Computation of Leak Areas of Circumferential Cracks in Piping for Application in Demonstrating Leak-Before-Break Behaviour

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INTRODUCTION

One of the key-points in the application of the Leak-Before-Break (LBB) concept in any structure is the computation of leak areas in order to determine the leak rates. This information is essential for the demonstration of the concept. In fact the two problems for which the evaluation of leak area is of importance, are

- . detectability of a small crack using the fluid flowing through this crack,
- . computation of the force exerted by the fluid flowing through the opening.

In the first case, one needs a lower bound of the leak area in order to establish conservative predictions while in the second case, it is the upper bound which permits conservative evaluations. In this paper, it is the first case which is of interest to us.

There exists various simplified methods used in U.S.A., GERMANY, JAPAN etc... In this paper, a new method is presented and the results obtained are compared with the other methods.

Starting from the Linear Elastic Fracture Mechanics (LEFM) theory applied to plates, one introduces the curvature effects in order to apply the theory on piping structures. Finally, the plasticity effects are considered. This paper deals essentially with circumferential cracks although the results can easily be applied to longitudinal cracks.

LINEAR ELASTIC THEORY

Starting from the definition of the Strain Energy Release Rate G as

$$G = - \frac{1}{2t} \frac{\partial P}{\partial a}$$

where P is the Potential Energy and $2a$ the crack length in a structure of thickness t , one can show that for the case of a uniform applied stress, σ , the crack opening area A is related to G through the relation :

$$G = \frac{\sigma^2}{4} \left(\frac{\partial A}{\partial a} \right)$$

or
$$A = \frac{4}{\sigma^2 E'} \int_0^a K_I^2(x) dx \quad \text{-----} \quad [1]$$

where K_I is Stress Intensity Factor (SIF), E' , ν have their usual significance.

In the case of an infinite plate with a central crack of length $2a$,

$$K_I = \sigma \sqrt{\pi a}$$

and the crack-opening-area (COA) becomes :

$$A_o = 2 \sigma \frac{\pi a^2}{E'}$$

(In the following $E' = E$ will be considered).

Case of a Cylindrical Shell

In general, for the case of a through crack of length $2a$ in a pipe subjected to a uniform stress σ , the SIF can be written as :

$$K_I = M(\lambda) \sigma \sqrt{\pi a}$$

where M represents the amplification factor (bulging factor) and λ , the shell parameter, R , t being respectively the radius and thickness of the pipe.

Using equation (1), the COA, A_c , in non-dimensional form becomes :

$$\alpha(\lambda) = \frac{A_c}{A_o} = \frac{2}{\lambda^2} \int_0^\lambda M^2(\lambda) \lambda d\lambda \text{ ----- [2]}$$

Depending on the expression of $M(\lambda)$ used, various cases can be considered :

- . Wüthrich (1983) considers results concerning $M(\lambda)$ of Erdogan and Deale (1979)
- . Tada-Paris Method (1983) : In this case, results of $M(\lambda)$ come from two sources :
 - Case of bending moment and axial forces : One uses Sanders' solutions (1982, 1983) but the results are obtained only for $R/t = 10$, although the solutions are valid for R/t between 5 and 15. For uniform stress, one obtains :

$$\alpha_t = 1 + \left[\frac{\theta}{\pi} \right]^{3/2} \left\{ 8.6 - 13.3 \left[\frac{\theta}{\pi} \right] + 24 \left[\frac{\theta}{\pi} \right]^2 \right\} \\ + \left[\frac{\theta}{\pi} \right]^3 \left\{ 22.5 - 75 \left[\frac{\theta}{\pi} \right] + 205.7 \left[\frac{\theta}{\pi} \right]^2 - 247.5 \left[\frac{\theta}{\pi} \right]^3 + 242 \left[\frac{\theta}{\pi} \right]^4 \right\}$$

