ABSTRACT

RAFIEI, AMIN. Coupled Analysis of Wave, Structure, and Sloping Seabed Interaction: Response and Instability of Seabed. (Under the direction of Dr. M.S. Rahman and Dr. M.A. Gabr).

Instability of seabed due to water surface waves may cause damage to coastal/offshore structures. Liquefaction-induced scour due to wave action may impact the stability and bearing capacity of the foundation element of coastal structures. The failure of geotextile sand containers (GSCs), as coastal protective systems, caused by instability of the foundation seabed during storm events has been reported, as for example during Hurricane Ike in 2008. Liquefaction in marine sediments may be instantaneous in which the total mean effective stress reaches zero, or it may occur due to progressive buildup of pore water pressure associated with nonlinear deformations (residual liquefaction). Of the consequences of strength degradation of soil deposit due to increase of pore pressure one is global shear failure of the entire slope in large scale. The wave-induced sliding of sloping seabed was reported after Hurricane Carla damaging South East of US. in 1961.

The interaction of wave, seabed, and structure has been studied mostly for flat or mildly sloping seabed ($< 5^\circ$) using decoupled approach. However, some of structures may be built on or anchored to the seabed with significant steepness. Moreover, numerous studies have investigated the internal stability of geotextile container, and only few are done on the stability due to wave action. The applied wave loading on such structures, as well, has been mostly obtained from simplified analytical solutions. The wave-induced strength degradation of soil deposit has been analyzed by assuming prescribed wave loading amplitudes on the bed surface (that is not influenced by the soil response). Yet, reduction of soil stiffness may cause smaller stress magnitudes transferred to the soil. Therefore, considering all the mentioned aspects, there is a need to study the interaction of wave-structure-sloping seabed using coupled analyses.

In this study, the wave-induced response and instantaneous liquefaction of sloping seabed supporting marine structures are evaluated by developing a coupled finite element model in COMSOL. The interactions of all domains (fluid, structure, and seabed) are included in the analyses. Introducing such coupled approach, the extents of liquefaction around a single GSC and the loading scheme acting on the structure are obtained. The impacts of waves on global shear failure of the sloping seabed are evaluated using strength reduction technique. The effects of slope steepness (resulting in static shear stresses) on the rate of buildup of pore pressure is investigated using experimentally derived models for simulating residual response of soils under cycling.

Biot's poroelastic theory for the sediment, u-p approximation, linear waves and potential flow for the fluid, and elastodynamic equations for the structure domains are solved simultaneously in a coupled finite element model. For the global shear failure analyses, the soil is deemed as elastic-perfectly plastic with Mohr-Coulomb yield condition and associated flow rule. Residual pore pressure modellings are carried out using empirical equations based on lab data in which the models are incorporated into the analysis of wave-sloping seabed interaction.

The results indicate that the coupled and decoupled analyses result in almost the same pore water pressure in sloping seabed for the tested wave soil characteristics. For steeper slopes, the size of the zones of instantaneous liquefaction diminishes with increasing slope angle. Instantaneous liquefaction may not occur in the immediate vicinity of the structure. Surface waves reduce the global stability of the sloping seabed. Such reduction is more pronounced for seabed slope with slightly unsaturated sediments.

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Coupled Analysis of Wave, Structure, and Sloping Seabed Interaction:
Response and Instability of Seabed

by
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DEDICATION

To my dearest parents Nasser, Mahtab, and my wonderful brothers Vahid, Ali

BIOGRAPHY

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CHAPTER 1. INTRODUCTION

1.1. Background

Instability of the seabed, caused by the gravitational and environmental (oceanic) forces (waves, tides and currents), and erosion (scour) are of significant concerns as they may endanger the stability and performance of coastal and offshore structures such as buried pipelines, offshore platform, Geotextile Sand Containers (GSCs), and Marine Hydrokinetic (MHK) devices. Wave-induced seabed instability has been identified as a major cause of damage to such structures [1-3]. Submarine landslide is another factor destabilizing the soil deposits and needs to be considered when the offshore facilities are built on a sloping seabed.

Based on the wave characteristics in the intermediate to shallow depth of ocean, the magnitude of the hydrodynamic pressure on seabed surface can be large enough to induce high pore pressure (the general sketch of the reference problem is shown in Figure. 1.1). Consequently, the effective stresses may be diminished resulting in a considerable loss of soil strength and possible instantaneous liquefaction in the extreme cases. Liquefaction in marine sediments may be instantaneous in which the total mean effective stress reaches zero, or it may occur due to a progressive buildup of pore water pressure associated with nonlinear deformations (residual liquefaction).

Scour near the foundation of marine structures due to waves and currents is a significant aspect in instability of such structures. Two types of scour have been identified in the previous studies: a) shear-induced scour due to pick up and transport of sediment and b) liquefaction-induced scour due to instability of soil due to pore pressure gradient [4].

After hurricane Katrina, numerous liquefaction-induced scour cases around the foundation of coastal structure led to substantial damages [4]. Moreover, the initiation of sediment transport is attributed to the generated bed shear stress and the magnitude of critical shear stress. This criterion does not inherently include the gradient of pore fluid pressure in sediments. Therefore, more advanced criteria are needed to account for both aspects. Lambrechts et al. [5] proposed a model based on experimental and field observations to include the effects of stress state in sediments on sediment transport.

With the aim of maintaining the development and sustaining the environment we need to exploit all the available sources of energy transitioning from non-renewable sources to renewable. It is estimated that 33% of the total electricity usage in US can be supplied by ocean renewable energy [6], with the energy from waves providing the largest share [7]. Generally, offshore foundation elements can potentially be built on sloping seabed. This is particularly the case for MHKs deployed near the edge of Continental Shelf with steep slopes; a location where strong Gulf Stream currents are available for exploiting the water kinetic energy. The seabed stability near the foundations of these structures is as significant factor impacting their performance that is must be investigated.

The interaction of wave, seabed, and structure has been studied mostly for flat or mildly sloping seabed ($< 5^\circ$) using a decoupled approach. However, as mentioned earlier, some of structures may be built on or anchored to the seabed with significant steepness. Moreover, numerous studies have investigated the internal stability of geotextile container, and only few are done on the stability due to wave action. The applied wave loading on such structures, as well, has been mostly obtained

from simplified analytical solutions. The wave-induced strength degradation of soil deposit has been analyzed by assuming prescribed wave loading amplitudes on the bed surface (that is not influenced by the soil response). Yet, reduction of soil stiffness may cause smaller stress magnitudes transferred to the soil. Therefore, considering all the mentioned aspects, there is a need to study the interaction of wave-structure-sloping seabed using coupled analyses.

1.2. Objectives

The objective of this study is to investigate the response and instability of a sloping seabed (with significant slope angles) due to wave action near the foundation of marine structures following a coupled approach. To this end, first, a modeling framework to analyze the interaction of coupled media including fluid, solid, and soil is developed by implementing governing equations and boundary conditions in mathematical module of COMSOL. Then, the developed numerical framework is employed to study few marine geotechnical problems, as for instance wave-induced liquefaction analysis of sediments around a marine structure built on sloping seabed.

The interactions among media (fluid, structure, and soil) is modeled rigorously in which the influences of domains' response on one another are included in the analyses. However, it is assumed that surface wave profile is kept unchanged during propagation. Seabed instability being examined herein is related to: a) increase in pore water pressure due to wave action that may lead to reduction of shear strength of sediments and liquefaction in severe cases and b) global instability of submarine slopes due to shear failure caused by combination of wave and gravitational forces.

The effects of soil and wave characteristics on the extent of instantaneous liquefaction both in free field condition and when the structure is placed on the seabed are investigated through parametric studies. Another goal of this work is to assess the liquefaction associated with progressive buildup of pore pressure. Numerical models (based on lab data) for pore pressure buildup with various levels of complications are employed. Pan and Yang's model [8] for residual pore pressure generation (capable of including effects of static shear stresses) is incorporated in 2D plane strain analysis of wave-sloping seabed interaction. Moreover, a more advanced numerical model (based on lab data) that simulates the residual pore pressure and deformation simultaneously is used [9]. This model is employed to capture nonlinear response of soil sample in undrained cyclic triaxial test. In addition, shear failure of sloping seabed is analyzed using finite element-based shear strength reduction method which has become a more reliable approach recently because of numerous advantages over conventional limit state equilibrium methods [10].

Analysis of response and extent of instantaneous liquefaction caused by waves within seabed sediments supporting Geotextile Sand Containers is also an objective of this study. These structures are used near coastlines to break the wave energy and protects the coastal infrastructures [11]. The significance of this investigation is due to: a) deployments locations of such devices causing significant forces on the device and the seabed; thus the coupling effects are stronger than the one for MHKs (placed on sloping seabed), b) The geometry is more complicated demanding more computational efforts in equating normal component of the stress vector for the geotextile layer with the fluid pressure, and c) GSCs are composite structures with a very thin geotextile layer (thickness in the order of 5mm), and a special treatment in finite element modeling is needed.

