ABSTRACT

GIANIKOS, ZACHARY NILE. Limit Cycle Characterization of an Aeroelastic Wing in a Bluff Body Wake. (Under the direction of Dr. Matthew Bryant).

This paper presents an experimental investigation aimed at characterizing the kinematics of a pitching-heaving aeroelastic wing placed downstream of a rectangular bluff body. The influence of the bluff body wake on the wing is two fold: a viscous wake which produces a velocity deficit downstream and an oscillating induced velocity field due to periodic vortex shedding. The latter effect is the focus of this paper, specifically, the interaction between the wake frequency and the wing limit cycle oscillation (LCO) frequency. Wind tunnel experiments showed that the presence of the upstream bluff body causes modulation of the LCO amplitude. The modulation resembles a beat phenomenon, however the modulation frequency is related to the third harmonic frequency of the LCO oscillation rather than the fundamental frequency. The modulation behavior also differs from that of a beat in that the spectral content contains sideband frequencies, characteristic of a multiplication between a carrier wave and a modulation wave rather than a simple sinusoidal superposition. Additionally, the streamwise spacing between the bluff body and the wing significantly influences the wing kinematics, with a closer spacing between the two bodies increasing the intensity of the amplitude modulation. For shedding frequencies sufficiently close to the LCO third harmonic, reducing this streamwise distance is shown to induce an alternation between two distinct modes of amplitude modulation, each with its own intensity and frequency.

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Limit Cycle Characterization of an Aeroelastic Wing in a Bluff Body Wake

by
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A thesis submitted to the Graduate Faculty of
North Carolina State University
in partial fulfillment of the
requirements for the Degree of
Master of Science

Aerospace Engineering

Raleigh, North Carolina

2019

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ACKNOWLEDGEMENTS

I would like to gratefully acknowledge funding support from by the Air Force Office of Scientific Research under award number FA9550-17-1-0301, monitored by Dr. Gregg Abate as well as additional support from the National Science Foundation under Award No. ECCS – 1509592 and program officer Radhakisan S. Baheti.

Throughout this process I have been extremely lucky to receive personal and academic support from professors, friends, family, and fellow students alike. I consider the teamwork mentality of the iSSRL to be a significant factor in my research success over the past two years. I felt at home from day one and no matter how many questions I had (of which there were plenty), the team was always more than happy to share their knowledge. Finally I would like to thank Dr. Bryant and Dr. Gopalarathnam in particular for their guidance and receptivity during dozens of project meetings and office visits.

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LIST OF SYMBOLS

β	fitting parameter for softening pitch stiffness model
$\phi_{\theta,h}$	pitch-heave phase difference [$^{\circ}$]
ρ	freestream air density [kg/m^3]
θ, θ_A	pitch displacement and pitch amplitude, respectively [$^{\circ}$]
AR	wing aspect ratio
b	airfoil semi-chord length [m]
c	airfoil chord length [m]
C_1, C_2, C_3	fitting parameters for softening pitch stiffness model
c_h, c_p	viscous damping coefficient for heave and pitch, respectively [kg/s]
C_L, C_m	lift and pitching moment coefficients, respectively
D	bluff body characteristic length (short side) [m]
E, T, U	total, kinetic, and potential energy, respectively [J]
F_f	force due to kinetic friction in the heave degree of freedom (DOF) [N]
f_{LCO}	fundamental frequency of limit cycle oscillation [Hz]
f_{shed}	bluff body shedding frequency [Hz]
h, h_A	heave displacement and heave amplitude, respectively [m]
I_{θ}	pitching inertia about elastic axis [kg m^2]

K_h	effective heave stiffness [N/m]
$K_{\theta_A}, K_{\theta_B}, K_{\theta_C}$	effective pitch stiffness for linear, hardening, and softening regimes, respectively [Nm/rad]
M_f	moment due to kinetic friction in the pitch DOF [Nm]
m_W, m_T	wing mass (rotating components), and total mass (all moving components), respectively [kg]
M_θ	restoring moment due to structural pitch stiffness
P	power [W]
S	wing planform area [m ²]
St	Strouhal number
t	time [s]
U_∞	freestream wind speed [m/s]
W_A, W_D	net work done by aerodynamics and work done by structural damping, respectively [J]
X_b	streamwise distance between bluff body trailing edge (TE) and wing leading edge (LE) [m]
x_p	non-dimensional pitching axis location (chord length fraction from LE)
x_θ	non-dimensional distance (by semi-chord) from pitching axis to rotational center of mass (COM) [kg · m ²]

Introduction

Unsteady wing-wake interaction research has undergone profound growth in recent years as researchers utilize knowledge of wake impingement to develop new multi-vehicle flight formations, design novel micro-air vehicles [26], and leverage inspirations from biological swimmers and flyers [13]. Examples of such interactions include close formation flight [2], wake-induced flutter [5, 23, 12], aerial refueling [3], and fish schooling dynamics [21]. Although diverse in applications, the fundamental thread through this area of research is the characterization of interactions between unsteady wakes and lifting bodies. Given the ubiquity of wake impingement in real systems, these interactions are pertinent in the characterization of limit cycle oscillations (LCO) of nonlinear aeroelastic systems.