But for the case of bending stress which varies from σ_b to $\sigma_b \cos \theta$ equation (1) is no longer valid. Reasoning with bounds on the crack opening-area, the authors propose the following expression for α in bending :

$$\alpha_b = \left[\frac{3 + \cos \theta}{4} \right] \alpha_t$$

- For the case of internal pressure, the authors use an approximation of the following solution (Rooke, 1976)

$$K_p = F_p \sigma_p \sqrt{\pi a} \text{ for the pressure stress } \sigma_p$$

where

$$F_p = (1 + 0.3225 \lambda_1^2)^{1/2} \text{ for } (0 \leq \lambda_1 \leq 1) \\ = 0.9 + 0.25 \lambda_1 \text{ for } (1 \leq \lambda_1 \leq 5)$$

and $\lambda_1 = a / \sqrt{Rt}$

Integrating this expression, one obtains

$$\alpha_p = 1 + 0.16 \lambda_1^2 \quad \text{for } (0 \leq \lambda_1 \leq 1)$$

$$= 0.2/\lambda_1^2 + 0.81 + 0.30 \lambda_1 + 0.03 \lambda_1^2 \quad \text{for } (1 \leq \lambda_1 \leq 5)$$

. BARTHOLOME-KASTNER METHOD (BARTHOLOME, 1986 ; KASTNER, 1981) : The B-K method represents a big simplification since it does not distinguish between :

- . the end effect due to internal pressure and the axial forces
- . the bending stress and the membrane stress.

In other words, in this method one considers a uniform stress

$$\sigma = \sigma_t + \sigma_b + \sigma_p$$

with a unique and simple expression for α as

$$\alpha(\lambda) = (1 + 0.117 \lambda^2)^{1/2}$$

Moreover, the results are based on an approximate expression of the crack edge field (asymptotic solution) in a plate which gives COA as :

$$(A_0)_{B-K} = \frac{7.54 \sigma a^2}{E}$$

with respect to theoretical value of

$$A_0 = \frac{2 \pi \sigma a^2}{E}$$

This shows that the B-K-method is already considering the opening area 20% higher than the theoretical elastic value.

. PROPOSED METHOD : it makes two modifications in the TADA-PARIS method :

- Case of non-uniform loading : Since the opening area is closely related to the SIF K , the method consists in finding an equivalent stress σ_{eq} , which when applied uniformly on the crack faces, would give the same SIF as that obtained under the given loading conditions. The computation of the opening area becomes then a simple problem of crack subjected to a uniform stress σ_{eq} . In the case of bending load,

$$K_b = F_b \sigma_b \sqrt{\pi a} = F_t \sigma_{eq} \sqrt{\pi a}$$

which gives

$$\alpha_b = \frac{F_b}{F_t} \alpha_t$$

It is interesting to note that for the case of $R/t = 10$, the difference between this method and that of TADA-PARIS is only about 3%.

- Case of $R/t \neq 10$: The proposed method generalises the applicability of the T-P method for a value of R/T between 5 and 15 by using Sanders' solutions (1982, 1983) in a polynomial form as a function of R/t , which gives :

$$\alpha_t = 1 + \left\{ \frac{\theta}{\pi} \right\}^{3/2} + \left\{ 1.14 A + 0.89 B \left\{ \frac{\theta}{\pi} \right\} + 0.727 C \left\{ \frac{\theta}{\pi} \right\}^2 \right\} + \left\{ \frac{\theta}{\pi} \right\}^3$$

$$\left\{ 0.4 A^2 - 0.67 AB \left\{ \frac{\theta}{\pi} \right\} + 0.29 (B^2 + 2AC) \left\{ \frac{\theta}{\pi} \right\}^2 - 0.5 BC \left\{ \frac{\theta}{\pi} \right\}^3 + 0.22 C^2 \left\{ \frac{\theta}{\pi} \right\}^4 \right\}$$