1.3. Scope

This dissertation consists of seven chapters. In chapter 2, the response of finitely steep sloping seabed under surface water waves is investigated, and the development of instantaneous liquefaction is evaluated (free field). An almost fully coupled approach is employed for fluid-soil interaction that captures the coupling process among the media rigorously. Subsequently, the impacts of such coupling scheme on sediments response is assessed. The extents of instantaneous liquefaction near bed surface as a function of seabed slope angle are investigated.

In chapter 3, with the inclusion of a structure mounted on the sloping seabed, the wave-induced response of sediments around the foundation element is evaluated, and the extent of instantaneous liquefaction is computed (full system). The impacts of various coupling schemes (coupled vs. decoupled) on the response of the seabed are demonstrated.

In chapter 4, the response and instability of foundation soil around a single GSC structure under progressive surface waves are analyzed. The distribution of loading on the structure under the wave action is obtained. The extent of liquefaction around the GSC is calculated under various wave characteristics.

In chapter 5, effects of storm waves on global shear failure of the sloping seabed with finite steepness are evaluated using strength reduction finite method. The variations of slope stability number (factor of safety) caused by wave action are elaborated through demonstration of effective stress paths within a wave cycle.

In chapter 6, the residual pore pressure response under cyclic wave loadings is investigated using experimentally-derived models. The impacts of seabed slope angle on temporal variation of pore water pressure is studied by incorporating Pan and Yang's model [8] into coupled wave-sloping seabed analyses. The progressive buildup of pore water pressure during undrained cyclic triaxial test is calculated (elemental behavior) employing more advanced Bouckovalas' model [9] capable of concurrent modeling of residual pore pressure and deformation of soil elements.

Lastly, the summary, conclusion, and suggested future works are presented in chapter 7. The overview of the current study is demonstrated in Figure. 1.2.

Figures

Figure 1.1. Schematic sketch of the reference problem

Modeling and Computing

- Poroelastic Response
- Progressive Buildup of Pore Pressure
- Strength Reduction Technique
- Interaction of Domains
- Coupling Scheme

Applications

- Wave-Sloping Seabed Interaction (Poroelastic Response) (Chapter 2)
- Wave-MHK-Sloping Seabed Interaction (Poroelastic Response) (Chapter 3)
- Wave-GSC-Sloping Seabed Interaction (Poroelastic Response) (Chapter 4)
- Wave-Sloping Seabed Interaction (Global Shear Failure) (Chapter 5)
- Wave-Sloping Seabed Interaction (Residual Response) (Chapter 6)

Figure 1.2. Overview of the current study

CHAPTER 2. COUPLED ANALYSIS FOR RESPONSE AND INSTABILITY OF SLOPING SEABED UNDER WAVE ACTION

The contents of this chapter have been published in the Applied Ocean Research Journal.

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Abstract

Wave-induced instability of seabed may cause damage to coastal and offshore structures. This issue has been investigated mostly for mildly sloping ($< 5^\circ$) seabed considering uncoupled or one-way coupled response of wave and seabed interaction. However, some of the marine structures are founded on seabed with steeper slopes. In this study, the wave-induced response and instability of sloping seabed are evaluated using a coupled finite element model. The interaction between fluid and porous seabed accounting for the effect of fluid motion on the seabed response, and conversely the effect of seabed response on the fluid motion (but not on the surface wave profile) is considered. The results indicate that the system response (fluid pressure, stresses, etc.) and the extent of instantaneously liquefied zone within the sloping seabed with significant steepness are lesser than those for horizontal seabed. Moreover, for typical sediment and wave characteristics, for the flat seabed, the response obtained from fully coupled analysis is not significantly different from those obtained by uncoupled analysis. For the sloping bed, such difference is slightly greater as compared to that for the flat bed.

2.1. Introduction

Evaluation of wave-seabed interaction is required for a safe design of coastal and offshore structures such as breakwaters, buried pipelines, offshore platform, and Marine Hydrokinetic (MHK) devices. Wave-induced seabed instability, rather than construction deficiency, has been identified as a major cause of damage to such structures [1-3]. Wave action on seabed surface in intermediate water depth, may alter the stress state within the soil skeleton and pore water in such a way that seabed soil may become unstable endangering the stability of these structures [12,13]. Instability in a soil mass may be caused by either the shear stresses exceeding the shear strength or the reduction of shear strength associated with the pore pressure changes. In relation to the second cause, we identify an extreme condition of ‘liquefaction’ in which the mean effective stress reduces to zero leading to the total loss of shear strength. Furthermore, two different types of liquefaction have been found to occur [14, 15]: (a) liquefaction due to the progressive development of pore pressure (associated with nonlinear deformation of soils and potential for accumulation of volumetric strain) due to cyclic shear stresses. (b) ‘instantaneous liquefaction’, which may occur in nearly saturated elastic soil (with some pore air) due to phase shift in stress response. In this study our focus is only on ‘instantaneous liquefaction’.

Okusa [16] evaluated the instantaneously liquefied zone due to surface waves in partly saturated sediments through an analytical study. Examination of wave-seabed interaction was reported in a study by Lee and Lan [17] in which the continuity of flux and dynamic pressure were applied at the interface of water and porous seabed domains. The effect of breaking solitary waves on pore pressure variation within sloping sandy seabed (slope less than 4°) has been investigated by Xiao et al. [18] with only the dynamic pressure from the wave model transferred to the seabed. The

results indicate a relatively good agreement between pore fluid pressure from numerical results and experimental measurements. Recently, the influence of randomness in sediment properties on liquefaction zone within the seabed, particularly the shear modulus, is investigated using stochastic finite element analysis and decoupled approach [19].

In previous studies, the wave-sloping seabed interaction is considered to be either decoupled or one-way coupled. The one-way coupled term refers to coupling of wave motion to seabed response in which there is no feedback from the soil to fluid [20]. In the work presented herein, a coupled analysis of interaction between fluid and porous seabed is considered. This accounts for the effect of fluid motion on the seabed response, and also the effect of seabed response on the motion in the fluid domain (with the wave profile kept unchanged). This should also be noted here that the continuities of both the fluid pressure and flux within the domains of fluid and porous bed are enforced at their interface.

As mentioned, the stress and deformation response for flat or mildly sloping seabed (slope less than five degrees) have been investigated [14,21,22]. However, the significance of using the more accurate coupled flow and deformation processes on sediments instability around the foundation of marine structures is yet to be investigated. Furthermore, offshore foundation elements can potentially be built on sloping seabed. This is particularly the case for Marine Hydrokinetic (MHK) devices deployed near the edge of Continental Shelf with steep slopes; a location where strong Gulf Stream currents are available for exploiting the water kinetic energy.

In this paper, the coupled response of sloping seabed to wave action is investigated with a focus on the evaluating instantaneous liquefaction. Two main aspects investigated herein are: (a) response of seabed with significant steepness of slope, and (b) wave-seabed interaction (semi-coupled approach with known wave profile at water surface). Biot's theory, with ' $\mathbf{u}$ (displacement)- $\mathbf{p}$ (pore water pressure)' as field variables, is used to formulate flow and deformation response of soils [23,24] in which the acceleration of pore fluid is neglected [22], and potential flow theory [25] is used to formulate the response of fluid domain. The governing equations are solved simultaneously using finite element method. The soil sub-model is verified by available closed-form solutions for the case of horizontal seabed [22]. The extent of liquefied zone is compared to results from analytical study [16]. The effect of slope steepness on seabed response is discussed, and the effect of soil characteristics on the hydrodynamic pressure for both coupled and decoupled solutions along sloping seabed surface is investigated. A correlation between seabed slope and extent of the instantaneous liquefaction zone (width and depth) is presented. This is noted that a semi (or almost fully) coupled analysis of wave-seabed interaction together with response and instability (instantaneous liquefaction) of finitely steep seabed slopes are being considered for the first time in this study.

2.2. Mathematical Model: Formulation

The idealization and formulation of the equations governing the response of seawater and seabed and their boundary conditions are presented herein for a two-dimensional problem considering plane strain condition. The modeled domains are shown in Figure. 2.1.

2.2.1. Wave-Seabed Interaction: Coupling of the Responses

The wave-seabed interaction has been considered with various levels of idealization (e.g. Rahman [14], Jeng et al. [20]), which may be categorized as:

(a) *Decoupled*. In this, the response in fluid domain is evaluated first considering the seabed to be impervious and nondeformable. Then the fluid pressure evaluated on the seabed is applied on the porous and deformable seabed to evaluate the response.

(b) *One-way Coupled*. In this approach, the dynamic pressure as well as water flux from the fluid model is obtained and applied on the seabed surface. The effects of flow and deformation within the seabed on the fluid section are not considered.

(c) *Semi-Coupled*. The effects of wave-induced motion in the water domain on the flow and deformation within the seabed soil domain and also their inverse (i.e. the effects of seabed response on the motion in water domain) are accounted. However, the wave profile is considered to remain unaffected. For this reason, this almost fully coupled analysis is being named as ‘semi coupled’. In this study, this approach is being used to formulate wave-seabed interaction.

