Hexahedral Mesh Generation of Nuclear Structures Using Intelligent Local Approach

Yoshitaka Wada, Shinobu Yoshimura and Genki Yagawa

The University of Tokyo, Japan

ABSTRACT

This paper describes an automated generation method of hexahedral mesh named Intelligent Local Approach (ILA). Here elements are generated sequentially, considering both local information on geometrical constraints and user's demand on quality of elements. Topological connectivity is first determined based on a heuristic category-based method. Then a fuzzy knowledge processing technique is utilized to determine precise location of node, considering various complicated geometrical constraints for high quality hexahedral elements. Fundamental performances of the ILA are demonstrated in detail through generating some hexahedral meshes of nuclear structures.

INTRODUCTION

Unstructured mesh generation for the FEM is in general a very labor-intensive and time-consuming process. Recently, the portion of the mesh generation process in the analysis increases, because the model of analysis becomes very complex and the complexity of mesh also increases. Numerous research activities have been devoted into the development of automatic mesh generation techniques. Fully automatic mesh generation techniques for triangular elements and tetrahedral elements have already been established. On the other hand, quadrilateral elements and hexahedral elements, the automatic generation of which is still an open problem, are strongly demanded for some problems. This is because compared with triangular and tetrahedral elements, those elements are more suitable to strongly nonlinear problems, and large aspect ratio elements are usable.

In Ref. 1, the present authors proposed a new automatic mesh generation method named Intelligent Local Approach (ILA), which can control size and aspect ratio of quadrilateral mesh. The ILA is a kind of heuristic method that is derived from the analysis of human expert's approach. This was successfully applied to the generation of several quadrilateral meshes. In the present study, we extend the ILA into an automatic generation of hexahedral mesh and apply it to nuclear structures.

FUNDAMENTAL PRINCIPLE

It is well known that automatic generation of hexahedral mesh is a very complicated and difficult task.

Nevertheless, human experts on mesh generation can generate hexahedral meshes for an arbitrarily shaped domain using their superior capability for image recognition and qualitative judgement, if the number of elements to be generated is small. The present authors have invented a new mesh generation method for quadrilateral and hexahedral meshes, basically (a) by analyzing such mesh generation processes by human experts, (b) by abstracting elemental processes hidden, and (c) by systematizing them into a complete procedure. The elemental processes abstracted are summarized as follows:

- (1) Imagine qualitatively a whole distribution of element size and that of aspect ratio over a whole geometry model.
- (2) Generate elements one by one, starting from an outer boundary or inner boundary of the geometry model.
- (3) Collect geometrical information such as angle, length and area of generated elements from a local region where one to few elements are to be generated.
- (4) Imagine "virtual elements" to be generated, and evaluate their goodness. Then determine actual node location and element, based on the fuzzy judgement for goodness of "virtual elements".
- (5) Regenerate old elements if they are not compatible to a newly generated element.

The mesh generation process based on the hypothesis above is named "Intelligent Local Approach (ILA)", and an actual system is developed. ILA's actual implementation to quadrilateral mesh generation and some examples can be found in Ref. 1.

ALGORITHMS

3.1 Front Boundary

In 2-D plane case, an initial front boundary is defined as a set of segments, which represent a geometry model. In 3-D solid case, the initial front boundary is defined as a set of quadrilateral surface patches, each of which consists of four edges and four nodes, as shown in Figure 1 (a). The outer boundary of the model is named an outer (front) boundary, while the inner boundary is named an inner (front) boundary as shown in Figure 1 (b). Starting from one surface patch of the initial front boundary, one to few elements are generated sequentially. The front boundary is then updated by adding new surface patches and by erasing a few surface patches covered by newly generated elements.

Figure 1. Front Boundary.

Figure 2. Search Area for 3-D Geometrical Information along Front Boundary.

3.2 Collection of Local Geometrical Information

After determining one surface patch from which mesh generation starts, geometrical information is collected from a local region near the surface patch. In hexahedral mesh generation, the geometrical information includes sizes of surfaces and angles of two adjacent surfaces.

In the current version of system, such geometrical information is collected along the front boundary as illustrated in Figure 2. The total number of collected sizes of surfaces is 37, while that of collected angles of any two adjacent surfaces is 48. These numbers are not definite values. If we increase these numbers, we can perform more flexible mesh generation. But we should remember that increasing these numbers also increases processing time.

Table 1. Definition of Division Category.