Previous experimental and computational studies have addressed the characteristics and mechanisms of aeroelastic LCO for undisturbed flow conditions [22, 19, 6]. The effect of upstream flow disturbances on the kinematic behavior of wings undergoing LCO, however, has received less attention. In studies of nonlinear aeroelastic systems, researchers found that the wing response dynamics depend heavily on the inflow conditions. Poirel et al. [20] found that longitudinally turbulent inflow can decrease the stability of the system by reducing the flutter

speed to a point lower than that for the non-excited case. In addition to reducing the flutter speed, the damping is also diminished for airspeed below the flutter point. In a numerical study of an aeroelastic airfoil with varying inflow, Venkatramani et al. [24] demonstrated intermittency, a phenomenon generally discussed in the context of nonlinear dynamical systems [8, 7]. Intermittency is characterized by an alternation between qualitatively different states, usually in the vicinity of a stability boundary. Venkatramani et al. [24] showed that fluctuations of the flow velocity over large time scales produced an "on-off" intermittency in which the system alternated between states of stability and instability about the flutter point. However, for shorter period velocity fluctuations, the system underwent amplitude-modulated oscillations without fully regaining stability.

The previously mentioned studies [20, 24] have considered effects of either isotropic turbulence or purely longitudinal freestream velocity fluctuations. Although useful conceptualizations, these conditions are rarely encountered outside of wind tunnel experiments or numerical simulations. Several studies in past decades have examined the effect of impinging Kármán vortex streets on downstream airfoils undergoing prescribed motion. These flow structures produce relatively high anisotropy and large-scale unsteadiness and therefore, are analogous to scenarios where the flow is disturbed by an upstream structure. One such study comes from Chen and Choa [4], who examined the effect of a wake shed from a cylindrical rod upstream of a NACA 0012 airfoil pitching upward at a constant rate and performed measurements of the unsteady forces acting on the airfoil. Chen and Choa [4] reported that the peak suction pressure at the leading edge of the airfoil with the cylinder placed upstream was increased by as much as 50% compared to the undisturbed case and that the peak suction was reached at a larger angle of attack. They attributed this effect to the upstream disturbance acting as a vortex generator that energizes the boundary layer, causing earlier turbulent transition and delayed separation of the dynamic stall vortex (DSV). Merrill and Peet [17] conducted a similar investigation re-

garding the effect of an impinging wake from a bluff body on the dynamic stall characteristics of a sinusoidally pitching airfoil using direct numerical simulation. They concluded that the impinging wake largely affects the forces and moments acting on the airfoil by altering the formation of vortex structures. For the conditions investigated, Merrill and Peet [17] concluded that dynamic stall for a pitching airfoil occurs later in the pitching cycle when in the presence of a turbulent wake due to the delayed formation and detachment of the DSV. Both of these studies were successful in identifying aerodynamic mechanisms of the impinging wake structures; however, since the airfoils in both cases underwent prescribed motion, they do not reveal effects on the aeroelastic kinematics and dynamics.

The aim of this paper is to investigate the kinematic response of an aeroelastic wing system undergoing LCO in the presence of an impinging bluff body wake. Experiments are first conducted without the presence of an upstream bluff body to establish the baseline case. Then, tests with the bluff body positioned upstream are performed, varying two parameters: the bluff body shedding frequency, which is varied implicitly via the freestream velocity, and the streamwise distance between the bluff body and the wing.

Experimental Methods

2.1 Aeroelastic Apparatus Design

A two-degree-of-freedom aeroelastic apparatus was constructed to investigate the effects of wing-wake interaction phenomena (Fig. 1). The wing is a 3D printed symmetric version of the SD 7003 with a chord length (c) of 150mm and aspect ratio (AR) of 4. Based on the results of Visbal and Garmann [25], endplates are attached to the tips of each wing to emulate 2D flow conditions. The endplates extend one chord length forward and aft of the wing section. The pitching axis was located at the mid-chord position for the experiments discussed in this paper. The aeroelastic system is supported within the wind tunnel test section by an external box structure which acts as a mechanical ground, isolated from the wind tunnel. The wing is supported by upper and lower PBC Linear guide rails and carriages (Fig. 2.1). Linear extension springs provide elastic restoring force and moment for both the heave and pitch degrees of freedom. A set of springs are attached to a cable-pulley system to create a restoring moment for the pitch degree of freedom. The spring stiffness properties are characterized later in Section 3.

Figure 2.1 CAD model of aeroelastic apparatus

Table 2.1 provides the mass and geometric system properties. These values were obtained via direct measurement except for the center of mass (COM) and moment of inertia (MOI) which were estimated via SolidWorks software.

Table 2.1 Mass and dimensional properties

Parameter	Description	Value
AR	aspect ratio	4
c	chord length [m]	0.15
I_θ	pitching inertia about elastic axis [$\text{kg} \cdot \text{m}^2$]	5.32e-03
m_T	mass of all moving parts [kg]	3.27
m_W	mass of all rotating parts [kg]	1.61
x_p	non-dimensional pitching axis location (chord length fraction from leading edge)	0.5
x_θ	non-dimensional distance from pitching axis to rotational COM	0.062

2.2 Upstream Bluff Body

During wing-wake interaction experiments an upstream rectangular cylinder was used to create the wake. The streamwise length of the bluff body is 152 mm and the width (D) is 76 mm. Okajima [18] found that the Strouhal number (St) for a rectangular cylinder with length to width ratio of 2 is approximately 0.085 for the Reynolds numbers tested herein. This allows for shedding frequencies closer to the natural frequencies of the aeroelastic system than for a circular cylinder where $St \approx 0.2$. The bluff body was constructed from laser cut birch plywood and attached to an ATI Gamma six-degree-of-freedom load cell (not shown) to measure shedding the frequency (Fig. 2.2).

Figure 2.2 Photograph of aeroelastic apparatus shown with bluff body mounted upstream

For this set of experiments, the bluff body was positioned at three different locations relative to the wing. A diagram of the test section layout is shown in Fig. 2.3. Table 2.2 defines the three bluff body positions tested.