(d) *Fully Coupled*. In this approach, in addition to considering the wave-seabed interaction as in semi-coupled approach, the waves are initially generated by wave maker theory [25], propagated towards the zone of interest, and the effect of all the interactions on wave profile is also considered.

2.2.2. Governing Equations of Flow

Intermediate zone of water depth and non-breaking wave condition are considered in this study, and linear wave theory is used in which the water is assumed as incompressible and inviscid with an irrotational flow expressed as [25]:

$$\nabla^2 \phi = 0 \quad (2.1)$$

where ϕ is velocity potential. The corresponding hydrodynamic pressure (p_f) and velocity (v_f) are given by:

$$v_f = -\nabla \phi \quad (2.2)$$

$$p_f = \rho_f \frac{\partial \phi}{\partial t} \quad (2.3)$$

in which ρ_f denotes water density. For severe storm conditions, nonlinear wave idealizations will be more realistic [26]. However, linear waves are adopted as the first approximation in this work.

2.2.3. Equations Governing Seabed Response

The seabed is assumed to be homogenous and isotropic, and the response is described by Biot's poroelasticity equations in partly dynamic form (u-p approximation as presented by Biot [23] and Zienkiewicz et al. [24]). The flow of pore fluid inside the sediment obeys Darcy's law and in this form the acceleration of pore water is neglected while the acceleration of solid particles is included. The combination of equilibrium and conservation of mass equations for pore water in a soil element can be written as [24]:

$$k \nabla^2 p - \gamma_w n \beta \frac{\partial p}{\partial t} + k \rho_f \frac{\partial^2 \varepsilon_v}{\partial t^2} = \gamma_w \frac{\partial \varepsilon_v}{\partial t} \quad (2.4)$$

where k , n are permeability and porosity of sediment, respectively and β , γ_w , and p denote

compressibility, unit weight, and pressure of pore water, and ε_v is volumetric strain of the soil element. Respectively, for partly saturated sediment, the compressibility takes the form as:

$$\beta = \frac{1}{K_w} + \frac{1 - S_r}{p_{w_0}} \quad (2.5)$$

in which K_w , S_r and p_{w_0} represent bulk modulus, degree of saturation, and absolute water pressure, respectively. Overall equilibrium of the soil element when the total stress is decomposed into effective stress and pore water pressure is given by:

$$\frac{\partial \sigma_x'}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} - \frac{\partial p}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2} \quad (2.6)$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma_z'}{\partial z} - \frac{\partial p}{\partial z} = \rho \frac{\partial^2 v}{\partial t^2} \quad (2.7)$$

Here, σ_x' and σ_z' indicate the components of normal stresses, τ_{xz} is shear stress, and u and v are the components of displacement in horizontal and vertical directions of the soil skeleton, respectively. The total density of seabed sediment (ρ) is expressed as:

$$\rho = \rho_f n + \rho_s (1 - n) \quad (2.8)$$

in which ρ_s and n denote the density of soil solids and porosity. For isotropic linear elastic soil skeleton the normal and shear stresses in plane strain condition are written as [27]:

$$\sigma_x' = 2G \left[\frac{\partial u}{\partial x} + \frac{\vartheta}{1 - 2\vartheta} \varepsilon_v \right] \quad (2.9)$$

$$\sigma_z' = 2G \left[\frac{\partial v}{\partial z} + \frac{\vartheta}{1 - 2\vartheta} \varepsilon_v \right] \quad (2.10)$$

$$\tau_{xz} = 2G \left[\frac{\partial u}{\partial z} + \frac{\partial v}{\partial x} \right] = \tau_{zx} \quad (2.11)$$

where G and ϑ are shear modulus and Poisson's ratio. The final form of governing equations is obtained by substitution of Eqs. (2.9) – (2.11) into Eqs. (2.6) – (2.7) and is expressed as:

$$G \nabla^2 u + \frac{G}{1 - 2\vartheta} \frac{\partial \varepsilon_v}{\partial x} = \frac{\partial p}{\partial x} + \rho \frac{\partial^2 u}{\partial t^2} \quad (2.12)$$

$$G \nabla^2 v + \frac{G}{1 - 2\vartheta} \frac{\partial \varepsilon_v}{\partial z} = \frac{\partial p}{\partial z} + \rho \frac{\partial^2 v}{\partial t^2} \quad (2.13)$$

2.2.4. Boundary Conditions for Fluid Domain

In intermediate water depth where the magnitude of wave height is smaller than mean water depth and wavelength; the potential flow theory may be assumed. At water surface, the progressive wave profile is computed and the corresponding dynamic pressure (p_f) is imposed as a prescribed boundary condition. For the case of horizontal seabed, the dynamic pressure at surface due to harmonic progressive wave is written as [14]:

$$p_f = \frac{\rho_f g H}{2} \cos(Kx - \omega t) \quad (2.14)$$

where $K = 2\pi/L$ is wave number (L being wavelength), $\omega = 2\pi/T$ wave circular frequency (T being wave period), and H denotes wave height.

By approaching shallower water, the wave height increases and the length decreases. For linear waves, the values of length are a function of period and water depth (d) and are obtained from the dispersion equation given by [25]:

$$\omega^2 = gK \tanh(Kd) \quad (2.15)$$

or

$$L = L_0 \tanh\left(2\pi \frac{d}{L}\right) \quad (2.16)$$

$$L_0 = \frac{gT^2}{2\pi} \quad (2.17)$$

where L_0 represents the wave length in deep water.

A range of wave steepness in deep ocean is reported by Horikawa [28] based on field observations and is expressed as (H_0 denotes wave height in deep water):

$$0.008 \leq \frac{H_0}{L_0} \leq 0.055 \quad (2.18)$$

In shallower water based on conservation of energy, wave amplitude can be computed as:

$$H = H_0 \left\{ \left[1 + \frac{4\pi d/L}{\sinh(4\pi d/L)} \right] \tanh \left(2\pi \frac{d}{L} \right) \right\}^{-1/2} \quad (2.19)$$

The steepness of waves grows higher as the water depth decreases. Such increase can reach to an extent where the waves start to break after a certain value. Such threshold for horizontal bathymetry is defined as:

$$\frac{H}{L} \geq \frac{1}{7} \tanh \left(2\pi \frac{d}{L} \right) \quad (2.20)$$

For the sloping seabed, the following empirical criterion using data from numerous experiments is adopted for the breaking wave height [29, 30]:

$$\left(\frac{H}{d} \right)_b = \frac{b}{1 + \frac{ad}{gT^2}} \quad (2.21)$$

where

$$a = 43.75(1 - \exp(-19m)) \quad (2.22)$$

$$b = \frac{1.56}{1 + \exp(-19.5m)} \quad (2.23)$$

, and m denotes the seabed slope. By implementing Eqs. (2.16) – (2.23), a range of wave steepness and water depth ratio (d/L) is obtained for intermediate and non-breaking zone [14].

At wave-seabed interface, the continuity of fluid pressure and flux are imposed as [21]:

$$p_f = p \quad (2.24)$$

$$v_f = v_{pf} \quad (2.25)$$

where v_{pf} is the velocity of pore fluid inside the sediment. The general equilibrium equation for pore water in i direction is expressed as [24]:

$$-\frac{\partial p}{\partial x_i} + \rho_f g_i - \frac{\rho_f g_i}{k_i} \frac{\partial \bar{w}_i}{\partial t} - \rho_f \frac{\partial^2 u_i}{\partial t^2} - \frac{\rho_f}{n} \frac{\partial^2 \bar{w}_i}{\partial t^2} = 0 \quad (2.26)$$

in which w_i is the relative average displacement of pore fluid in i direction with respect to that of solid skeleton of the soil. For partly-dynamic form of Biot's equations [24], the acceleration of pore fluid is neglected. The actual relative velocity is defined as $1/n \times \partial \bar{w}_i / \partial t$ where n is soil porosity. Therefore, the actual relative pore water velocity ($\partial w_i / \partial t$) neglecting the gravitational body force term ($\rho_f g_i$) takes the form as:

$$\frac{\partial w_i}{\partial t} = -\frac{1}{n} \frac{k_i}{g_i} \frac{\partial^2 u_i}{\partial t^2} - \frac{1}{n} \frac{k_i}{\rho_f g_i} \frac{\partial p}{\partial x_i} \quad (2.27)$$

Thus, the actual pore water velocity (v_{pf} in Eq. (2.25)) with the substitution of $v_{pf} = -\partial \phi / \partial x_i$ in z direction is given by:

$$-\frac{\partial \phi}{\partial z} = -\frac{1}{n} \frac{k}{g} \frac{\partial^2 v}{\partial t^2} - \frac{1}{n} \frac{k}{\rho_f g} \frac{\partial p}{\partial z} + \frac{\partial v}{\partial t} \quad (2.28)$$

For the case of sloping seabed, the spatial derivative in Eq. (2.27) is normal gradient to the water-seabed interface.