Division Category	Number of Divisions	An Angle of Two Surfaces (degree)
C1	1	40-105
C12	1 or 2	105-155
C2	2	155-200
C3	3	200-300
C4	4	300-360

3.3 Categories of Angle Division

Considering the local geometrical information collected, we categorize a geometrical situation. Table 1 summarizes five kinds of categories of angle division, C1, C2, C3, C4 and C12. In C1, the angle of two adjacent surfaces should not be divided. In C2, C3 and C4, the angle should be divided into two, three or four angles, and then one, two or three surfaces are newly generated, respectively. The angle of C12 is dealt as that of either C1 or C2, depending on its circumstance. C1 has the first priority among the five categories. After multiple angles are collected from a local region and classified into the above division categories, an element is generated first in the C1 category angle. When the angle is less than 40°, those surfaces are merged into one surface in order to erase sharp angle and avoid creating irregular elements.

3.4 Patterns of Element Generation

Based on the categories of angle division given in the previous subsection, we define four kinds of 2-D patterns of element generation as shown in Figure 3. "2-D pattern 0" shown in Figure 3 (a) is the pattern that a new element is automatically determined without generating any nodes or edges. In "2-D pattern 1" shown in Figure 3 (b), an edge connecting nodes *a* and *b* are generated to generate an element. In "2-D pattern 2", one node, two edges and one element are generated as shown in Figure 3 (c). In "2-D pattern 3" shown in Figure 3 (d), the angle of the two adjacent edges *l* and *m* is divided into two, three or four angles, and then one, two or three edges are newly generated, respectively.

In 3-D case, we define 3-D patterns of element generation through the analogy with the 2-D patterns. Nine independent patterns, which are derived considering full combinations of the four 2-D patterns, are given in Figure 4. "3-D Pattern 0" is basically the same as "2-D Pattern 0". In this pattern, a new element is automatically determined without generating any nodes or edges. In "3-D Pattern 1", an element can be generated by newly generating two surfaces, but without generating edges and nodes.

Figure 3. 2-D Patterns for Creation of Connectivity.

In "3-D Pattern 2" and "3-D Pattern 2a", an element can be generated by only generating one surface. In "3-D Pattern 3" and "3-D Pattern 4", an element can be also generated by only generating some surfaces. In "3-D Pattern 5", "3-D Pattern 6" and "3-D Pattern 7", an element is generated by generating both new nodes and edges. We define priority of the 3-D patterns. "3-D Pattern 0" is the easiest pattern to create an element. The top priority is given to this, and the next highest priority is given to "3-D Pattern 1".

These 3-D patterns shown in Figure 4 are not all of possible 3-D patterns. We should also consider variation of edges. There are 41 more patterns in addition to the previous 9 patterns. Figure 5 (a) and (b) show some examples of such additional 3-D patterns derived from 3-D pattern 5 and 3-D pattern 6, respectively. In 3-D solid case, element connectivity is determined sequentially by converting any of 3-D patterns into a simpler 3-D pattern repeatedly through adding new surfaces and edges.

2D Pattern 0	2D Pattern 1	2D Pattern 2	2D Pattern 3
2D Pattern 0	3D Pattern 0	<p>P0-P0</p> 3D Pattern 1	<p>P0-P2</p> 3D Pattern 2a	<p>P0-P1</p> 3D Pattern 1
2D Pattern 1	<p>P1-P0 is the same as P0-P0.</p>	<p>P1-P1</p> 3D Pattern 2	<p>P1-P2</p> 3D Pattern 3	<p>P1-P1</p> 3D Pattern 4
2D Pattern 2	<p>P2-P0 is the same as P0-P2.</p>	<p>P2-P1 is the same as P1-P2.</p>	<p>P2-P2</p> 3D Pattern 5	<p>P2-P1</p> 3D Pattern 6
2D Pattern 3	<p>P3-P0 is the same as P0-P3.</p>	<p>P3-P1 is the same as P3-P1.</p>	<p>P3-P2 is the same as P2-P3.</p>	<p>P3-P3</p> 3D Pattern 7

Figure 4. Basic 3-D Patterns for Creation of Connectivity.

(a) One of Derivatives of 3-D Pattern 5 (b) One of Derivatives of 3-D Pattern 6
Figure 5. Examples of Derivatives of Basic Nine 3-D Patterns

3.5 Determination of Node Location Using Fuzzy Knowledge Processing

After determining the pattern of node and element generation as described in subsections 3.3 and 3.4, the location of node is determined so as that the shapes of newly generated elements are appropriate. For the judgement, we employ the following fuzzy knowledge processing [2,3].

At first, we explain in detail the method to determine an appropriate location of node in 2-D plane case, which is a basis for the method of 3-D solid case. Then the method of 3-D case is explained.