Figure 2.3 Schematic of bluff body spatial configuration (wing deflections not to scale)

Table 2.2 Bluff body spatial configurations

Configuration	X_b [m]	X_b/D
1	N/A	N/A
2	0.51	6.5
3	0.43	5.6
4	0.33	4.3

2.3 Wind Tunnel and Data Collection Methods

The pitch and heave kinematics are measured via a National Instruments NI PXIe-6363 I/O module using a LabVIEW based data acquisition system with a nominal sampling rate of 50 kHz. Two U.S. Digital E6-10000 encoders with 0.07-degree resolution measure the wing's angular position while two Renishaw LM10 magnetic linear encoders with 0.03 mm resolution measure the heave position. Aeroelastic experiments were conducted in the North Carolina State University closed-return subsonic wind tunnel with a 0.81 m x 1.14 m x 1.17 m test section. The freestream velocity for LCO experiments ranged from 6 m/s to 12 m/s, which corresponds to an airfoil chord based Reynolds number range of 70×10^3 to 120×10^3 . The freestream windspeed (U_∞) was monitored via a dynamic pressure transducer. For each test case the wing was given initial deflections of 45 degrees, increasing the dynamic pressure in increments of 0.1 psf until self-sustained oscillations began. Once LCO was established, the wing was allowed to oscillate for one minute to avoid measuring transient behavior. Next, the wing position data was recorded for two minutes and this process was repeated for increasing dynamic pressure.

System Characterization

The aeroelastic apparatus is modeled as a coupled two degree-of-freedom mass-spring-damper system. The equations of motion were derived previously by Kirschmeier et al. [10] and are reproduced in equations (3.1) and (3.2).

$$m_T \ddot{h} + m_W b x_\theta \dot{\theta}^2 \sin(\theta) - m_W b x_\theta \ddot{\theta} \cos(\theta) + k_h h + c_h \dot{h} + F_f \text{sign}(\dot{h}) = C_L \frac{1}{2} \rho U_\infty^2 c S \quad (3.1)$$

$$I_\theta \ddot{\theta} - m_W b x_\theta \cos(\theta) \ddot{h} + M_\theta(\theta) + c_p \dot{\theta} + M_f \text{sign}(\dot{\theta}) = C_m \frac{1}{2} \rho U_\infty^2 c^2 S \quad (3.2)$$

Initial characterization of the effective stiffness in the pitch and heave degrees-of-freedom were obtained via direct measurements from the ATI Gamma 6 axis load cell and Instron 4400R, respectively. The heave spring stiffness was measured to be constant over the range of the range of heave deflections in the present experiments. The restoring moment response of the pitch DOF showed three distinct stiffness regions, labeled A, B, and C in Fig. 3.1. The transition from region A to C is caused by one side of the pulley losing tension for large deflections, reducing the effective stiffness by half. There are two transition angles marking the beginning of regions B and C. These transition angles for the present system are 60 deg and 64 deg,

respectively and can be prescribed by fixing the amount of pre-tension in the pitch extension springs. Wind-off free decay tests were also used to characterize the pitch nonlinearity. During pitch-only and coupled pitch-heave free decay tests, a shift in the oscillation frequency was observed at small pitch angles (≤ 6 deg). A power-law softening model was incorporated into the piecewise stiffness model to capture this. Based on these observations, the form of the structural pitching moment equation is defined as:

$$M_{\theta}(\theta) = \begin{cases} K_{\theta_A}(\theta)\theta & \text{if } \theta < \theta_{P_1} \\ \text{where } K_{\theta_A}(\theta) = C_1\theta^{2\beta} + C_2\theta^{\beta} + C_3 & \\ K_{\theta_B}(\theta_{T_1} + (\theta - \theta_{P_1})) & \text{if } \theta_{P_1} \leq \theta < \theta_{P_2} \\ K_{\theta_C}(\theta_{T_2} + (\theta - \theta_{P_2})) & \text{if } \theta_{P_2} \leq \theta_p \\ \text{where } K_{\theta_C}(\theta) = \frac{K_{\theta_A}(\theta_{P_1})}{2} & \end{cases} \quad (3.3)$$

where $\theta_{P_{1,2}}$ are the transition angles and $\theta_{T_{1,2}}$ are the modified transition angles which continuity in the stiffness transition regions. The modified transition angles are defined as:

$$\theta_{T_1} = \frac{K_{\theta_A}(\theta_{P_1})\theta_{P_1}}{K_{\theta_B}}; \quad (3.4)$$

$$\theta_{T_2} = \frac{K_{\theta_B}(\theta_{T_1} + \theta_{P_2} - \theta_{P_1})}{K_{\theta_C}}; \quad (3.5)$$

Figure 3.1 shows the spring torque responses compared to the pitch stiffness model described in Eq. 3.3. The piecewise model matches the experimental response well and captures the three stiffness regions: A, B, and C.

Figure 3.1 Spring torque measurements with load cell compared to Eqn. 3.3

Appropriate values for the fitting terms in Eq. 3.3 as well as the stiffness and damping terms in Eqs. 3.1 and 3.2 were obtained in MATLAB using the Delayed Rejection Adaptive Metropolis (DRAM) Markov chain Monte Carlo (MCMC) algorithm as described by Kirschmeier et al. [11]. These parameters are provided below in Table 3.1.

