Probabilistic Fracture Analysis of a Straight Pipe with Through-Wall Circumferential Crack Using R6 Method

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ABSTRACT

Deterministic fracture analysis of cracked structures has been successful in demonstrating the behavior of structures under observed or postulated flaws. However, crack size, postulated accidental loading, mechanical and fracture parameters are statistically distributed and one can never be absolutely sure that the values worse than the chosen conservative limits will not be encountered. These uncertainties encourage us to take a statistical or probabilistic approach to the structural integrity. R6 is a well-known engineering assessment procedure for the evaluation of the structural integrity of flawed structure. This paper presents the determination of failure probabilities of a straight pipe with a through-wall crack in the circumferential direction, using R6 method. The pipe is subjected to an inplane bending moment. The scatter in the crack size, mechanical and fracture properties is considered. The probability of failure is studied for variation in applied loads. Monte Carlo simulation is used to find the failure probability. The results are compared with approximate analytical formulations and those available in literature.

1. INTRODUCTION

The mechanical integrity of nuclear reactor piping is a matter of great importance for both economical and safety reasons. In practice this integrity is assured by careful attention to all aspects of design, manufacturing, installation and operation of the piping system. It is well recognized that cracks act to magnify the stress in the piping component. The crack under severe load can become unstable and thereby cause the failure at flawed sections of the pipe. The cracks therefore are the important consideration for safety analysis. Although every effort is made to ensure that crack existence is minimized during fabrication and operation, still their undetected presence can never be completely ruled out. The maximum load bearing capacity of a flawed pipe is determined by applying principles of fracture mechanics. However many of the input parameters for analysis are statistically distributed and one can never be absolutely

sure that values worse than the chosen conservative will not be encountered. The uncertainties in input parameters encourage us to take a statistical or probabilistic approach for determination of critical load.

2. R6 METHOD

Structures made from materials with sufficient toughness may not be susceptible to brittle fracture, but they can fail by plastic collapse if they are overloaded. Dowling and Townley [1] and Harrison, et al [2] introduced the concept of a two-criteria Failure Assessment Diagram (FAD) to describe the interaction between fracture and collapse. This simple engineering assessment procedure for the evaluation of structural integrity for flawed structures was improved and was called as the R-6 procedure [3]. The FAD is shown in Figure 1. It effectively considers the crack tip failure criteria along with the net section collapse. In Figure 1, the x-axis represents the parameter L_r , which denotes nearness to plastic collapse and the y-axis represents the parameter K_r , which measures the nearness to crack growth initiation or normalized crack initiation. The FAD is bounded by these two axes and the curve is called Failure Assessment Line (FAL). The usage of FAD approach (i.e. R-6 method) lead to quick failure assessment. In this method, the analysis involves assessment of given conditions and representing them by a single point, called "assessed point", on FAD and examining where the assessed point lies inside the area bounded by the FAL and X/Y axes. If point falls inside the envelope, it will indicate that crack will not grow. The co-ordinates of assessed point (K_r' , L_r') are evaluated using Equation 2. The FAL used for the present study is the Option 2 FAL of R6 method and is given by Equation 1.

Figure 1 The R6 Failure Assessment Diagram

$$K_r = \left\{ \frac{E \varepsilon_{ref}}{L_r \sigma_y} + \frac{L_r^3 \sigma_y}{2 E \varepsilon_{ref}} \right\}^{\frac{1}{2}} \text{ for } L_r \leq L_r^{\max}$$

$$K_r = 0 \text{ for } L_r > L_r^{\max}$$

Where

- K_r = Measure of proximity to LEFM failure
 L_r = Measure of proximity to plastic yielding
 σ_y = Lower Yield or 0.2% proof stress
 ε_{ref} = True strain from the uniaxial tensile stress-strain curve at true stress $L_r \sigma_y$
 E = Young's Modulus
 $L_r^{\max}$ = cut-off defined by :

$$\frac{\text{uniaxial flow stress } \bar{\sigma}}{\text{uniaxial 0.2\% proof stress, } \sigma_y} \quad (1)$$

- L_r' = applied load / yield stress based limit load
 K_r' = stress intensity factor / fracture toughness

(2)

3. DESCRIPTION

The objective of this study is to find out the failure probability of straight pipe with through the wall circumferential (TWC) crack of size 2θ , subjected to in-plane bending load (M). The cracked pipe is shown in Figure 2. The variation in probability of failure with respect to change in applied load is studied. The failure modes considered are Crack Growth Initiation and Unstable Crack Growth, using R6 method, (Option 2). The input data is obtained from a similar study done by Rahman [4]. The input variables[4] are log-normally distributed. There are cases where the input variables are co-related, i.e. a co-variance matrix exists. Failure probability is determined using Monte-Carlo Simulation (MCS).