(1) Geometrical Parameters for Goodness of Element Shape

As shown in Figure 8, we employ the following six kinds of parameters for the judgement of goodness of element shape, i.e. (a) ratios of opposite edges, S_1 / S_3 and S_2 / S_4 , (b) ratio of diagonal lines, l_1 / l_2 , (c) square root of area, $\sqrt{a}$, (d) ratios of opposite angles, θ_1 / θ_3 and θ_2 / θ_4 , (e) average of ratios of opposite edges, $(S_1 / S_3 + S_2 / S_4) / 2.0$, (f) ratio of summations of adjacent edges, $(S_1 + S_2) / (S_3 + S_4)$. These geometrical parameters become 1.0 for a rectangular element. l_0 means length of standard element evaluated from the field of element size specified by a user, which will be explained subsequently.

(2) Information for Control of Mesh Subdivision

The control of both element size and aspect ratio is a key issue for any automatic mesh generation techniques. In the ILA, a user can specify the following two kinds of "fields": (a) the field of element size and (b) that of aspect ratio. A field of priority of element creation is also specified within the ILA. The three fields are stored over a uniform Cartesian grid, i.e. a background cell, enveloping a geometry model. The values at any location are linearly interpolated from the grid point values.

(2a) Field of Element Size

Element size is a scalar value. The reference point for this information is defined as a central point of edge, a gravity center of surface, or that of volume. Even a complicated distribution over a whole geometry model can be easily specified using the fuzzy knowledge processing as described in the FuzzyMesh method [4-6].

(2b) Field of Aspect Ratio

Aspect ratio is also scalar. Priority is given to this information near the boundary. On the other hand, this information is often neglected in the region far from the boundary.

(2c) Field of Priority of Element Creation

In the ILA, elements are generated sequentially. The field of priority of element creation directs the sequence of element generation. One possible method is that elements are generated along boundaries, from the outer to the inner of the geometry model. Another method is that elements are generated in the one direction as if water was fulfilled in vase. In the latter method, the field of priority of element

Figure 6. Geometrical Parameters of Quadrilateral Elements.

Figure 7. Typical Membership Function.

creation plays the same role as "Gravitational potential field" in mechanics. According to the latter method, unmeshed area always contains a part of the outer boundary of the geometry model. Therefore, geometrical constraint is still weak to some extent, even at the last moment of the mesh generation.

(3) Fuzzy Knowledge Processing

Node location is precisely determined, considering the geometrical parameters defined in subsection 3.5(1) and the field information defined in subsection 3.5(2). Taking the case of category C2 as a typical example, the present fuzzy knowledge processing is explained in the following.

(3a) 2-D Case

Figure 7 shows a typical membership function employed in this study. The horizontal axis denotes any of geometrical parameters, while the vertical axis does the degree of membership, i.e. appropriateness.

At the first step, a local x - y region where a new element is generated is mapped onto a ξ - η grid space as shown in Figure 8 (a). Here thick solid lines both in the x - y space and in the ξ - η grid space denote the segment with which a new element is to be generated. At the second step, we presume a virtual node at each grid point, and evaluate goodness of shape of each virtual element using its geometrical parameters. For each parameter, a degree of membership is calculated grid point by grid point. Figure 8 (b) illustrates the distribution of degree of membership μ_a for "ratio of angle" parameter of category C2 plotted in the ξ - η grid space. Figure 8 (c) illustrates the similar distribution for "ratio of opposite edges" parameter μ_i . If we consider only the two geometrical parameters, we superpose the μ_a and μ_i distributions by the product operation of the fuzzy set theory [2,3], and get the $(\mu_a \wedge \mu_i)$ distribution as shown in Figure 8 (d). In really, we generate more distributions for the six kinds of geometrical parameters of one virtual element, and superpose them. If multiple virtual elements are generated at once, the multiple superposed distributions for these elements are further superposed into the one distribution. Node location to be generated is finally determined as the grid point where the degree of membership of the last superposed distribution takes the maximum as illustrated in Figure 8 (d). In the ILA, the process of the determination of node location takes most computation time. However, this process can be extremely speeded up by employing parallel processing [7,8], since this can be perfectly parallelized in grid point wise.

(3b) 3-D Case

The method for 2-D case can be straightforwardly extended to 3-D case. However, such a straightforward method requires too many grid points of evaluation, i.e. the order of n^3 , where n is the division number in one dimension, and is not efficient. Therefore we adopt the following alternate method, which is explained taking "3-D Pattern 5" as an example.