Table 3.1 Structural stiffness and damping properties

Parameter	Description	Value
β	fitting parameter for softening model	-4.35×10^{-1}
C_1	fitting parameter for softening model	6.86×10^{-4}
C_2	fitting parameter for softening model	9.74×10^{-2}
C_3	fitting parameter for softening model	3.47
c_h [kg/s]	viscous damping coefficient for heave DOF	1.73
c_p [kg/s]	viscous damping coefficient for pitch DOF	3.36×10^{-3}
F_f [N]	force due to kinetic friction in the heave DOF	1.02
M_f [Nm]	moment due to kinetic friction in the heave DOF	8.19×10^{-3}
k_h [N/m]	effective stiffness in heave DOF	2.17×10^3
K_{θ_H} [Nm/rad]	pitch stiffness for hardening regime	5.92

Results and Discussion

4.1 LCO Response with Steady Inflow

The LCO response was first characterized without the presence of the upstream bluff body. Figure 4.1(a) and 4.1(b) show the mean LCO amplitude vs windspeed for heave and pitch, respectively. For each DOF, the mean amplitude is defined by taking the mean of the set of upper peaks from the corresponding position signal.

Figure 4.1 (a) Non-dimensional mean heave amplitude and (b) mean pitch amplitude vs windspeed for no-bluff-body case. Error bars represent one standard deviation.

From Fig. 4.1, it can be seen that the LCO amplitude increases with windspeed with small error bars indicating nearly constant amplitude. Fig. 4.1(b) shows a distinct inflection point which corresponds to the transition from the linear to the softening nonlinear pitch stiffness regime. From Fig. 3.1 we see that this hardening transition region exists when the pitch amplitude is between 60 and 65 degrees. Fig. 4.1(a) contains this inflection point as well, reflecting the coupling between the two DOF. The trends seen in Fig. 4.1 are also reflected in the pitch angle time histories (Fig. 4.2). Note that the trends in the pitch and heave responses are similar and therefore, the remainder of the analysis in this paper will focus primarily on the response of the pitch DOF.

Figure 4.2 Pitch angle time histories for no-bluff-body case.

As shown in Fig. 4.3(a), the fundamental LCO frequency is approximately equal to 4 Hz and is invariant with windspeed within 2%. The dominant frequency for each windspeed was determined via FFT with a frequency resolution of $\Delta f = 1/N\Delta t = 0.017\text{Hz}$ where $N\Delta t$ is the total sampling time of 60 seconds. The frequency curves for the pitch and heave degrees of freedom lie on top of each other which is characteristic of LCO initiated by a modal convergence flutter instability. The average phase difference between pitch and heave position ($\bar{\phi}_{\theta,h}$) was calculated using cross correlation and is shown in Fig. 4.3(b). We see that for this configuration, $\bar{\phi}_{\theta,h}$ trends from 15 degrees (pitch-leading) to 0 degrees (in-phase).

(a)

(b)

Figure 4.3 (a) Fundamental LCO frequency for pitch and heave vs windspeed and (b) mean pitch-heave phase difference $\bar{\phi}_{\theta,h}$ vs windspeed for no-bluff-body case.

Exploring the frequency content of these responses via their power spectral densities (PSD) (Fig. 4.4) reveals that in addition to the fundamental frequency peaks, there exist both even and odd superharmonics which are characteristic of nonlinear oscillations. For increasing wind-speed, the fundamental frequency of oscillation remains constant.

Figure 4.4 Pitch angle PSD plotted vs windspeed for no-bluff-body case (PSD magnitude = arbitrary log scale)

4.2 LCO Response with Disturbed Inflow

4.2.1 Amplitude Modulation and LCO Superharmonic Frequency Analysis

It was shown previously that nonlinearities are introduced to the system through the pitch stiffness (Eq. 3.3), nonlinear equations of motion (Eqs. 3.1, 3.2), and the aerodynamics. While the system response in clean inflow conditions showed classical constant-amplitude LCO (Fig. 4.2), nonlinearities can result in more complex dynamics when a system is introduced to external disturbances. For example, a one DOF linear system exhibits a single resonance when the forcing frequency matches the natural frequency of the system, while in a nonlinear system, subharmonic and superharmonic resonances can exist [16].

For the present system, the bluff body shedding frequency is close three times the fundamental LCO frequency. In order to investigate the effects of wing-wake frequency interaction, the ratio f_{shed}/f_{LCO} is defined as the independent variable for each test case. Figure 4.5 presents pitch angle time histories, spanning the range of windspeeds tested, for $X_b/D = 4.3$. For windspeeds corresponding to $f_{shed}/f_{LCO} \leq 2.9$, the deviation in LCO amplitude is relatively small compared to the mean amplitude. However, as the bluff body shedding frequency migrates closer to $3f_{LCO}$, the intensity of the amplitude modulation increases. As the wind-speed is increased further, the shedding frequency surpasses $3f_{LCO}$ and for $f_{shed}/f_{LCO} > 3.1$, the modulation intensity diminishes again. This symmetry about $f_{shed}/f_{LCO} = 3$ indicates that the bluff body shedding frequency, rather than the freestream windspeed, dominates the LCO response in this regime.

Figure 4.5 Pitch angle time histories for $X_b/D = 4.3$ case.

The frequency content for a set of responses with the bluff body positioned at $X_b/D = 4.3$ is provided in Fig. 4.6. Similar to the PSDs for the no-bluff-body case in Fig. 4.4, the peaks representing the fundamental and superharmonic frequencies are present, however, with the

addition of a peak corresponding to the bluff body shedding frequency. The shedding frequency increases linearly from 9.5 Hz to 13.5 Hz with windspeed, following the expected trend for a Strouhal number of 0.085 [18]. The shedding frequency that appears in the PSD also matches the dominant frequency in the PSD of the bluff body load cell data. Additionally, there are two sideband peaks which are symmetric about the fundamental frequency. These sidebands are characteristic of amplitude-modulated signals and here, the frequency difference between the fundamental frequency and one of the sideband peaks is equal to the modulation frequency.