Coefficients A, B, C represent Sanders' solutions (Brust, 1987)

$$\text{For } \alpha_t \begin{cases} A_t = -2.02917 \pm 1.67763 (R_m/t) + 0.07987 (R_m/t)^2 + 0.00176 (R_m/t)^3 \\ B_t = 7.09987 - 4.42394 (R_m/t) + 0.21036 (R_m/t)^2 - 0.00463 (R_m/t)^3 \\ C_t = 7.79661 + 5.16676 (R_m/t) - 0.24577 (R_m/t)^2 + 0.00541 (R_m/t)^3 \end{cases}$$

$$\text{For } \alpha_b \begin{cases} A_b = -3.26543 + 1.52784 (R_m/t) - 0.072698 (R_m/t)^2 + 0.0016011 (R_m/t)^3 \\ B_b = 11.36322 - 3.91412 (R_m/t) + 0.18619 (R_m/t)^2 + 0.004099 (R_m/t)^3 \\ C_b = -3.18609 + 3.84763 (R_m/t) + 0.18304 (R_m/t)^2 + 0.00403 (R_m/t)^3 \end{cases}$$

PLASTICITY EFFECTS

The effect of plasticity is to increase the opening of the crack and gives areas greater than those in elastic case. To take account of these effects, one can use

- either one of available analytical models like DUGDALE, IRWIN
- or an approach based on J-integral
- or any numerical method like Finite elements. However the cost of computation in this case is certainly much larger than any simplified method unless one has EPRI-handbook type solutions obtained from numerical results (Kumar & al, 1984)

IRWIN Plasticity model : This model uses the elastic solution with an effective crack length

$$a_{eff} = a_0 + r_y$$

where a_0 is the initial crack length and

$$r_y = \frac{1}{\beta \pi} \left[\frac{K}{\sigma_y} \right]^2 ; \quad \beta = \begin{cases} 2 & \text{in Plane-stress conditions} \\ 6 & \text{in Plane-strain conditions.} \end{cases}$$

TADA-PARIS METHOD : is based on IRWIN-model in plane-stress. The effective crack angle is given by :

$$\theta_{eff} = \theta_0 + \frac{K^2}{(2 \pi R \sigma_y^2)}$$

which is obtained using an iterative approach.

BARTHOLOME-KASTNER METHOD : also uses IRWIN-model, however σ_y is replaced by σ_F (Flow-Stress) of the material ($\sigma_F = (\sigma_y + \sigma_u)/2$).

The asymptotic solution in the case of an infinite plate with a central crack of length $2a_0$ subjected to a uniform stress, σ , gives the displacement field which when integrated gives the opening area as in non-dimensional form as

$$\gamma = \frac{A}{A_0} = \frac{1 - x^3}{(1 - x^2)^2} ; \quad x = \frac{1}{\sqrt{2}} \left[\frac{\sigma}{\sigma_F} \right]$$

representing a correction factor accounting for the plasticity effects.

(Note : The above expression for γ represents the present version of the B-K method (8) and gives a value higher than the preceding version (Kastner, 1981)

$$\gamma = (1 + x^2)^{3/2} - (x^2)^{3/2}$$

The difference in the two versions is due to the fact that Ref. (Kastner, 1981) does not update the value of K after plasticity correction).

. JAERI METHOD (ISOZAKI, 1988) is a modification of the PARIS-TADA method replacing σ_y by a flow-stress σ_F^* defined as follows

$$\sigma_F^* = \left(\frac{3 \sigma_y + \sigma_u}{4} \right)$$

The results thus obtained correlate well with experimental data obtained on 3", 6" and 12" pipes with 60°, 90° and 150° angle cracks.

. DUGDALE Plasticity Model : IRWIN-plasticity model is valid for small-scale yielding conditions unless the fracture parameters are updated after the plasticity effects. DUGDALE-model has a more extended application.

The effective length of the crack in Griffith configuration is given by :

$$a_{eff} = a_0 \sec \left(\frac{\pi}{2} s \right) ; s = \sigma / \sigma_F$$

Integrating the displacement field given by this model and writing opening area in a non-dimensional form

$$\gamma(s) = \frac{A_{el-pl}}{A_{el}}$$

one obtains solutions given in table I.

Case of non-uniform loading : one of the problems of this model is that it is applicable under uniform loading conditions since parameter s represents the ratio of uniform stress to the flow stress. This difficulty can be overcome by using reference stress concept (PENNY, 1971)

$$\sigma^* = \frac{\text{applied load}}{\text{limit load } (\sigma_F)} \cdot \sigma_F$$

Thus whatever the applied loading condition, one can associate a reference stress σ^* to it and the parameter

$$s = \frac{\sigma^*}{\sigma_F} = \frac{\text{applied load}}{\text{limit load } (\sigma_F)}$$

can be obtained to apply the DUGDALE model.