Firstly a plane consisting of two edges (S_7 - S_2) are mapped onto 2-D plane as illustrated in Figure 9. Secondly one node is generated in the plane using the same method explained in subsection 3.5 (3a). Thirdly this node is mapped back to the 3-D space. The same process is repeatedly applied to both the plane consisting of (S_7 - S_4) and that of (S_7 - S_6). Thus we can obtain three candidate locations of nodes, the mean position of which is finally adopted as a new node to be generated. Then one element is generated in the 3-D space. Since the number of grid points of evaluation in this method is the order of $3 \times n^2$, this method is more efficient than the straightforward 3-D method.

(a) Mapping from Local x - y Space to ξ - η

(b) μ_a Distribution over the Grid

(c) μ_l Distribution over the Grid

(d) $\mu_a \wedge \mu_l$ Distribution Derived from Fuzzy Product

Figure 8. Node Generation Based on Fuzzy Knowledge Processing in 2-D Case.

Figure 9. Node Generation in 3-D Pattern 5.

Number of Nodes: 1026
Number of Elements: 795
Generation Time: 61.5 sec.

(a) Initial Mesh of HTTR Carbon Block

Number of Nodes: 8333
Number of Elements: 6360
Generation Time: 68.4 sec.

(b) Fine Mesh of HTTR Carbon Block

Figure 10. Examples of Hexahedral Mesh.

EXAMPLES

Mesh generations of examples given here are performed on a PC with one Pentium II 333MHz, 384 MBytes memory and 8 GBytes disk. Operating system is FreeBSD 2.2.8 and compiler is egcs-1.1.1 release. According to these experiences, 160 MBytes memory is necessary as ILA system's working area. Examples show meshes, numbers of nodes and elements and generation times.

Figure 10 (a) shows that for a simplified model of carbon block of HTTR (High Temperature Engineering Test Reactor). Figure 10 (b) shows a fine mesh of HTTR. This mesh is obtained by subdividing each element of the coarse mesh into 8 smaller elements. In the model, both element size and aspect ratio are specified as 1.0 over the whole domain.

In the current version of system, the capability of controlling aspect ratio is not completely implemented yet, though this capability was verified for mesh generation of quadrilateral elements as in Ref. 1. This is another function to be implemented in the next version of the system.

CONCLUSIONS

We presented a new mesh generation method named Intelligent Local Approach for hexahedral elements. The ILA consists of the following fundamental features:

- (1) Sequential mesh generation.
- (2) Collection of geometrical information from a local region.
- (3) Explicit specification of user's demand on element size and aspect ratio as fields information.
- (4) Heuristic determination of local element connectivity, i.e. patterns of element generation.
- (5) Fuzzy knowledge processing for multiple criteria on goodness of hexahedral elements.

Further research is needed to generate a mesh with more complex geometry.

References

- [1] Yoshimura, S., Wada, Y. and Yagawa, G., "Automatic Mesh Generation of Quadrilateral Elements Using Intelligent Local Approach", *Computer Methods in Applied Mechanics and Engineering*, 1999, in Print.
- [2] Zadeh, L. A., "Fuzzy Algorithms, Information and Control", Vol.12, 1968, pp. 94-102.
- [3] Zadeh, L. A., "Outline of a New Approach to the Analysis of Complex Systems and Decision Process", *IEEE Transactions on Systems, Man and Cybernetics*, Vol.SMC-3, 1973, pp. 28-44.
- [4] Yagawa, G., Yoshimura, S., Nakao, K. and Soneda, N., "Automatic Two- and Three-Dimensional Mesh Generation Based on Fuzzy Knowledge Processing", *Computational Mechanics*, Vol.9, 1992, pp. 333-346.
- [5] Yagawa, G., Yoshimura, S. and Nakao, K., "Automatic Mesh Generation of Complex Geometries Based on Fuzzy Knowledge Processing and Computational Geometry", *Integrated Computer-Aided Engineering*, Vol.2, 1995, pp. 265-280.
- [6] Yagawa, G., Kawai, H., Yoshimura, S. and Yoshioka, A., "Mesh-invisible Finite Element Analysis System in a Virtual Reality Environment", *Computer Modeling and Simulation in Engineering*, Vol.2, 1996, pp. 289-314.
- [7] Yagawa, G., Yoshioka, A., Yoshimura, S. and Soneda, N., "A Parallel Finite Element Method with Supercomputer Network", *Computers and Structures*, Vol.47, 1993, pp. 407-418.
- [8] Yagawa, G. and Shiota, R., "Parallel Finite Elements on a Massively Parallel Computer with Domain Decomposition", *Computing Systems in Engineering*, Vol.4, 1993, pp. 495-503.