Simplified Estimation Method for First Excursion Probability of Secondary System with Gap

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ABSTRACT This paper deals with the first excursion probability of the secondary system with gap such as pipings, tanks and other mechanical equipment having mechanical supports and joints installed on the primary system such as buildings. An analytical method for the secondary system with gap is presented. This method is based on equivalent linearization method. As input excitations, artificial time histories are used. Using the proposed method, some numerical examples are shown. It is found that when the tolerance level is normalized by the maximum standard deviation of response of the linear secondary system, the first excursion probability is independent of mass ratio of the secondary system to the primary system, the damping ratio and the natural period of the secondary system.

INTRODUCTION

The important secondary system such as pipings, tanks and other mechanical equipment installed on the primary system such as building in industrial facilities should be designed so as to maintain their functions even if they are subjected to excess seismic loadings. Importance of probabilistic safety assessment of such secondary system has been pointed out. Some studies on probabilistic safety assessment have been carried out [1], [2]. Response of the secondary system is greatly amplified depending on dynamic characteristics of the primary system. It is well known that the secondary system has gap in mechanical joints and supports [3], [4].

Many studies on the first excursion probability have been carried out [5], [6]. Studies on nonlinear system is also carried out [7], [8]. However, main purposes of these studies are derivation of theoretical equations for obtaining the first excursion probability. And few studies on a practical estimation method of the first excursion probability have been carried out.

On the other hand, design response spectrum which is the maximum response of single-degree-of-freedom system is used for aseismic design [9]. For aseismic design of the secondary system, design method using modal analysis [10] or some factor [11] based on the design response spectrum is used.

In this paper, a simplified estimation method for the first excursion probability of the secondary system is proposed. For simplicity, the secondary system and the primary system are simulated by single-degree-of-freedom system respectively. First, an analytical method for the secondary system with gap is presented. This method is based on equivalent linearization method. As input excitations, two kinds of artificial time histories are used.

One is nonstationary filtered white noise and the other is artificial time histories compatible to the design response spectrum. Next, by using the proposed method, some numerical examples are shown. From the numerical examples, it is found that when the tolerance level is normalized by the maximum standard deviation of response of the linear secondary system, the first excursion probability is independent of the damping ratio and the natural period of the secondary system.

ANALYTICAL MODEL AND INPUT EXCITATION

An analytical model is shown in Fig.1. The secondary system and the primary system are simulated by single-degree-of-freedom system respectively for simplicity. It is assumed that the secondary system has gap. Equations of motion are

$$\left. \begin{aligned} \ddot{z}_s + 2h_s \omega_s \dot{z}_s + f = -\ddot{y} - \ddot{z}_p \\ \ddot{z}_p + 2h_p \omega_p \dot{z}_p + \omega_p^2 z_p - \gamma (2h_s \omega_s \dot{z}_s + f) = -\ddot{y} \end{aligned} \right\} \quad (1)$$

where z_s is relative displacement of the secondary system to the primary system ($x_s - x_p$), z_p is relative displacement of the primary system to the ground ($x_p - y$), h_s is the damping ratio of the secondary system, h_p is the damping ratio of the primary system, f is nonlinear restoring force caused by gap, ω_p is the natural circular frequency of the primary system and γ is mass ratio of the secondary system to the primary system. It is assumed that absolute value of relative displacement z_s of the secondary system to the primary system first crosses the tolerance level B_D .

As input excitations $\ddot{y}$, two kinds of artificial time histories are used. One is nonstationary filtered white noise. Fig.2 shows envelope function. The other is artificial time histories compatible to the design response spectrum. Fig.3 shows target response spectrum[12] and Fig.4 shows envelope function[13] which represents nonstationary amplitude characteristics of earthquake excitations.