For a system of horizontal seabed and progressive surface waves, the computational domain can be truncated with a length equal to a coefficient of wavelength, and periodic boundary conditions can be employed on the lateral boundaries. However, this approach cannot be followed for the sloping seabed since the values of field variables will not be the same on the lateral edges of the domains. As an ideal approach, a radiation boundary condition should be devised and employed on lateral sides to suppress the spurious reflections from far field to the zone of interest. Development of such boundaries will be complicated and is not the focus of this study given the details of the analysis being in time domain and the regions are fully coupled. Also, to the author's knowledge, the efficient lateral boundary conditions have not been developed yet. Alternatively, the authors used the boundary conditions suggested by Ye and Jeng [31] to impose on the two ends. Ye and Jeng [31] proposed horizontally fixed and no-flow lateral boundary conditions for seabed domain for the cases where periodic boundary conditions are not valid (for instance in analysis of combination effects of waves and currents on seabed response.) They suggested employing an adequately large domain to mitigate the effect of lateral boundaries on sediment response at the middle of the domain. Accordingly, a large domain is selected spanning from deep water to approximately breaking zone limit in shallower water. Thus, zero hydrodynamic pressure on the two end boundaries is applied as a reasonable assumption for the case of inclined seabed.

A parametric study to determine the adequacy of domain length is performed to avoid the fictitious boundary effects on the fluid and soil response.

2.2.5. Boundary Conditions for Soil Domain

At the bottom of seabed with the finite depth, a rigid and impermeable base is assumed where zero displacement for the soil particles and no flow conditions are applied along the boundary

$$u = v = \frac{dp}{dn} = 0 \quad (2.29)$$

The continuity of pressure and flux at the interface of soil and water domains is introduced similar to the conditions mentioned for the fluid section, Eqs. (2.24) and (2.25). Also, zero shear stresses as well as zero normal effective stresses are generated, which is then expressed as:

$$\sigma_z' = \tau_{xz} = 0 \quad (2.30)$$

For lateral boundaries, and similar to the fluid section, periodic conditions of the field variables are implemented for horizontal seabed. Transmitting boundary condition along the lateral edges is the ideal form of condition for the case of sloping seabed. As mentioned before, development of such boundaries (consistent or local radiation boundaries in time domain) is beyond the scope of this work. As an alternative, horizontally fixed and impermeable boundary conditions expressed are implemented along the lateral sides. In order to avoid the effects of such fictitious boundaries on system response, in the form of spurious reflection for instance, an adequately large domain is selected [31].

$$u = \frac{dp}{dx} = 0 \quad (2.31)$$

The domain geometry with applied boundary conditions is shown in Figure. 2.1.

2.2.6. Soil Instability: Instantaneous Liquefaction

In soils with slight unsaturation (due to the presence of air bubbles or gases) the total mean effective stresses may reduce to zero (*i. e.* $\bar{\sigma}' = 0$) leading to what is termed as ‘instantaneous liquefaction’. The liquefaction criterion can be expressed as:

$$\begin{aligned} \bar{\sigma}' &= 0 \quad \text{or} \\ \bar{\sigma}'_g + \bar{\sigma}'_w &= 0 \end{aligned} \quad (2.32)$$

The first term represents the geostatic mean effective stress and the second is wave-induced mean effective stress. For flat seabed in plane strain condition, the above equation takes the form as:

$$\frac{1}{3} [(\rho_t - \rho_f)gz(1 + 2K_0) + (v + 1)(\sigma'_x + \sigma'_z)] = 0 \quad (2.33)$$

in which ρ_t and K_0 denote total density and coefficient of lateral earth pressure at rest, respectively. For the case of sloping seabed, geostatic stresses are computed based on elasticity solutions of infinite sloping grounds [32] given by:

$$\sigma_{g,z} = \gamma z (1 + K \sin^2 \beta) \quad (2.34)$$

$$\sigma_{g,x} = K \gamma z \cos^2 \beta \quad (2.35)$$

where

$$K = \frac{K_0 + r_u(1 - K_0)}{\cos^2 \beta - K_0 \sin^2 \beta} \quad (2.36)$$

In the above, K_0 is the coefficient of lateral earth pressure at rest for level ground, and r_u is the pore water pressure ratio ($r_u = u/\gamma z$).

For saturated sediment, the second term of LHS of Eq. (2.32) ($\bar{\sigma}'_w$) is always equal to zero because the wave-induced normal effective stresses are completely out-of-phase. Therefore, the total mean effective stress will be equal to geostatic mean effective stress ($\bar{\sigma}' = \bar{\sigma}'_g$) which has non-zero values. However, in slightly unsaturated sediment, the smaller phase shift between wave-induced effective stresses may produce a negative value of the wave-induced mean effective stress leading to the possibility of the total mean effective becoming zero and thus the state of ‘instantaneous liquefaction’.

Okusa in an analytical study demonstrated the phase shift in the wave-induced effective stress response of partly saturated sediments by evaluating the effect of compressibility of pore fluid using Skempton parameter (B) [16]. He indicated that lower Skempton parameter (corresponds to higher compressibility of pore fluid and higher unsaturation level) leads to reduction of phase shift of wave-induced normal stress components (see Eqs. (2.38), (2.39), and (2.42)). This, in turn, can result in larger mean effective stresses caused by waves ($\bar{\sigma}'_w$) and more pronounced instantaneous liquefaction.

As mentioned, the effect of pore fluid compressibility (β) on the soil response (e.g. pore water pressure, mean effective stress) is considered using the Skempton parameter (B) [16] defined as:

$$B = \frac{1}{1 + n\beta/\alpha} \quad (2.37)$$

where α denotes compressibility of soil skeleton $\alpha = 3(1 - 2\nu)/E$ with E being Young's Modulus. Eq. (2.37) indicates that for a given soil, higher compressible pore fluid leads to greater reduction of Skempton parameter and consequently more unsaturation.

The wave-induced normal effective stresses for slightly unsaturated and infinitely deep sediments are given by [16]:

$$\begin{aligned} \sigma'_x = & (1 - B'_1) \exp(-Kz) \cos(Kx - \omega t) - Kz \exp(-Kz) \cos(Kx - \omega t) \\ & - \frac{\nu(1 - B'_1)}{1 - \nu} \exp(-\kappa_1 z) \cos(Kx - \kappa_2 z - \omega t) \end{aligned} \quad (2.38)$$

$$\begin{aligned} \sigma'_z = & (1 - B'_1) \exp(-Kz) \cos(Kx - \omega t) + Kz \exp(Kz) \cos(Kx - \omega t) \\ & - (1 - B'_1) \exp(-\kappa_1 z) \cos(Kx - \kappa_2 z - \omega t) \end{aligned} \quad (2.39)$$

in which K and ω are wave number and wave frequency, respectively, and κ_1 and κ_2 are the roots of

$$\frac{\pm k^2 + [k^4 + (\omega/c_v)^2]^{1/2}}{2} \quad (2.40)$$

and B'_1 is obtained from:

$$\frac{1}{B'_1} = 1 + \frac{n\beta}{m_v} \quad (2.41)$$

where m_v and c_v are coefficients of compressibility and consolidation, respectively. Following the expressions for normal stresses, the mean effective stress due to wave action takes the form as:

$$\begin{aligned} \bar{\sigma}'_w = \frac{(1 + \vartheta)}{3} \{ & 2(1 - B'_1) \exp(-kz) \cos(kx - \omega t) \\ & - (1 - B'_1)/(1 - \vartheta) \exp(-\kappa_1 z) \cos(kx - \kappa_2 z - \omega t) \} \end{aligned} \quad (2.42)$$

For slightly unsaturated condition, B'_1 is less than unity resulting in $\bar{\sigma}'_w$ to have non-zero values. Figure. 2.2 demonstrates the temporal variations of wave-induced normal effective stresses for a generic point inside the seabed for various saturation levels $S_r = 0.95$, $S_r = 1.0$ ($\sigma'_{m,w}$ denotes mean effective stress due to wave action). As indicated, the mean effective stress caused by waves for fully saturated sediment deposits is equal to zero.

2.2.7. Coupled Analysis: Steps

In this study, as mentioned before, the semi-coupled approach is adopted, where equations governing the fluid (Eq. (2.1)) and seabed (Eqs. (2.4), (2.12)-(2.13)) domains are coupled based on interface conditions (Eqs. (2.24)-(2.25)) at common surface of the two media. Then, the set of equations are solved simultaneously, and at each time step all the field variables (velocity potential of fluid ϕ , sediment displacement components u and v , and pore water pressure p) are obtained (Figure. 2.3). With such direct coupling, the interaction effects are captured rigorously in terms of

the impact of fluid motion on the seabed response, and conversely the impact of seabed response on the fluid motion.

2.3. Numerical Results and Discussion

The numerical model is solved using finite element method in time domain within COMSOL [33]. The governing equations and boundary conditions are introduced into the model. The equations are solved in time domain following Backward Differentiation Formulas method (BDFs) with the desirable stability properties [34].

The model is verified for the case of horizontal seabed and then the effect of slope on seabed response and development of instantaneously liquefied zone are studied.