Figure 4.6 Pitch angle PSD plotted vs windspeed for $X_b/D = 4.3$ case (PSD magnitude = arbitrary log scale)

4.2.2 Quantification of LCO Amplitude and Amplitude Modulation Intensity

Figure 4.5, qualitatively illustrates the system's windspeed-dependent (and therefore shedding frequency-dependent) response. To quantify this behavior, Fig. 4.7 presents amplitude distri-

bution plots in which the set of peak pitch displacements, i.e. the pitch amplitudes, for each windspeed are overlaid. For the bluff body cases (Figs. 4.7(b), 4.7(c), 4.7(d)) there is an additional horizontal axis with the corresponding f_{shed}/f_{LCO} . Note that the darker shaded regions correspond to a tighter grouping of amplitudes while the lighter regions represent more disperse amplitudes. Fig. 4.7(a) shows the no-bluff-body amplitude response in which the pitch amplitude increases with windspeed with tight groupings indicating a constant pitch amplitude. Figure 4.7(b), contains the pitch amplitudes for the case where $X_b/D = 6.5$. Here, the trend resembles Fig. 4.7(a) for sufficiently low values of f_{shed}/f_{LCO} ; i.e., the mean amplitude increases with windspeed and the individual cycle amplitudes are tightly grouped. However, this trend is interrupted as the bluff body shedding frequency nears $3f_{LCO}$. For values of f_{shed}/f_{LCO} close to three, the dispersion in pitch amplitude increases significantly. A similar behavior is exhibited by the case shown in Fig. 4.7(c) in which $X_b/D = 5.6$; however, here there is a stronger influence from the bluff body, causing the cycle amplitude to drop below 20 degrees at its minimum. Figure 4.7(d), in which the bluff body is in the closest upstream position ($X_b/D = 4.3$), shows a significant disruption in the pitch amplitude trend with cycle amplitudes ranging between approximately 10 and 75 degrees as the systems nears $f_{shed}/f_{LCO} = 3$.

Figure 4.7 Pitch amplitude distributions for (a) no-bluff-body, (b) $X_b/D = 6.5$, (c) $X_b/D = 5.6$, (d) $X_b/D = 4.3$.

In Fig. 4.8, the amplitude modulation intensity is quantified via the coefficient of variation (CV) or the standard deviation of amplitude divided by the mean. The CV serves as a measure of dispersion in LCO amplitude or how heavily the amplitude is modulated at a given point in time. The local mean and standard deviation were obtained by dividing the time history into 10 second windows and interpolating to produce the contours seen in Fig. 4.8. Figure 4.8(a) shows a uniformly small CV across the time and windspeed domains on the order 1×10^{-3} , reflecting

a non-modulated, steady LCO. For the bluff body cases, for windspeeds in which f_{shed}/f_{LCO} was not close to an integer value, the LCO exhibited a relatively small degree of amplitude modulation, similar to the no-bluff-body case. However, as f_{shed} approaches and passes through an integer multiple of f_{LCO} , the system exhibits a drastic change in behavior. The system returns to a relatively low modulation intensity for $f_{shed}/f_{LCO} \geq 3.2$, demonstrating the frequency-dominated nature of the phenomenon. This effect is reflected in Figure 4.8(b) which shows two distinct bands of increased modulation intensity on either side of the $f_{shed}/f_{LCO} = 3$ line. These regions of increased CV correspond directly to the regions of increased pitch amplitude dispersion from Fig. 4.7(b). Similar behavior is exhibited in Figs. 4.8(c) and 4.8(d), however with increased modulation intensity for decreased X_b/D . The global maximum CV increased 300% between Figs. 4.8(b) and 4.8(c) and 530% between Figs. 4.8(b) and 4.8(d). Additionally, as the distance X_b/D decreases, the modulation intensity becomes more time-dependent in the $f_{shed}/f_{LCO} \approx 3$ regime.

Figure 4.8 Pitch amplitude CV contours for (a) no-bluff-body, (b) $X_b/D = 6.5$, (c) $X_b/D = 5.6$, (d) $X_b/D = 4.3$.

4.2.3 Time-Frequency Analysis of LCO Modulation

Further characterization of the LCO amplitude modulation was performed via a time-frequency wavelet analysis of the LCO envelope. The envelope is defined as a spline interpolation over the positive peaks of the LCO signal. The continuous wavelet transform (CWT)

of the LCO envelope is shown as a series of scalograms which illustrate the evolution of the modulation frequency over time.

Fig. 4.9 contains the CWTs and time histories corresponding to $f_{shed}/f_{LCO} = 2.8$ for the three bluff body positions (a) $X_b/D = 6.5$, (b) $X_b/D = 5.6$, (c) $X_b/D = 4.3$. In this regime, where f_{shed}/f_{LCO} is sufficiently far from an integer value, the distance X_b/D has little influence on neither the modulation intensity nor the modulation frequency. The intensity of the modulation for the three bluff body positions is uniform with respect to time and the dominant modulation frequency is approximately 0.9 Hz. By examining the PSD of the pitch angle time history for $X_b/D = 4.3$ (Fig. 4.6), we see that the LCO third harmonic frequency is 12.2 Hz and the shedding frequency is equal to 11.3 Hz. The difference between these two frequencies ($f_{shed} - 3f_{LCO}$) is equal to the modulation frequency found in Fig. 4.9(c), indicating behavior analogous to a beat phenomenon except that the interaction is with a harmonic frequency rather than the fundamental. Extending the same analysis to the other configurations (Figs. 4.9(a) and 4.9(b)), the modulating frequency is equal to $f_{shed} - 3f_{LCO}$ in both cases.