THEORETICAL ESTIMATION METHOD

The first excursion probability at time t $P_f(t)$ is obtained by the following equation.

$$P_f(t) = 1 - \exp \left\{ -2 \int_0^t \nu(t) dt \right\} \quad (2)$$

It is assumed that instants at which $z_s(t)$ crosses B_D is statistically independent, $\nu(t)$ is given as:

$$\begin{aligned} \nu(t) = \frac{1}{2\pi} \frac{\sqrt{D}}{\sigma_{z_s}^2} \left[\exp \left\{ -\frac{B_D^2}{2\sigma_{z_s}^2} \left(\frac{1 + \kappa_{z_s} \dot{z}_s}{D} \right)^2 \right\} \right. \\ \left. + B_D \kappa_{z_s} \dot{z}_s \sqrt{\frac{\pi}{2D\sigma_{z_s}^2}} \exp \left(-\frac{B_D^2}{2\sigma_{z_s}^2} \right) \{1 + \text{erf}(C)\} \right] \end{aligned} \quad (3)$$

where,

$$C = \frac{B_D \kappa_{z_s} \dot{z}_s}{\sqrt{2D\sigma_{z_s}^2}}, \quad D = \sigma_{z_s}^2 \sigma_{\dot{z}_s}^2 - \kappa_{z_s} \dot{z}_s^2$$

$$\operatorname{erf}(u) = \frac{2}{\sqrt{\pi}} \int_0^u \exp(-y^2) dy \quad (4)$$

$\sigma_{z_s}^2$ is variance of relative displacement, $\sigma_{\dot{z}_s}^2$ is variance of relative velocity, $K_{z_s \dot{z}_s}$ is covariance of relative displacement and relative velocity.

For nonstationary filtered white noise, when input excitation to the base rock is white noise, power spectral density of ground motion is assumed to be given as:

$$G(\omega) = \frac{(2h_g \omega_g \omega)^2 + \omega_g^4}{(\omega^2 - \omega_g^2)^2 + (2h_g \omega_g \omega)^2} G_0 \quad (5)$$

where h_g and ω_g are the damping ratio and the predominant circular frequency of the ground model and G_0 is power spectral density function of input excitations to the base rock.

For nonstationary artificial time history, the expected value of power spectral density function $G(\omega)$ is given as Eq.(5) [14]. When target response spectrum shown in Fig.3 and envelope function shown in Fig.4 are used, $h_g=0.5$, $T_g(2\pi/\omega_g)=0.285s$ and $G_0=1.94 \times 10^{-3} (1/s)$ [14].

The Fokker-Planck equation for joint probability density p with respect to z_s , $\dot{z}_s$, z_p , $\dot{z}_p$, and relative displacement z_g of the ground to the base rock, relative velocity $\dot{z}_g$ is obtained as follows.

$$\begin{aligned} \frac{\partial p}{\partial t} = & \frac{\partial p}{\partial z_p} \dot{z}_p + 2h_p \omega_p p - \frac{\partial p}{\partial \dot{z}_p} \{-2h_p \omega_p \dot{z}_p - \omega_p^2 z_p + \gamma(2h_s \omega_s \dot{z}_s + f) \\ & + I(t)(2h_g \omega_g \dot{z}_g + \omega_g^2 z_g)\} - \frac{\partial p}{\partial z_s} \dot{z}_s + 2h_s \omega_s (1+\gamma)p \\ & - \frac{\partial p}{\partial \dot{z}_s} \{-2h_s \omega_s (1+\gamma)\dot{z}_s - (1+\gamma)f + 2h_p \omega_p \dot{z}_p + \omega_p^2 z_p\} \\ & - \frac{\partial p}{\partial z_g} \dot{z}_g + 2h_g \omega_g p + \frac{\partial p}{\partial \dot{z}_g} (2h_g \omega_g \dot{z}_g + \omega_g^2 z_g) + \frac{\partial^2 p}{\partial \dot{z}_g^2} \frac{\pi G_0}{2} \end{aligned} \quad (6)$$

In order to use Eq.(2), second moments with respect to z_s and $\dot{z}_s$ are to be obtained. Applying the partial integral method to Eq.(6), moment equations with respect to z_s , $\dot{z}_s$, z_p , $\dot{z}_p$, z_g and $\dot{z}_g$ are obtained. Moment equations consist of 21 differential equations of the first order [14]. P_f is obtained by solving moment equations and using Eq.(3) and Eq.(2).