2.3.1. Model Verifications for Horizontal Seabed: Decoupled Approach

With the aim of verification of fluid and soil sub-models, the domains' response is obtained and compared to analytical solutions. A fluid domain with horizontal rigid and impermeable base is considered. The modeled wave characteristics are presented in Table 2.1. Rectangular elements with the size of 0.4 m are used for spatial discretization (considering the effects of mesh fineness on solution convergence). The response distribution within the depth for maximum dynamic pressure and maximum velocity components are illustrated in Figure. 2.4. As shown, there is a good agreement among the results of this study and the closed-form solutions [25].

Regarding the seabed model verification, sediment response is obtained under a prescribed hydrodynamic pressure applied along the seabed surface (decoupled approach). The results are compared with analytical solutions [22]. The hydrodynamic pressure distribution along the bottom of the fluid domain is given as follows [25]:

$$p_f(x, t) = P_0 * \cos\left(\frac{2\pi}{L}x - \omega t\right) \quad (2.43)$$

in which

$$P_0 = \frac{\gamma_w H}{2 \cosh\left(\frac{2\pi}{L}d\right)} \quad (2.44)$$

A rectangular domain with the length equal to one wavelength is modeled and the input data regarding seabed and wave characteristics are shown in Table 2.1. Rectangular elements (structured mesh) are employed for discretization of the domain and maximum element size is selected as 0.4 m after conducting a convergence analysis on mesh density.

The comparisons of solutions for maximum pore water pressure and shear stress inside the seabed depth is illustrated in Figure. 2.5. The magnitude of dynamic pressure (P_0) is computed from Eq. (2.44) and $z/D = 0$ denotes the seabed surface. Based on the figure, seabed response obtained from the model converges closed from solutions considerably. Moreover, the boundary conditions along seabed surface, Eqs. (2.24) and (2.30), are satisfied appropriately.

2.3.2. Horizontal Seabed: Fully Coupled Approach

Fluid and soil domains for the case of horizontal seabed are modeled as coupled system, by imposing the continuity of pressure and flux. Both domains are discretized by Lagrange rectangular elements with a maximum size 0.4 m (selected after a convergence study on the mesh resolution). Also, at the interface of domains, a matching mesh is employed in which the elements of each region have the same common nodes; therefore, no data exchange mechanism is required to transfer information from one domain to another [35].

The distribution of maximum pore water pressure in sediment and maximum hydrodynamic pressure of fluid section is illustrated in Figure. 2.6 based on the data of Table 2.1. The reduction of pressure with depth and pressure continuity at the interface are apparent from Figure. 2.6. Also, as shown in Figure. 2.7, for a wide range of sediment and wave characteristics, the coupled and decoupled approaches yield approximately the same response with a very slight difference. This behavior is in concert with findings from a previous work [17] in which the maximum pore pressure obtained from the various approaches differs only by the order of 2% for a given wave condition.

2.3.3. Sloping Seabed: Decoupled vs. Fully Coupled Approach

An inclined seabed with the slope of 15° is modeled, as presented in Figure. 2.8. The parameters used in analyses are tabulated in Table 2.2. The wave height for the given wave period $T=7 \text{ sec}$ varies from 4.2m in deep water to 3.9 m close to breaking zone limit, as expressed by equation (19). It is noted from shoaling of waves (Eq. (2.19)) that by approaching the shallower water, the wave height initially decreases slightly and then increases. However, as the study focus is on

intermediate water, the increase in wave height (happening in shallow water particularly) has not been observed within the given range of water depths. Also, the adopted wavelength in the deep zone is 76.5m which decreases to 61.8m based on (16). The domains are discretized by rectangular elements (near the water-sloping seabed interface) and triangle elements away from the interface. The maximum size of rectangular and triangle elements is selected as 0.2m and 2m, respectively, following parametric studies on mesh fineness. Figure. 2.9 demonstrates the temporal variations of shear stress for a generic point inside the seabed for various mesh resolution (i.e. study of size of rectangular elements near the water-seabed interface while keeping the maximum size of triangle elements constant). As shown in Figure. 2.9, there is a very slight difference (less than 2%) in the response when element size varies from 0.3m to 0.1m. As mentioned previously, the domain length is large enough to reduce the effect of the lateral boundary reflection on the system response. On the other hand, for the critical wave conditions, the unnecessary long domain generates two challenges: a) high computational costs, and, b) increase in the area of applied surface waves that results in higher induced dynamic pressure. As a consequence, the larger seabed domain is needed to avoid the multiple reflections at the boundaries. Accordingly, domain length equal to 220m is selected, which is almost three times of L_0 .

The distribution of maximum pore pressure and shear stress with depth (on a vertical line located 40m away from right boundary) is illustrated in Figure 2.10. With such distance, the wave characteristics are close to critical condition and the influence of the implemented boundary conditions of the right side is minimized. Negligible shear stress and high magnitude of pore water pressure at seabed surface are observed that become small at seabed bottom. The hydrodynamic pressure variation along bed surface for an arbitrary time $t = 50s$ is plotted in Figure 2.10 and the

data are compared with results from decoupled approach. As shown in Figure 2.10, the dynamic pressure becomes greater with approaching shallower water due to increase in wave steepness.

In addition, for the given wave and soil characteristics, the dynamic pressure obtained from coupled and decoupled solutions are almost the same in magnitude and distribution.

2.3.4. Effect of Slope Steepness on Seabed Response

In order to investigate the impact of variation in seabed slope on system response, four slope angles of the seabed ($\beta = 0^\circ$, $\beta = 5^\circ$, $\beta = 10^\circ$, and $\beta = 15^\circ$) are studied. The sediment properties in all cases are the same and the seabed response with depth is evaluated. Two ways of comparison are possible (Figure 2.11): a comparison at two different locations but with the same water depth [see point 1 for different slope angles (a) and (b)], and a comparison at the same distance from the deep-water zone (left side of the domain) but with different water depths [see point 2 for different slope angles (a) and (b)].

Regarding the first comparison, as show in Figure 2.12, by increasing the seabed slope, the maximum pore water pressure and shear stress decrease, particularly close to the seabed surface. This is similar to the results obtained by Zhang and Jeng [36]. The main reason of such behavior is related to the flow regime, particularly adjacent to the interface of the domains, and shoaling effects of waves. In the case of horizontal seabed, the flow direction is parallel to the seabed surface and water flux into the subsurface domain is insignificantly small.

Consequently, the velocity of water particles normal to the interface will be small. Therefore, the energy of the fluid medium is slightly propagated through the seabed and the seabed acts as a rigid

with low permeability interface. However, by increasing the slope steepness, a higher flux between domains develops (with the normal velocity component of fluid rapidly increasing) and water percolation through the seabed is greater. This process leads to generation of smaller hydrodynamic pressure along the seabed surface and hence lower pore water pressure within the seabed sediments.

In relation to the shoaling effect, Figure 2.13 illustrates wave profile at a given time for two slope angles $\beta = 15^\circ$ and $\beta = 30^\circ$ where the domain lengths are selected as 220m and 240m, respectively (selection of domain's length is dictated by the effects of lateral boundary conditions). As shown in Figure 2.13, for higher steepness near the wave breaking zone (on the right end), for the similar range of water depths, the wavelength is reduced significantly (higher water depth ratio d/L) while the difference in wave height is not remarkable. Also, if the general form of dynamic pressure amplitude for the sloping seabed assumed to be similar to that of horizontal bed (Eq. (2.44)), the increasing rate of denominator ($\cosh(\frac{2\pi}{L}d)$) is higher than that of nominator (H). Thus, the dynamic pressure along the seabed surface for steeper slope may be smaller.

On the other hand, for points at the same distance from the deep water with various slope angles, point 2 indicated in Figure 2.11, (assuming constant wave height H_0 and water depth d_0 on the left side of the domain), smaller water depth for steeper slopes results in stronger waves with more remarkable wave steepness. Therefore, the corresponding dynamic pressure on bed surface may be greater [37].

The distribution of maximum hydrodynamic pressure on seabed as a function of the slope magnitude is demonstrated in Figure 2.14 using coupled and decoupled approaches. The difference

between the hydrodynamic pressure values obtained using both approaches as a function of slope magnitude is also shown in Figure 2.14. The shear modulus and permeability of sediment are chosen as $G = 10^8 Pa$ and $k = 10^{-2} m/s$, and the same wave characteristics are applied for each slope case. The ΔP and P_0 denote the dynamic pressure difference and the pressure obtained from decoupled approach, respectively. It is apparent that by increasing the slope magnitude, the dynamic pressure difference increases because larger flux into the seabed is developed albeit such increase is not significant.

2.3.5. Effect of Sediment Permeability and Degree of Saturation on Hydrodynamic Pressure

Parametric studies are performed to investigate the effect of sediment characteristics (permeability and degree of saturation) on the hydrodynamic pressure amplitude on seabed surface obtained from coupled and decoupled approaches. The variation of dynamic pressure difference with respect to sediment permeability under the same wave conditions is illustrated in Figure 2.15. A sloping seabed with the slope angle $\beta = 15^\circ$, shear modulus $G = 10^8 Pa$, and wave period $T = 7 \text{ sec}$ is chosen for this analysis. The hydrodynamic pressure for a generic point at $x = 200 \text{ m}$ (Figure 2.8) on the seabed surface is obtained for three permeability values $k = 10^{-3} \text{ m/s}$, $k = 10^{-5} \text{ m/s}$, and $k = 10^{-7} \text{ m/s}$.