Figure 4.9 Time-frequency content of pitch angle envelope (bottom) with corresponding pitch angle time history (top) for (a) $X_b/D = 6.5$, (b) $X_b/D = 5.6$, and (c) $X_b/D = 4.3$ where $f_{shed}/f_{LCO} = 2.8$.

Fig. 4.10 contains envelope CWTs and pitch angle time history plots for $f_{shed}/f_{LCO} = 2.9$. Figure 4.10(a) shows an amplitude modulation that is relatively uniform in intensity and frequency with a dominant modulation frequency equal to $f_{shed} - 3f_{LCO}$. Compared to Fig. 4.9(a), the modulation is lower in frequency due to the smaller difference between $3f_{LCO}$ and f_{shed} . When the bluff body is moved to $X_b/D = 5.6$ (Fig. 4.10(b)), $f_{shed} - 3f_{LCO}$ is still present in the envelope CWT, however it is no longer the only modulation frequency. In this case, the LCO response alternates between two modulation modes, represented by the two dominant modulation frequencies seen in the envelope CWT. Further evidence of this is provided by Fig. 4.10(c) in which $X_b/D = 4.3$ and the two modes are even more pronounced. Among the two distinct modulation modes present, the first is a low frequency mode that persists during the first 60 seconds of the trial and causes a relatively intense modulation of the LCO amplitude. The second is a higher frequency mode representing $f_{shed} - 3f_{LCO}$ which dominates between $t = 80$ sec. and $t = 120$ sec. and produces a less intense modulation. Figure 4.10(c) also shows transient period between the two modes ($t = 60$ sec. to $t = 80$ sec.) in which neither frequency dominates entirely.

Figure 4.10 Time-frequency content of pitch angle envelope (bottom) with corresponding pitch angle time history (top) for (a) $X_b/D = 6.5$, (b) $X_b/D = 5.6$, and (c) $X_b/D = 4.3$ where $f_{shed}/f_{LCO} = 2.9$.

The pitch angle time histories and envelope CWTs for $f_{shed}/f_{LCO} = 3.1$ are shown in Fig. 4.11. Here, the system exhibits responses similar to the case in Fig. 4.10 where $f_{shed}/f_{LCO} = 2.9$. The amplitude modulation is uniform in frequency and intensity for $X_b/D = 6.5$ with a dominant modulation frequency equal to $f_{shed} - 3f_{LCO}$ (Fig. 4.11(a)). For the case of $X_b/D = 5.6$, the modulation becomes less uniform, with lower frequency content appearing sporadically throughout the time history (Fig. 4.11(b)). For the case of $X_b/D = 4.3$, the low frequency modulation dominates the signal along with the $f_{shed} - 3f_{LCO}$ (Fig. 4.11(c)). Referencing Fig. 4.7(d), it is clear that the cause of the large amplitude dispersion corresponds due to the lower frequency mode. Comparing Fig. 4.11(c) to Fig. 4.10(c), the switching between the two primary modulation modes does not occur over a consistent period, as evidenced by a more rapid alternation between modes seen in Fig. 4.11(c).

Figure 4.11 Time-frequency content of pitch angle envelope (bottom) with corresponding pitch angle time history (top) for (a) $X_b/D = 6.5$, (b) $X_b/D = 5.6$, and (c) $X_b/D = 4.3$ where $f_{shed}/f_{LCO} = 3.1$.

As discussed previously, the amplitude CV decreases as f_{shed} increases sufficiently past $3f_{LCO}$, demonstrating that the order of magnitude of the intensity of modulation is symmetric about $3f_{LCO}$. Figure 4.12 shows how this symmetry is also reflected in the frequency of the

modulation. Figures 4.9 and 4.12 both show a low intensity modulation with a dominant modulation frequency equal to $f_{shed} - 3f_{LCO}$, that is invariant with respect to bluff body position.

Figure 4.12 Time-frequency content of pitch angle envelope (bottom) with corresponding pitch angle time history (top) for (a) $X_b/D = 6.5$, (b) $X_b/D = 5.6$, and (c) $X_b/D = 4.3$ where $f_{shed}/f_{LCO} = 3.25$.

4.2.4 Instantaneous Phase and Aeroelastic Energy Analysis

The efficiency of aeroelastic systems is often discussed in literature in the context of energy harvesting and bio-inspired vehicle locomotion. Several studies that have examined the efficiency of pitch-heave aeroelastic systems have concluded that the phase angle difference between the two degrees of freedom is an important parameter for predicting efficiency. Anderson et al. [1] conducted an experiment in which the propulsive efficiency of an airfoil undergoing prescribed motion was determined as a function of the phase angle between heave and pitch. They found the optimal phase angle for propulsion to be about 75 degrees since this condition produces favorable timing of the leading edge vortex (LEV) shedding, facilitating the formation of a thrust-producing reverse Kármán street. Similarly, McKinney and DeLaurier [15] found that a phase angle of about 90 degrees is optimally efficient for power extraction in a pitching-heaving aeroelastic system. Here, efficiency is defined as the ratio of the total power extracted

to the total power available in the oncoming flow passing through the swept area normal to the mean wind vector. Later, Kinsey and Dumas [9] showed that LEV shedding is an important mechanism by which favorable synchronization is achieved between the lift force and the heaving velocity, thus controlling the power input to the system.