NONLINEAR CHARACTERISTICS

As nonlinear characteristics, bilinear restoring force-deformation relation shown in Fig.5 is used. f in Eq.(6) is equivalently linearized as follows.

$$f = 2h_c \omega_c \dot{z}_s + \omega_c^2 z_s \quad (7)$$

When nonlinear effect is not so great, it is assumed that the effect of gap is great near the main shock. It is also assumed that $z_s(t)$ near the main shock is stationary random process and its distribution of amplitude is the Rayleigh distribution. Hence, h_c and ω_c are obtained by stationary random process theory.

$$\left. \begin{aligned} h_c &= 0 \\ \omega_c^2 &= \omega_s^2 + b \omega_s^2 \exp(-\mu^{-1}) - b \omega_s^2 \sqrt{\pi} \mu^{-1} \operatorname{erfc}(\mu^{-1}) \end{aligned} \right\} \quad (8)$$

where

$$\begin{aligned} \mu &= 2 \sigma_{zs} / d, \\ \operatorname{erfc}(u) &= 1 - \frac{2}{\sqrt{\pi}} \int_0^u \exp(-v^2) dv \end{aligned} \quad (9)$$

and ω_s is the natural circular frequency of the secondary system for elastic range.

For nonlinear characteristics as shown in Fig.5, clearance d and stiffness ratio of impacted part to the secondary system b should be determined. d is normalized by the maximum standard deviation of the secondary system without nonlinear characteristics $|\sigma_{s1}|_{\max}$ as follows.

$$d^* = d / |\sigma_{s1}|_{\max} \quad (10)$$

The tolerance level B_D is normalized by $|\sigma_{s1}|_{\max}$ as follows.

$$\delta_t = B_D / |\sigma_{s1}|_{\max} \quad (11)$$

RESULTS OF FIRST EXCURSION PROBABILITY

From Eq. (2), P_f is function of time t . Important secondary system should maintain their function after earthquake excitation decays. Hence, P_f at the end of earthquake excitation is obtained.

Since many parameters should be determined, some values of parameters are fixed. The damping ratio of the primary system is 0.05. P_f for the case where the natural period of the secondary system $T_s (2\pi/\omega_s)$ coincides with that of the primary system $T_p (2\pi/\omega_p)$ is obtained.

From Table 1 to Table 6, results for nonstationary filtered white noise excitations are shown. Table 1, Table 2 and Table 3 show P_f for $b=5$ and $d^*=1.0$. Table 1 shows P_f for some values of mass ratio γ . Table 2 shows P_f for some values of the damping ratio of the secondary system h_s . Table 3 shows P_f for some values of the natural period of the secondary system T_s and the primary system T_p . From Table 1, P_f becomes large when γ increases. However, variation of P_f is not so great when $\gamma > 0.01$. From Table 2, P_f becomes large when h_s increases. However, variation of P_f is not so great when $h_s < 0.05$. In actual system, the condition $h_s < 0.05$ is usually satisfied. From Table 3, P_f becomes large when T_s and T_p becomes long. However, variation of P_f is not so great.

Table 4, Table 5 and Table 6 show P_f for $b=10$ and $d^*=1.0$. Table 4 shows P_f for some values of γ . Table 5 shows P_f for some values of h_s . Table 6 shows P_f for some values of T_s and T_p . From Table 4, variation of P_f is not so great when $\gamma > 0.01$. From Table 5, variation of P_f is not so great when $h_s < 0.05$. From Table 6, variation of P_f is not so great for different value of T_s and T_p .

From Table 7 to Table 9, results for artificial time history excitations are shown. Table 7, Table 8 and Table 9 show P_f for $b=5$ and $d^*=1.0$. Table 7 shows P_f for some values of γ . Table 8 shows P_f for some values of h_s . Table 9 shows P_f for some values of T_s and T_p . From Table 7, P_f becomes large when γ increases. However, variation of P_f is not so great when $\gamma > 0.01$. From Table 8, P_f becomes large when h_s increases.

However, variation of P_f is not so great when $h_s < 0.05$. From Table 9, P_f becomes large when T_s and T_p becomes long. However, variation of P_f is not so great.

From these tables, when the tolerance level is normalized by the maximum standard deviation of the secondary system without nonlinear characteristics as Eq.(11), the first excursion probability of the secondary system becomes large as mass ratio and the damping Ratio of the secondary system increases and the natural period becomes long. However, mass ratio is greater than 0.01 or the damping ratio is less than 0.05, variation of the first excursion probability is not so great.