Results indicated that an increase in permeability leads to higher dynamic pressure difference between coupled and decoupled solutions with the pressure amplitude obtained from the coupled approach being less than that of the decoupled solution. This is mostly due to the development of

water flux into the seabed that causes dissipation of water energy and smaller dynamic pressure; however, such difference in amplitude is small.

Similarly, the distribution of dynamic pressure difference as a function of degree of saturation of sediments (for three saturation conditions $S_r = 1, S_r = 0.98$ and $S_r = 0.95$) is indicated in Figure 2.15 for the sloping seabed ($\beta = 15^\circ$). The same wave condition with the wave period $T = 7 \text{ sec}$ is imposed in each case, and the shear modulus and permeability of sediments are selected as $G = 5 \times 10^7 \text{ Pa}$ and $k = 10^{-5} \text{ m/s}$, respectively. As shown in Figure 2.15, slight unsaturation leads to more compressibility of the soil layer, and consequently the seabed acts as a flexible base that dissipates the water energy and reduces the dynamic pressure in comparison with the rigid base; although, such reduction is not significant.

2.3.6. Wave-Induced Instantaneous Liquefaction

In this section, the extent of liquefied zone within the horizontal seabed is obtained for a given set of soil and wave characteristics and verified by comparing to results from an analytical study [16]. Then, the effect of slope angle on the extent of liquefaction is examined.

2.3.7. Model Verification for Horizontal Seabed: Instantaneous Liquefaction

Contour of zero mean effective stress for given soil and wave properties (Table 2.3) is indicated in Figure 2.16 and is compared with the analytical solution by Okusa [16]. As shown, the finite element model predicts the development of instantaneous liquefied zone with a good accuracy. The scale for horizontal axis is 1/14, and the width and maximum depth of liquefied zone are obtained as 83m and 1.9m. As mentioned earlier, for fully saturated soil, the instantaneously

liquefaction may not occur. This is also noted from the results that the depth of liquefied zone within the saturated seabed is measured as nearly zero.

2.3.8. Effect of Slope Steepness on Zones of Instantaneous Liquefaction

The effect of seabed slope on extension of instantaneous liquefied zones is studied considering three slopes ($\beta = 0^\circ$, $\beta = 10^\circ$, and $\beta = 15^\circ$). The sediment characteristics in all cases are the same and the liquefaction properties (width, depth, area) are evaluated under the same critical wave characteristics. It is assumed that seabed material is slightly unsaturated with the degree of saturation $S_r = 0.95$ uniformly through the depth, and the coefficient of lateral earth pressure at rest for the horizontal seabed is assumed as $K_0 = 0.5$.

The extent of the zone near the seabed surface, where critical wave conditions exist is indicated in Table 2.4. By increasing the slope, the wave-induced liquefied zone is getting smaller (and the corresponding depth decreases). This is because the seabed response is a function of the slope, as was shown earlier, and an increase in slope results in a decrease of induced stresses within the sediment. Therefore, in comparison with the horizontal case, at lower depth from the seabed surface the induced mean effective stress will be equal to geostatic mean stress of the seabed. Consequently, the depth of instantaneous liquefaction reduces for steeper slopes.

2.4. Conclusions

Response of sloping seabed under surface water waves and the development of instantaneous liquefaction are investigated through numerical models implemented within COMSOL [33]. Linear wave theory, Biot's equations for partly dynamic condition, and the coupled flow and linear

elastic deformation are used to formulate the model. The model is verified with analytical solutions for the case of horizontal seabed. A semi (or almost fully) coupled analysis of wave-seabed interaction together with response and instability (instantaneous liquefaction) of finitely steep seabed slopes have been considered for the first time in this study.

Based on the results obtained in this study, the following conclusions are made:

1. For the horizontal seabed, coupled and decoupled analyses yield nearly the same response. This observation is valid for a wide range of sediment properties and wave characteristics and is consistent with earlier findings.
2. For the sloping seabed, the pore water pressure, particularly close to surface, decrease with increasing slope steepness. Moreover, the shear stress amplitudes for steeper slopes appeared to be smaller. For the steeper slopes, larger water flux into the seabed and weaker wave profile lead to a lower pore pressure.
3. The coupled and decoupled analyses result in almost the same pore water pressure also in sloping seabed under critical wave conditions and various sediment properties. However, the differences in results from the two approaches become larger for steeper slopes as the flux between regions increases. Yet, the magnitude of flux into sloping bed is not large enough to produce a significant difference in seabed response.
4. For steeper slopes, the geometry (width and depth) of the zones of wave-induced instantaneous liquefaction diminishes with increasing slope angle.

The approach used in this study may be extended for the analyses of the wave-seabed-structure interaction for structures such as breakwater, platform, pipeline, and foundation systems supporting Marine Hydrokinetic devices (MHK). An extension to use a model for nonlinear soil deformation is possible. Consideration of nonlinear soil deformation may lead to larger differences between the coupled and uncoupled response.

Tables

Table 2.1.

Input data for horizontal seabed condition

Parameter	Description	Value
$L(m)$	Wavelength	32
$T(s)$	Wave Period	5
$H(m)$	Wave Height	3
$d(m)$	Water Depth	7
$D(m)$	Seabed Thickness	20
$k(m/s)$	Sediment Permeability	0.0001
n	Sediment Porosity	0.35
$K_w(Pa)$	Water Bulk Modulus	2×10^9
$\rho_f(Kg/m^3)$	Fluid Phase Density	1000
$\rho_s(Kg/m^3)$	Solid Phase Density	2600
ν	Poisson's Ratio	0.3
$G(Pa)$	Shear Modulus of Sediments	10^7

Table 2.2.

Input data for sloping seabed condition

Parameter	Description	Value
$T(s)$	Wave Period	7
$k(m/s)$	Sediment Permeability	0.0001
n	Sediment Porosity	0.35
$K_w(Pa)$	Water Bulk Modulus	2×10^9
$\rho_f(Kg/m^3)$	Fluid Phase Density	1000
$\rho_s(Kg/m^3)$	Solid Phase Density	1720
ν	Poisson's Ratio	0.3
$G(Pa)$	Shear Modulus of Sediments	5×10^7

Table 2.3.
Input data for verification of instantaneous liquefaction

Water Depth (m)	Wave Conditions			Sediment Properties				
	Height (m)	Period (s)	Length (m)	Sediment type	Compressibility (m^2/N)	Permeability (m/s)	Poisson's ratio	Bulk density (t/m^3)
20	5	15	197.42	Loose Sand	9.18×10^{-8}	10^{-4}	0.3	1.5

Table 2.4.
Effect of slope on wave-induced momentary liquefaction

Seabed Slope (deg)	Width (m)	Depth (m)	Area (m^2)
$\beta = 0$	36.3	1.0	23.9
$\beta = 10$	36.5	0.9	16.8
$\beta = 15$	26.4	0.7	12.9

Figures

Figure 2.1. Schematic figure of the reference problem

Figure 2.2. Temporal variations of sediment normal stress components and their average caused by wave action for different degree of saturation: $S_r = 1.0$ vs. $S_r = 0.95$

Figure 2.3. Computation chart of the coupled model

Figure 2.4. Response comparison of the fluid domain: maximum velocity and dynamic pressure profiles

Figure 2.5. Horizontal seabed response obtained from decoupled approach: FEM model vs. analytical solution

Figure 2.6. Variations of maximum pore water pressure (p) and dynamic pressure (p_f) in [Pa]

(a)

(b)

(c)

(d)

Figure 2.7. Comparison of coupled and decoupled approaches for horizontal seabed under critical wave condition: a) maximum pore water pressure, b) maximum shear stress, c) maximum vertical effective stress, d) hydrodynamic pressure distribution along seabed surface at $t = 50$ s

Figure 2.8. Problem dimensions and wave characteristics for the sloped seabed condition

Figure 2.9. Variations of shear stress with time for different mesh resolution

Figure 2.10. Sloping seabed response and comparison with decoupled approach: a) maximum pore water pressure and maximum shear stress, b) hydrodynamic pressure distribution along seabed surface at $t=50$ s

Figure 2.11. Possible comparison approaches (1 and 2) to evaluate effects of seabed steepness on the response for (a) sloping seabed with steep slope and (b) sloping seabed with mild slope

Figure 2.12. Effect of slope on seabed response

Figure 2.13. Comparison of wave profile at $t = 7 \times T$ sec for various slope angles

Figure 2.14. Variation of maximum hydrodynamic pressure with respect to slope magnitude for coupled and decoupled approaches

Figure 2.15. Hydrodynamic pressure difference between coupled and decoupled approach vs. a) permeability, b) degree of saturation

Figure 2.16. Instantaneous liquefied zone, FEM model vs. analytical solution [6]

CHAPTER 3. RESPONSE AND INSTABILITY OF SLOPING SEABED SUPPORTING MARINE STRUCTURES: WAVE-STRUCTURE-SOIL INTERACTION ANALYSIS

Some of the contents of this chapter have been published in the proceedings of Geo-Congress.