Plasticity Effects in Cylindrical Shells : to a first approximation, one can combine plasticity and bulging effects through a multiplicative function $\gamma(s) \cdot \alpha(\lambda)$ which would give opening area as (WÜthrich, 1983)

$$A = \gamma(s) \cdot \alpha(\lambda) A_0$$

This relation does not account for the interaction between plasticity and bulging since both phenomena are considered independently. It is therefore conservative from the point of view of the objectives enunciated in the beginning

VALIDATION/APPLICATIONS

The analysis of the methods presented here has been conducted using data on pipes with various geometrical parameters such as :

. Experimental results on thick large pipes (ϕ 850 mm, t = 40 mm) tested at MPA, Stuttgart in West Germany. The loading is internal pressure with or without applied moment,

- . Finite-element computations and EPRI Handbook solution on a large thin pipe (ϕ 750 mm, t = 12 mm) under bending load,
- . EPRI handbook solution on a certain number of piping configurations typical of PWR-plants ;

Table II gives all the geometrical, material and loading details and compares the results obtained by different methods.

Finally an application of these methods is made on a benchmark problem (HASAGANA & al, 1987) where the experimental results obtained at JAERI on a 6" pipe in 304 SS under bending, were already compared with GE/EPRI handbook solutions and TADA-PARIS method.

Fig. 1 shows the results of crack opening displacement as a function of applied bending stress.

CONCLUSION

The method proposed in this paper is reliable in a broader range of pipe radius to thickness R/t , and of loading conditions than previously proposed methods, namely TADA-PARIS and BARTHOLOME-KASTNER. In all the cases where correct input data is available, it gives a lower bound for the crack opening area. It is therefore recommended for applications in the demonstration of Leak-Before-Break concept in components.

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TABLE I
Plasticity Correction Factor γ (s)

(s)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
γ (s) Irwin (B-K 1)	1.007	1.027	1.059	1.10	1.149	1.205	1.268	1.336	1.408
γ (s) B-K (B-K 2)	1.01	1.038	1.086	1.155	1.248	1.374	1.542	1.771	2.097
γ (s) Dugdale	1.008	1.031	1.069	1.123	1.198	1.302	1.450	1.680	2.117

FIGURE 1

TABLE II

CRACK OPENING AREA IN cm^2									
No	ORIGIN	a_n	t	2a	Exp. (or γ , K.) value	JAERI Method	EPRI Method	PARIS Method	IRWIN-PARIS Method
1	HPA	401	42	1008	2040	1029		2112.0	1391
2	tests conducted	401	42	1078	2450	2112		2572	2714
3	in	401	42	1260	6654	2978		4023	3070
4	FIG.	401	42	1008	2116	2140		2469	2591
5		401	42	1008	2264	2711		2805	3131
6	This large pipes	376.25	11.9	500	(462.5)	359	462.5	441	612
7	This	82.45	3.4	78.9		3.69	4.8	4.64	4.30
8	small pipes	82.45	3.4	138		10.9	20.0	16.1	12.5
9		54.14	6.02	71.3		5.57	5.8	4.4	9.55
10		382	61	650		828	760	556	878
11	Piping	404.35	67	560		1082	692	566	1271
12	typical	417.35	72.5	80		7.13	8.0	6.45	7.26
13	of	185.23	35.95	308		355	243.5	161	439
14	PN's	185.23	35.95	260		102	83.5	69.9	105
15		405.1	74.4	223.2		71.0	56.0	51.8	84.1
16		204	51.0	153.0		17.9	15.72	14.2	18.5
17		123.8	25.4	172.85		24.9		22.1	24.90
18		123.8	25.4	183.7		32.5		28.0	32.5
19		123.8	25.4	112.4		19.0		15.2	19.0
20		123.8	25.4	123.6		22.7		18.4	22.7