CONCLUSIONS

A simplified estimation method for the first excursion probability of the secondary system with gap is examined. When the tolerance level is normalized by the maximum standard deviation of the secondary system without nonlinear characteristics as Eq.(11), variation of the first excursion probability is not so great independent of mass ratio, the damping ratio and the natural period of the secondary system and the primary system.

REFERENCES

1. Lin, Y.K., *Probabilistic Theory of Structural Dynamics*, Krieger, 1967.
2. Sundararajan, C., "Probabilistic Assessment of Pressure Vessel and Piping Reliability," *Trans. ASME, Journal of Pressure Vessel Technology*, Vol.108, No.1, 1986, pp.1-13.
3. Aoki, S. and Suzuki, K., "Dynamic Response Reduction Effect of the Piping due to Gap and Friction," *ASME Application of Modal Analysis to Extreme Loads*, PVP-Vol.150, 1987, pp.17-21.
4. Cross, W.B. and Jones, N.P., "Seismic Performance of Joint-Pocket Connections," *Proc. ASCE, Journal of Structural Engineering*, Vol.119, No.10, 1993, pp.2986-3007.
5. Yang, J.B., "Approximation to First Passage Probability," *Proc. ASCE, Journal of Engineering Mechanics*, Vol.99, No.8, 1975, pp.361-372.
6. Lennox, W.C. and Fraser, D.A., "On the First-Passage Distribution for the Envelope of a Nonstationary Narrow-Band Stochastic Process," *Trans. ASME, Journal of Applied Mechanics*, Vol.41, No.3, 1974, pp.793-797.
7. Ang, A.H-S., "Probabilistic Seismic Safety and Damage Assessment of Structures," *Proc. of 9WCEE*, Vol. VIII, 1988, pp.717-728.
8. Lin, Y.K. and Cai, Q.C., *Probabilistic Structural Dynamics*, MacGraw-Hill, 1995.
9. Clough, R.A. and Penzien, J., *Dynamics of Structures*, MacGraw-Hill, 1975.
10. Gupta, A.J., *Response Spectrum Method*, Blackwell, 1990.
11. Aoki, S. and Suzuki, K., "Conventional Earthquake Response Estimation Technique for Mechanical Appendage Structure System (Proposition of Floor Response Amplification Factor)," *Bulletin of the JSME*, Vol.25, No.204, 1982, pp.969-976.
12. Japanese Ministry of International Trade and Industry, *Guidelines for Aseismic Design of High Pressure Gas Facilities*, 1983.
13. Jennings, P.C., Housner, G.W. and Tsai, N.S., "Simulated Earthquake Motion Generation," EERL, California Institute of Technology, 1968.
14. Aoki, S. and Suzuki, K., "First Excursion Probability Estimation of Mechanical Appendage System Subjected to Nonstationary Earthquake Excitations," *Proceedings of 4th International Conference on Structural Safety and Reliability*, Vol. I, 1985, pp.201-210.

Fig.1 Analytical model of nonlinear system

Fig.2 Envelope function for nonstationary filtered white noise

Fig.3 Target response spectrum for 5% damping ratio of critical

Fig.4 Envelope function for artificial time history

Fig.5 Bilinear restoring force-deformation relation

Table 1 First excursion probability

($h_s=0.01$, $T_s=0.5s$, $h_p=0.05$, $T_p=0.5s$, $h_g=0.4$, $T_g=0.5s$, $b=5$, $d^*=1.0$)

δ_t	γ				
	0	0.01	0.02	0.05	0.1
1.0	0.997	0.998	0.999	0.999	0.999
1.5	0.442	0.583	0.639	0.691	0.718
2.0	2.98×10^{-2}	6.16×10^{-2}	7.91×10^{-2}	0.102	0.117
2.5	7.95×10^{-4}	2.41×10^{-3}	3.58×10^{-3}	5.48×10^{-3}	7.06×10^{-3}

Table 2 First excursion probability

($\gamma=0$, $T_s=0.5s$, $h_p=0.05$, $T_p=0.5s$, $h_g=0.4$, $T_g=0.5s$, $b=5$, $d^*=1.0$)

δ_t	h_s			
	0.01	0.02	0.05	0.1
1.0	0.997	0.996	0.998	0.999
1.5	0.442	0.455	0.562	0.704
2.0	2.98×10^{-2}	3.30×10^{-2}	5.62×10^{-2}	0.109
2.5	7.95×10^{-4}	9.41×10^{-4}	0	5.81×10^{-3}