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A. Rafiei, M.A. Gabr, M.S. Rahman, Coupled Analysis of Wave, Structure, and Sloping Seabed Interaction: Response and Instability of Seabed, Geo-Congress, Philadelphia, Pennsylvania, 2019.

Abstract

Seabed instability due to water waves may cause significant damages to marine structures. Liquefaction-induced scour due to wave action may impact the stability and bearing capacity of the foundation element of coastal structures. The interaction of wave, seabed, and structure has been studied mostly for flat or slightly sloping seabed ($< 5^\circ$). However, some of structures may be built on or anchored to the seabed with significant steepness. In this paper, the wave-induced response and instantaneous liquefaction of sloping seabed supporting marine structures is evaluated by developing a coupled finite element model. Poroelasticity equations of the seabed in conjunction with governing equations for other domains (structure and fluid) are solved simultaneously using finite element method. The influence of coupling scheme being employed in this work on the pore water pressure is investigated, and the results are compared with decoupled solutions. The development of instantaneously liquefied zones near the foundation is studied in terms of steepness angle of seabed and wave conditions. The results indicated that seabed response and the extent of liquefied zone near the structure reduce for larger slope angles.

3.1. Introduction

Evaluation of response and instability of seabed considering wave-structure-seabed interaction is needed for a safe design of coastal and offshore facilities such as breakwaters, buried pipelines, offshore platform, and Marine Hydrokinetic (MHK) devices. Wave-induced seabed instability, rather than structural breakdown, has been identified as a major cause of damage to such structures [1-3]. After hurricane Katrina, liquefaction-induced scour around the foundation of coastal structures (resulting from soil instability due to wave action) led to substantial damages [4].

For shallower water depths, wave-induced instability of sediments within the seabed can stem from: (a) shear stress (due to wave loading on the bed surface) exceeding the shear strength of soil, and (b) reduction of shear strength caused by increase of pore water pressure as waves propagate along the water surface. Regarding the second cause, two extreme cases may occur: (i) instantaneous liquefaction in which the mean effective stress of nearly saturated sediments reaches zero as the phase shift of the normal stress components [16, 38]; (ii) the liquefaction caused by progressive buildup of pore water pressure associate with tendency of fully saturated soil elements for volume reduction under cyclic wave loadings [15]. The focus of this study is on instantaneous liquefaction of sediments around the structure.

The analysis of wave and seabed interaction has been carried out extensively during last decades mostly for mildly sloping seabed (slopes less than five degrees) following decoupled [18, 36] and rarely coupled [39, 39] approaches. In the decoupled analysis, the seabed response (displacement of soil particles and pore pressure) does not impact the fluid motion (above the seabed). However,

the interaction of the soil and water should be fully considered; thus, the deformability and porosity of sediments influence the water flow in the fluid domain and vice versa [17].

Xiao et al. [18] in a decoupled numerical analysis indicated the high potential for failure of sloping sand beach due to tsunami-induced instantaneous liquefaction. Rafiei et al. [38] demonstrated that the extent of instantaneous liquefaction along the bed surface becomes smaller for steeper slopes. The wave characteristics can impact the extent of instantaneous liquefied zones around mounted breakwaters significantly as shown by Jeng et al. [20]. Moreover, the response of gently sloping seabed supporting a breakwater and subjected to tsunami wave loadings has been examined in an uncoupled analysis [40]. The results indicated that instantaneous liquefaction is less probable near the structure's foundation as tsunami wave loadings induce compressional stresses in the seabed.

As mentioned, in the earlier works considered the wave-structure-seabed interaction using a decoupled or semi-coupled analysis (e.g. [20], [35]). However, the significance of using more accurate coupling scheme for response and instability of seabed around the structure's foundation is yet to be investigated. Furthermore, offshore foundation elements may have to be built on sloping seabed. This is particularly the case for Marine Hydrokinetic (MHK) devices deployed near the edge of Continental Shelf with steep slopes, a location where strong Gulf Stream currents are available for exploiting the water kinetic energy.

The extents of wave-induced instantaneous liquefaction have been investigated around the marine structures (built on gently sloping seabed) considering sediments as poroelastic materials [16, 38].

A coupled analysis of interaction between fluid, structure, and porous sloping seabed is considered. This accounts for: (a) the effect of fluid motion on the response of structure and seabed, (b) the effect of seabed response on the motion in the fluid domain (with the wave profile kept unchanged) and on structure response (displacements and stresses), and also (c) the effect of structure response on fluid and seabed. To this end, all the interface conditions among the domains (i.e. continuity of displacement, traction, and flux) are enforced.

In this paper, the response of domains subjected to wave, hydrostatic, and geostatic forces is studied, and the extent of instantaneous liquefaction is evaluated. This study is the continuation of our recent work on coupled analysis for response and instability of sloping seabed under wave action [38], with the addition of placement of structure mounted on the sloping seabed. The main aspects of the analyses are: (a) seabed response with significant slope angles, (b) wave-structure-sloping seabed interaction (almost fully coupled approach with known wave profile at water surface), and Partly-dynamic poroelasticity theory (u-p approximation) in which the acceleration of pore fluid is neglected is used to simulate the coupled flow and deformation of sediments [23, 24]. Potential flow theory is employed to formulate the fluid domain [25], and elastodynamic equations are adopted for the structure response. The governing equations are solved simultaneously using finite element method. The soil and fluid sub-models are verified by available closed-form solutions for the case of horizontal seabed [22, 25]. Angular distortion of the foundation due to wave loadings within a wave cycle is obtained as a function of soil properties. The impacts of seabed steepness on sediment response and instantaneous liquefaction around the foundation is examined. The effects of soil characteristics on the distribution of hydrodynamic pressure along the bed surface near the structure are evaluated for coupled and decoupled solutions.

This study for the first time presents an almost fully coupled analysis of wave-structure-seabed interaction together with response and instability (instantaneous liquefaction) of finitely steep seabed slopes.

3.2. Mathematical Model

The idealizations, governing equations, and boundary conditions for the problem consisting of fluid, structure, and sloping seabed domains are presented herein for two-dimensional plane strain coupled analyses. The schematic sketch of the reference problem is illustrated in Figure 3.1.

3.2.1. Wave-Structure-Seabed Interaction: Almost Fully Coupled Response

The interaction of fluid flow (caused by surface waves), structure, and sloping seabed is considered with inclusion of all the interface conditions (along S_1 to S_4 in Figure 3.1). However, it is assumed that the structure and seabed response do not impact the wave profile at surface significantly, and the wave profile is kept unchanged. Such almost fully coupled method includes the direct interactive processes among domains rigorously.

3.2.2. Equations Governing the Fluid Flow

Intermediate zone of water depth and non-breaking wave condition are considered in this study, and potential flow theory with linear waves is used assuming incompressible and inviscid water with irrotational flow expressed by [25]:

$$\nabla^2 \phi = 0 \quad (3.1)$$

where ϕ is velocity potential. The corresponding hydrodynamic pressure (p_f) and velocity (v_f) are given by:

$$v_f = -\nabla\phi \quad (3.2)$$

$$p_f = \rho_f \frac{\partial\phi}{\partial t} \quad (3.3)$$

in which ρ_f denotes water density. Although for severe storm conditions nonlinear wave idealizations will be more realistic [41], linear waves are adopted as the first approximation in this study.

3.2.3. Governing Equations for Structure

The structure and the footing (Figure 3.1) are considered as homogenous and isotropic media that both follow elastodynamic equations in plane strain condition. For structure, we can write:

$$G_{st}\nabla^2 u_{st} + \frac{G_{st}}{1-2\vartheta_{st}} \frac{\partial \varepsilon_{v,st}}{\partial x} + F_{B,x} = \rho_{st} \frac{\partial^2 u_{st}}{\partial t^2} \quad (3.4)$$

$$G_{st}\nabla^2 v_{st} + \frac{G_{st}}{1-2\vartheta_{st}} \frac{\partial \varepsilon_{v,st}}{\partial z} + F_{B,y} = \rho_{st} \frac{\partial^2 v_{st}}{\partial t^2} \quad (3.5)$$

in which G_{st} , ϑ_{st} , and ρ_{st} are shear modulus, Poisson's ratio, and density of the structure domain, respectively, u_{st} , v_{st} are displacement components, $\varepsilon_{v,st}$ is volumetric strain of the structure element, and F_B denotes the body force vector per unit volume.