Figure 2 Through-wall circumferential crack in a straight pipe

vectors, repeated deterministic evaluation of failure is performed. If 'h' be the total number of trials in which failure occurs and 'N' be the total number of trials then the probability of failure (P_f) is given by Equation 6.

$$P_f = h/N \quad (6)$$

The step by step procedure to find out the probability of failure is listed below:

Let 'N' represents the counter for number of trials and 'h' represents counter for number of failures. For a given applied moment 'M', the probability of failure is determined by following the given steps.

1. Using random number generator, generate a value for each of the random parameter of the analysis. Increment 'N'
2. Construct the FAD using the value of tensile properties.
3. Calculate the assessed point and plot.
4. If the point lies in unsafe region increment 'h'

This is to be repeated such that saturated value of ' P_f ' is obtained. The same exercise is repeated for different loads. For calculating the probability of failure with unstable crack growth as failure mode, in step 3 above the J-R curve is also used to assess safety of the pipe.

Approximate theoretical calculations are also made to validate the results. These calculations are done for J-Tearing based analysis. The crack growth initiation is said to occur when the following condition is satisfied

$$J - J_{IC} > 0 \quad (7)$$

Thus if the distribution of J and J_{IC} is known, the distribution of $J - J_{IC}$ can be calculated. From that distribution, probability of the condition given by equation 10 can be obtained. J is a function of crack angle and Ramberg-Osgood constants.

$$J = J(\theta, \alpha, n) \quad (8)$$

Mean μ_J and variance σ_J^2 of J can be obtained as

$$\mu_J = J(\mu_\theta, \mu_\alpha, \mu_n) + \frac{1}{2} \left[\frac{\partial^2 J}{\partial \theta^2} \sigma_\theta^2 + \frac{\partial^2 J}{\partial \alpha^2} \sigma_\alpha^2 + \frac{\partial^2 J}{\partial n^2} \sigma_n^2 \right]$$

$$\sigma_J^2 = \left(\frac{\partial J}{\partial \theta} \right)^2 \sigma_\theta^2 + \left(\frac{\partial J}{\partial \alpha} \right)^2 \sigma_\alpha^2 + \left(\frac{\partial J}{\partial n} \right)^2 \sigma_n^2 \quad (9)$$

Similar calculations are done for J-Tearing analysis.

5. RESULTS

The initiation load M_i can be defined as the bending moment, which corresponds to the initiation of crack growth in a pipe. For crack growth initiation studies moment is varied from 0.5 MN-m to 3 MN-m. in steps of 0.1 MN-m. Converged results are obtained at 10^5 trials. For

For each of these trials the random variables are generated and assessment point is plotted on R6 FAD. The failure probability is calculated from the number of assessed points that fell in the unsafe region of the FAD. Figure 3 shows the variation of probability of failure based on crack growth initiation (R6 Option 2, Category 1) with respect to change in applied bending moment. This is shown in Figure 3. The results match well with those given by Rahman [4] and theoretical approximations. Rahman's [4] results are based on J-Tearing methodology.

The unstable load M_u can be defined as the bending moment, which corresponds to the unstable crack growth in a pipe. For unstable crack growth studies moment is varied from 1 MN-m to 3 MN-m. in steps of 0.1 MN-m. Converged results are obtained at 10^4 trials. For each of these trials the random variables are generated and assessment point is plotted on R6 FAD. The J-R curve is used to assess crack growth stability. Figure 4 shows the variation of probability of failure based on unstable crack growth (R6 Option 2, Category 3) with respect to change in applied bending moment. Figure 4 also shows the comparison of these results with those of Rahman [4] and those obtained by theoretical approximations. R6 predicts higher probabilities of failure than those obtained by Rahman. This may be partly due to conservatism in Option 2 of R6 over Option 3 R6. Option 3 of R6 is equivalent to J-Tearing analysis.

Figure 3: Comparison of results for initiation of crack growth

Figure 4: Comparison of results for unstable crack growth

6. REFERENCES

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