Table 3 First excursion probability

($\gamma=0$, $h_p=0.05$, $T_s=T_p$, $h_g=0.4$, $T_g=0.5s$, $b=5$, $d^*=1.0$)

δ_t	$T_s=T_p$			
	0.3s	0.5s	0.8s	1.0s
1.0	0.998	0.997	0.985	0.973
1.5	0.467	0.442	0.384	0.347
2.0	2.76×10^{-2}	2.98×10^{-2}	3.02×10^{-2}	2.81×10^{-2}
2.5	5.55×10^{-4}	7.95×10^{-4}	1.09×10^{-3}	1.11×10^{-3}

Table 4 First excursion probability

($h_s=0.01$, $T_s=0.5s$, $h_p=0.05$, $T_p=0.5s$, $h_g=0.4$, $T_g=0.5s$, $b=10$, $d^*=1.0$)

δ_t	γ				
	0	0.01	0.02	0.05	0.1
1.0	0.978	0.990	0.993	0.996	0.997
1.5	0.213	0.330	0.384	0.445	0.481
2.0	6.27×10^{-3}	1.55×10^{-2}	2.13×10^{-2}	3.01×10^{-2}	3.70×10^{-2}

Table 5 First excursion probability

($\gamma=0$, $T_s=0.5s$, $h_p=0.05$, $T_p=0.5s$, $h_g=0.4$, $T_g=0.5s$, $b=10$, $d^*=1.0$)

δ_t	h_s			
	0.01	0.02	0.05	0.1
1.0	0.978	0.978	0.988	0.995
1.5	0.213	0.226	0.307	0.440
2.0	6.27×10^{-3}	7.16×10^{-3}	1.33×10^{-2}	2.96×10^{-2}

Table 6 First excursion probability

($\gamma=0$, $h_p=0.05$, $T_s=T_p$, $h_g=0.4$, $T_g=0.5s$, $b=10$, $d^*=1.0$)

δ_t	$T_s=T_p$			
	0.3s	0.5s	0.8s	1.0s
1.0	0.996	0.978	0.943	0.913
1.5	0.253	0.213	0.185	0.164
2.0	5.94×10^{-3}	6.27×10^{-3}	6.78×10^{-3}	6.39×10^{-3}

Table 7 First excursion probability

($h_s=0.01$, $T_s=0.5s$, $h_p=0.05$, $T_p=0.5s$, $h_g=0.5$, $T_g=0.285s$, $b=5$, $d^*=1.0$)

δ_t	γ				
	0	0.01	0.02	0.05	0.1
1.0	0.999	1.000	1.000	1.000	1.000
1.5	0.555	0.808	0.859	0.899	0.912
2.0	4.11×10^{-2}	0.136	0.179	0.229	0.254
2.5	1.04×10^{-3}	6.91×10^{-3}	1.09×10^{-2}	1.68×10^{-2}	2.04×10^{-2}

Table 8 First excursion probability

($\gamma=0$, $T_s=0.5s$, $h_p=0.05$, $T_p=0.5s$, $h_g=0.5$, $T_g=0.285s$, $b=5$, $d^*=1.0$)

δ_t	h_s			
	0.01	0.02	0.05	0.1
1.0	0.999	1.000	1.000	1.000
1.5	0.555	0.606	0.752	0.880
2.0	4.11×10^{-2}	5.30×10^{-2}	1.05×10^{-1}	0.207
2.5	1.04×10^{-3}	1.57×10^{-3}	4.57×10^{-3}	1.43×10^{-2}

Table 9 First excursion probability

($\gamma=0$, $h_p=0.05$, $T_s=T_p$, $h_g=0.5$, $T_g=0.285s$, $b=5$, $d^*=1.0$)

δ_t	$T_s=T_p$			
	0.3s	0.5s	0.8s	1.0s
1.0	1.000	0.999	0.992	0.981
1.5	0.691	0.606	0.427	0.380
2.0	5.30×10^{-2}	5.30×10^{-2}	3.09×10^{-2}	2.82×10^{-2}
2.5	1.14×10^{-3}	1.57×10^{-3}	8.80×10^{-4}	0