In order to examine the influence of the bluff body in this context, we begin by performing a phase plane analysis of the LCO. Figure 4.13 contains the pitch-heave displacement phase portraits for (a) no-bluff-body ($U_\infty = 10.4m/s$) and (b) $X_b/D = 6.5$, (c) $X_b/D = 5.6$, and (d) $X_b/D = 4.3$ for similar windspeed ($f_{shed}/f_{LCO} = 2.9$). In Fig. 4.13(a), the no-bluff-body motion trajectory maintains little deviation from the closed orbit defined by the thin ellipse. This is in contrast to the trajectories for the three bluff body cases (Figs. 4.13(b), 4.13(c), 4.13(d)) in which the paths overlap and fill a large portion of the phase space.

Figure 4.13 Phase portraits for (a) no-bluff-body ($U_\infty = 10.4 \text{ m/s}$) and (b) $X_b/D = 6.5$, (c) $X_b/D = 5.6$, and (d) $X_b/D = 4.3$ ($f_{shed}/f_{LCO} = 2.9$).

Since the phase portraits shown in Fig. 4.13 contain the entire time history of the LCO, it is difficult to gain a detailed understanding of how the phase space trajectory evolves with time. To determine the phase angle difference as a function of time, the instantaneous pitch-heave phase difference, $\phi_{\theta,h}$, was calculated as a function of time via the discrete-time analytic signal which is a complex-valued function comprised of the real-valued LCO signal and its Hilbert transform. The algorithm used for computing the analytic signal of a discrete function

is described by Marple [14]. The form of the analytic signal is:

$$z(t) = z_r(t) + iz_i(t) = x(t) + HT[x(t)] \quad (4.1)$$

where $z_r(t)$ is the real component of $z(t)$, $z_i(t)$ is the imaginary component of $z(t)$, $x(t)$ is the original signal and $HT[\]$ is the Hilbert transform operation. Next, the instantaneous phase angle is computed as follows:

$$\phi(t) = \arctan \frac{z_i(t)}{z_r(t)} \quad (4.2)$$

where $\phi(t)$ is the instantaneous phase angle. For this system, the instantaneous phase difference between pitch and heave is calculated as the difference $\phi_{\theta,h} = \phi(t)_{pitch} - \phi(t)_{heave}$.

Fig. 4.14 shows $\phi_{\theta,h}$ for the same four configurations displayed in Fig. 4.13. Figure 4.14(a) demonstrates that the instantaneous phase difference $\phi_{\theta,h}$ is constant within 1.5 degrees in the absence of an upstream bluff body. Figs. 4.14(b), 4.14(c), and 4.14(d) show significant modulation of $\phi_{\theta,h}$ that occurs as the wing oscillates through the bluff body wake. The modulation of $\phi_{\theta,h}$ creates a path through the phase space that is time-dependent when the bluff body is upstream. This is qualitatively reflected by the phase portraits in Fig. 4.13. Note that the frequency of the $\phi_{\theta,h}$ modulation is dependent on the bluff body spatial configuration. For the case where $X_b/D = 6.5$ (Fig. 4.14(b)), the phase difference oscillates at a frequency equal to $f_{shed} - 3f_{LCO}$ for all time shown. For both the $X_b/D = 5.6$ and $X_b/D = 4.3$, (Figs. 4.14(c) and 4.14(d), respectively) the frequency of phase modulation is not uniform with time. Comparing the frequency of the phase modulation in both cases to their respective amplitude modulation frequencies (Figs. 4.10(b)) and 4.10(c)), we find that they are equal. Also, note that the time periods of intense $\phi_{\theta,h}$ modulation ($t = 0$ sec to $t = 60$ sec) seen in Fig. 4.14(d) correspond directly to the time periods of intense amplitude modulation seen in Fig. 4.10(c). Conversely, the time period of low intensity phase modulation ($t = 80$ sec to $t = 120$ sec) corresponds to the period of low

intensity amplitude modulation, indicating a correlation between the magnitude of the phase difference $\phi_{\theta,h}$ and the LCO amplitude.

Figure 4.14 θ (top) and $\phi_{\theta,h}$ (bottom) vs time for (a) no-bluff-body ($U_\infty = 10.4 \text{ m/s}$) and (b) $X_b/D = 6.5$, (c) $X_b/D = 5.6$, and (d) $X_b/D = 4.3$ ($f_{shed}/f_{LCO} = 2.9$).

A phase portrait is shown in Fig. 4.15, depicting the phase relationship between $\phi_{\theta,h}$ and the LCO amplitude. The orange trajectory represents the no-bluff-body configuration from Fig. 4.14(a) and shows that there is an insignificant change in either $\phi_{\theta,h}$ or the pitch amplitude

(θ_A) with time. The blue curve represents the $X_b/D = 6.5$ case from Fig. 4.14(b) and shows a strong correlation between $\phi_{\theta,h}$ and θ_A . The phase portrait maintains an elliptical shape, with the arrow indicating the direction of time. When read in the counterclockwise direction, this plot shows that $\phi_{\theta,h}$ leads θ_A by approximately 90 degrees.

Figure 4.15 Phase portrait of θ_A vs $\phi_{\theta,h}$ for no-bluff-body ($U_\infty = 10.4m/s$) and $X_b/D = 6.5$ ($f_{shed}/f_{LCO} = 2.9$).

