) \frac{(a-x)^{2/3}}{\sqrt{a}} \right] \quad (3)$$

where the stress intensity factor of the crack with the length a is given by

$$K_r = \sigma_0 F(a/t) \sqrt{\pi a} \quad (4)$$

where σ_0 is a stress parameter of the stress field $\sigma_r(x)$ of the reference problem. Usually $\sigma_r(x) = \sigma_0 = \text{constant}$ is used as the reference stress. The unknown function $G(a/t)$ in eqn.3 is determined by the requirement of self-consistency of eqn.(1), i.e. K has to be K_r if $\sigma(x)$ is equal to $\sigma_r(x)$, or

$$K = K_r = \frac{H}{K_r} \int_0^a \sigma_r(x) \frac{\partial u_r(a,x)}{\partial a} dx \quad (5)$$

If $\sigma_r(x) = \sigma_0 = \text{constant}$ is used, equation (5) yields

$$G(a/t) = \left[(R+a) \sqrt{2\pi} \int_0^a F(a/t) a da - 4F(a/t) a^2 \left(\frac{2}{3} R + \frac{4}{15} a \right) \right] \\ * \left[a^2 \left(\frac{2}{5} R + \frac{4}{35} a \right) \right]^{-1} \quad (6)$$

Three groups of the shell geometries with $R/t = 5, 10$, and 20 are studied. For each group the crack length varies as $a/t = 1/16, 1/8, 1/4, 3/8, 1/2, 5/8, 3/4$, and $7/8$. The cracks are either internal or external and are rotational symmetric. The finite element program ABAQUS (1986) is utilized in this study. The eight node axisymmetric quadrilateral element is used and the element numbers range from 514 to 706 for various cases. Singular elements are used around the crack tip. Due to symmetry only $1/4$ of the shell needs to be modeled. A finite element model is shown in fig.2. A typical computer run took about ten minutes CPU on a VAX 11/780 computer. ABAQUS calculates the J integral (Rice, 1968) along contours around the crack tip by using the virtual crack extension method. It is noted that the J integral has to be modified from its regular form (Karlsson and Backlund, 1978) to include the effect due to the pressure loading on the crack face in the current study. ABAQUS program has incorporated this modification in the J contour integration. The stress intensity factor is then obtained from the relation $K = (J^*E/(1-\nu^2))^{1/2}$ for the axisymmetric problem.

The stress intensity factors of the various geometry combinations were first calculated from the finite element method. A uniform pressure applied to the crack face was used as the loading. A polynomial of $\gamma = a/t$,

$$F(\gamma) = C_0 + C_1\gamma + C_2\gamma^2 + C_3\gamma^3, \quad (7)$$

was fitted to the data from each of the three groups. It yields the reference stress intensity factor, K_r in (4), with σ_0 being the applied uniform pressure. The results are listed in table 1. The maximum error from the polynomial fit to the data was less than two percents for the cases with internal crack and three percents for the cases with external crack.

With the $F(a/t)$ function determined by the finite element method and $G(a/t)$ from eqn.(6) the weight function can be constructed with eqns.(2) and (3).

RESULTS

The weight functions obtained are arranged in a form as :

$$h(a, x) = \frac{1}{K_I} \frac{\alpha + \gamma(x/a)}{\alpha + \gamma} \frac{1}{\sqrt{1 - (x/a)}} \left[m_1 + m_2 \left(1 - \frac{x}{a}\right) + m_3 \left(1 - \frac{x}{a}\right)^2 \right] \quad (8)$$

where

$$m_1 = \sqrt{2} \sigma_0 F$$

$$m_2 = \frac{\sigma_0}{2\sqrt{2}} (4F + 3G + 8F'a)$$

$$m_3 = \frac{\sigma_0}{2\sqrt{2}} (2G'a - G)$$

where the $F(a/t)$ functions are given in (7) and table 1 for the three groups of the shell geometry and $G(a/t)$ is given in (6). The $()'$ denotes the differentiation with respect to a/t .

To check the accuracy of the weight functions the stress intensity factors determined from the weight functions are compared with those directly obtained from the finite element method. Two cases are investigated --- (i) the uniform pressure applied to the crack face, and (ii) the constant pressure inside the spherical vessel.

For the case (i) the stress intensity factors are plotted in fig.3 for both the internal and the external cracks. It is noted that this case is actually the self-consistency test. The errors are from the approximations in $u_r(a, x)$ and the polynomial fit $F(a/t)$.

For the case (ii) the principle of superposition in linear elasticity is used. The stress intensity factor, K , is equal to the summation of K^1 and K^3 where K^1 is the stress intensity factor associated with a uniform pressure loading on the crack face and K^3 is associated with the $\sigma(x)$ loading which is the stress distribution within the wall of the uncracked spherical vessel. The results are plotted in fig.4 along with the K 's from the direct finite element calculation.

The data in figs. 3 and 4 show that the maximum deviation of the stress intensity factors, calculated from the weight functions given by eqn.(8), from the finite element results is less than 6.5% for the range of the shell parameters studied. It may be concluded from this good agreement that the weight functions derived in this paper provide useful results with reasonable accuracy for estimating stress intensity factors for complete, part-through internal and external cracks in spherical shells.

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Figure 1. Geometry of the Crack

Figure 2. The Finite Element Model

Figure 3. Stress Intensity Factors for Uniform Pressure on Crack Face

Figure 4. Stress Intensity Factors for Constant Internal Pressure

TABLE 1(a) - Values of constants, internal crack

	$\gamma=5$	$\gamma=10$	$\gamma=20$
C_0	1.03773	1.12156	1.09948
C_1	1.15749	0.169787	0.389511
C_2	-1.562	2.3474	2.84598
C_3	2.72839	0	0

TABLE 1(b)- Values of constants, external crack

	$\gamma=5$	$\gamma=10$	$\gamma=20$
C_0	0.991341	1.01021	1.05177
C_1	0.146293	0.097887	0.032983
C_2	-0.0155949	0.00664533	0.0310324
C_3	0.0026697	0.00115497	-0.00018225

TABLE 1 VALUES OF CONSTANTS IN $F(\gamma)$