The previously mentioned studies [1, 15, 9] focused primarily on the wing efficiency for the purposes of exploiting the dynamics to extract power from the structure. However, since the aeroelastic system in the present study is not designed for energy harvesting, any net power into the structure manifests as a change in the kinematics. For the case of a constant amplitude aeroelastic LCO, the net aerodynamic work is balanced by the losses due to structural damping forces over a given cycle. Thus, it follows that the average net power over a cycle is equal to zero. This differs from the conditions in which the average net power over a cycle is either

positive or negative, where the amplitude will grow or decay, respectively. For the present system, the available power is considered to be constant for a fixed freestream windspeed and the bluff body alters the distribution of kinetic energy in the flowfield. Therefore, any net power into the structure must be due to a change in aeroelastic efficiency. We define the total mechanical energy of the aeroelastic system, E , as the sum of the kinetic and potential energy, $T + U$. From this definition, the net work done on the structure (W_{net}) includes contributions from the aerodynamics (W_A) and the structural damping (W_D) and is defined as:

$$W_{net} = W_A - W_D = T + U - E_0 \quad (4.3)$$

where E_0 is the initial energy of the wing at the $t = 0$ reference point for the aerodynamic and damping work. This definition can be expanded by incorporating the definitions for T and U for this system, previously derived by Kirschmeier et al. [11]:

$$W_{net} = \frac{1}{2}m_T\dot{h}^2 + \frac{1}{2}I_\theta\dot{\theta}^2 - m_Wbx_\theta\dot{h}\dot{\theta}\cos\theta + \frac{1}{2}K_hh^2 + \int_0^\theta M_\theta(\theta)d\theta - E_0 \quad (4.4)$$

where $M_\theta(\theta)$ is the function defining the pitch nonlinearity (Eq.3.3). Next, the net instantaneous power is calculated as the numerical time derivative of work via a central difference scheme:

$$P_{net}(t_i) \approx \frac{W_{net}(t_i + \Delta t) - W_{net}(t_i - \Delta t)}{2\Delta t} \quad (4.5)$$

where Δt is the sampling period. Since we are interested in power on the time scale of the amplitude modulation rather than the intracycle power, the mean power over a cycle ($\bar{P}_{net}$) is

calculated by applying a central moving average to the instantaneous power:

$$\bar{P}_{net}(t_i) = \frac{1}{N} \sum_{j=-\frac{1}{2}(N-1)}^{\frac{1}{2}(N-1)} P_{net}(t_{i+j}) \quad (4.6)$$

where N is size of the sliding window which is set to cover a single wing cycle period (0.25 sec via PSD) and t_i is the i^{th} data point.

Figure 4.16(a) shows $\phi_{\theta,h}$ and $\bar{P}_{net}$ as functions of time for the $X_b/D = 6.5$, $f_{shed}/f_{LCO} = 2.9$ case. We see that $\bar{P}_{net}$ oscillates at the same frequency as both $\phi_{\theta,h}$ and the LCO amplitude. Also, $\phi_{\theta,h}$ and $\bar{P}_{net}$ are in phase with each other; i.e., maximum $\phi_{\theta,h}$ corresponds to maximum $\bar{P}_{net}$. Note that this is in contrast to Fig. 4.15 where $\phi_{\theta,h}$ leads θ_A by approximately 90 degrees. This result makes sense when we consider that $\bar{P}_{net}$ is the time derivative of the average total energy, $\bar{E}$. That is, maximum and minimum $\phi_{\theta,h}$ correspond to the maximum rates of amplitude increase and decrease, respectively, as seen in (Fig. 4.16(b)).

Figure 4.16 (a) $\phi_{\theta,h}$ and $\bar{P}_{net}$ vs time and (b) θ_A vs time for $X_b/D = 6.5$, $f_{shed}/f_{LCO} = 2.9$ case.

Conclusion

5.1 Conclusion

The aim of this research is to elucidate the complex interactions between a nonlinear aeroelastic wing and an oscillating wake shed from an upstream bluff body. The main conclusions stem from the characterization of LCO amplitude modulation and can be summarized as follows:

1. The cases in which the bluff body was present showed amplitude-modulated responses in both the pitch and heave degrees of freedom. The intensity of this modulation, defined by the coefficient of variation of the amplitude, increases sharply as the difference between the LCO third harmonic and the bluff body shedding frequency decreases. It is possible that similar interaction exists for shedding frequency regimes near integer multiples other than three, thus indicating the potential for future work in further exploration of the system dynamics.
2. The streamwise distance between the bluff body and the wing influenced the intensity of the modulation. For a given shedding frequency to LCO frequency ratio, a shorter distance between the wing and the bluff body resulted in greater deviations in amplitude

and increased the variation of the modulation intensity itself with time. The greater influence from the bluff body at closer distances is attributed to increased strength of the impinging wake vortices.

3. The modulation frequency was equal to $f_{shed} - 3f_{LCO}$ for the $X_b/D = 6.5$ case for all frequency ratios f_{shed}/f_{LCO} . This was true for all bluff body spatial configurations when $f_{shed}/f_{LCO} \leq 2.8$ or $f_{shed}/f_{LCO} \geq 3.2$. However, when $2.8 \leq f_{shed}/f_{LCO} \leq 3.2$, decreasing X_b/D was shown to cause intermittent alternation between two modulation modes, each with distinct characteristic intensity and frequency.
4. The mechanism of amplitude modulation was revealed through instantaneous phase and energy analysis. The presence of the bluff body alters the dynamics such that instantaneous pitch-heave phase difference modulates with time. This phase difference manifests as an effective change in the aeroelastic efficiency of the wing. That is, for a limit cycle to maintain constant amplitude, the phase difference between pitch and heave must be such that the average power over a cycle is zero. When the bluff body alters this phase difference, the balance of power into and out of the structure over a cycle is disrupted, resulting in either amplitude growth or decay.

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