3.2.4. Equations Governing Seabed Response

The seabed is assumed homogenous and isotropic, and the response is described by Biot's poroelasticity equations in partly dynamic form (u-p approximation as presented by Biot [23] and Zienkiewicz et al. [24]). The flow of pore fluid inside the sediment obeys Darcy's law, and in this form, the acceleration of pore water is neglected while the acceleration of solid particles is included in calculations. The combination of equilibrium and conservation of mass equations for pore water in a soil element can be written as [24]:

$$k \nabla^2 p - \gamma_w n \beta \frac{\partial p}{\partial t} + k \rho_f \frac{\partial^2 \varepsilon_v}{\partial t^2} = \gamma_w \frac{\partial \varepsilon_v}{\partial t} \quad (3.6)$$

where k , n are permeability and porosity of sediment, respectively and β , γ_w , and p denote compressibility, unit weight, and pressure of pore water, and ε_v is volumetric strain of the soil skeleton element. Respectively, for partly saturated sediment, the compressibility takes the form as:

$$\beta = \frac{1}{K_w} + \frac{1 - S_r}{p_{w_0}} \quad (3.7)$$

in which K_w , S_r and p_{w_0} represent bulk modulus, degree of saturation, and absolute water pressure, respectively. Overall equilibrium of the soil element when the total stress is decomposed into effective stress and pore water pressure is given by:

$$\frac{\partial \sigma_x'}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} - \frac{\partial p}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2} \quad (3.8)$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma_z'}{\partial z} - \frac{\partial p}{\partial z} = \rho \frac{\partial^2 v}{\partial t^2} \quad (3.9)$$

Here, σ_x' and σ_z' indicate the components of normal effective stresses, τ_{xz} is shear stress, and u and v are the components of displacement in horizontal and vertical directions of the soil skeleton, respectively. The total density of seabed sediment (ρ) is expressed as:

$$\rho = \rho_f n + \rho_s (1 - n) \quad (3.10)$$

in which ρ_s and n denote the density of soil solids and porosity. For isotropic linear elastic soil skeleton, the normal and shear stresses in plane strain condition are written as [27]:

$$\sigma_x' = 2G \left[\frac{\partial u}{\partial x} + \frac{\vartheta}{1 - 2\vartheta} \varepsilon_v \right] \quad (3.11)$$

$$\sigma_z' = 2G \left[\frac{\partial v}{\partial z} + \frac{\vartheta}{1 - 2\vartheta} \varepsilon_v \right] \quad (3.12)$$

$$\tau_{xz} = 2G \left[\frac{\partial u}{\partial z} + \frac{\partial v}{\partial x} \right] = \tau_{zx} \quad (3.13)$$

where G and ϑ are shear modulus and Poisson's ratio. The final form of governing equations is obtained by substitution of Eqs. (3.11) – (3.13) into Eqs. (3.8) – (3.9) and is expressed as:

$$G\nabla^2 u + \frac{G}{1-2\vartheta} \frac{\partial \varepsilon_v}{\partial x} = \frac{\partial p}{\partial x} + \rho \frac{\partial^2 u}{\partial t^2} \quad (3.14)$$

$$G\nabla^2 v + \frac{G}{1-2\vartheta} \frac{\partial \varepsilon_v}{\partial z} = \frac{\partial p}{\partial z} + \rho \frac{\partial^2 v}{\partial t^2} \quad (3.15)$$

The imposed shear modulus of sediments G is a function of effective octahedral stress (σ_{oct}') and initial porosity (n) of soil elements within the seabed. The following empirical equation is adopted for the analyses that is presented as [42]:

$$G = (6507 - 134n) \frac{p_{w_0}}{2(1 + \vartheta)} \left(\frac{\sigma_{oct}'}{p_{w_0}} \right)^{0.5} \quad (3.16)$$

For the analysis of wave-poroelastic soil interaction, the effective octahedral stress is assumed to be caused by geostatic action ($\sigma_{oct}' = \sigma_{oct-g}'$).

3.2.5. Boundary Conditions for Fluid Domain

The modeled fluid domain is assumed to be at the intermediate water depths prior to wave breaking zone. Progressive linear waves are assumed herein. The waves profile is computed, and the corresponding dynamic pressure (p_f) is imposed along the water surface (as prescribed boundary condition). For the case of horizontal seabed, such dynamic pressure is written as [25]:

$$p_f = \frac{\rho_f g H}{2} \cos(Kx - \omega t) \quad (3.17)$$

where $K = 2\pi/L$ is wave number (L being wavelength), $\omega = 2\pi/T$ wave circular frequency (T being wave period), and H denotes wave height.

By approaching shallower water, the wave height increases and the length decreases. For linear waves, the wavelength and period are correlated within dispersion equation given by [25]:

$$L = L_0 \tanh \left(2\pi \frac{d}{L} \right) \quad (3.18)$$

$$L_0 = \frac{gT^2}{2\pi} \quad (3.19)$$

where L_0 represents the wavelength in deep water.

A range of wave steepness in deep ocean is reported by Horikawa [28] based on field observations and is expressed as (H_0 denotes wave height in deep water):

$$0.008 \leq \frac{H_0}{L_0} \leq 0.055 \quad (3.20)$$

In shallower water based on conservation of energy, wave amplitude is computed as:

$$H = H_0 \left\{ \left[1 + \frac{4\pi d/L}{\sinh (4\pi d/L)} \right] \tanh \left(2\pi \frac{d}{L} \right) \right\}^{-1/2} \quad (3.21)$$

In lower depths near shoreline, waves tend to break after certain ratio of wave height to water depth. For the sloping seabed, the following empirical criterion for such threshold is adopted which is based on numerous experimental data [29, 30]:

$$\left(\frac{H}{d}\right)_b = \frac{b}{1 + \frac{ad}{gT^2}} \quad (3.22)$$

where

$$a = 43.75(1 - \exp(-19m)) \quad (3.23)$$

$$b = \frac{1.56}{1 + \exp(-19.5m)} \quad (3.24)$$

, and m denotes the seabed slope.

By implementing Eqs. (3.18) – (3.24), for a given wave period, the wave height and wavelength are computed corresponding to each water depth.

At wave-seabed interface (S_1 in Figure3.1), the continuity of fluid pressure and flux are imposed as [21]:

$$p_f = p \quad (3.25)$$

$$v_f = v_{pf} \quad (3.26)$$

where v_{pf} is the velocity of pore fluid inside the sediment. For partly-dynamic formulation of Biot's poroelastic theory, v_{pf} in vertical direction takes the form as [38]:

$$-\frac{\partial \phi}{\partial z} = -\frac{1}{n} \frac{k}{g} \frac{\partial^2 v}{\partial t^2} - \frac{1}{n} \frac{k}{\rho_f g} \frac{\partial p}{\partial z} + \frac{\partial v}{\partial t} \quad (3.27)$$

Along the common interface of fluid and structure (S_2 in Figure 3.1) the continuity of normal velocity to the interface applies (Eq. (3.28)), and the wave-induced dynamic pressure is equal to normal traction of structure (Eq. (3.29)):

$$v_{f,n} = v_{st,n} \quad (3.28)$$

$$p_f = \Gamma_{st,n} \quad (3.29)$$

in which $v_{f,n} = -\partial \phi / \partial n$ and $v_{st,n}$ are the normal component of velocities for fluid and structure, respectively, and $\Gamma_{st,n}$ indicates normal traction of the structure. Also, tangential component of the traction vector along fluid-structure interface ($\Gamma_{st,t}$) is set to zero (as fluid does not sustain any shear stress):

$$\Gamma_{st,t} = 0 \quad (3.30)$$

For a system of horizontal seabed and progressive surface waves, the computational domain can be truncated with a length equal to a coefficient of wavelength, and periodic boundary conditions can be employed on the lateral boundaries. However, this approach cannot be followed for the sloping seabed since the values of field variables will not be the same on the lateral edges of the domains. In this condition, a large domain is selected spanning from deep water to approximately breaking zone limit in shallower water, and zero hydrodynamic pressure on the two end boundaries

is applied as a reasonable assumption [38, 31]. A parametric study to determine the adequacy of domain length is performed to avoid the fictitious boundary effects on the fluid and soil response.

3.2.6. Boundary Conditions for Soil Domain

Along the common interface of water and seabed (S_1 in Figure 3.1), the continuity of pressure (Eq. (3.25)) and flux (Eq. (3.27)) is imposed, and vertical effective stress as well as shear stress are set to zero:

$$\sigma_z' = \tau_{xz} = 0 \quad (3.31)$$

At the soil-footing interface (S_4 in Figure 3.1), the displacements of the domains are equal (Eqs. (3.32)-(3.33)), traction vector (Eq. (34)) is continuous, and no flow condition (Eq. (3.35)) is applied (impermeable footing):

$$u = u_{ft} \quad (3.32)$$

$$v = v_{ft} \quad (3.33)$$

$$\Gamma \cdot n = \Gamma_{ft} \cdot n \quad (3.34)$$

$$\frac{\partial p}{\partial n} = 0 \quad (3.35)$$

where u_{ft} and v_{ft} are foundation displacement components, Γ and Γ_{ft} are total stress tensor of soil (combination of effective stress and pore water pressure tensors) and stress tensor of footing domain, respectively, and n denotes outward normal vector to the interface.

For horizontal seabed, along the lateral sides, periodic boundary conditions are imposed. While, for the two ends of the sloping seabed, horizontally fixed and impermeable boundaries are assumed that are expressed as [38]:

$$u = \frac{\partial p}{\partial x} = 0 \quad (3.36)$$

Also, the bottom of the seabed with finite depth is assumed as rigid and impermeable given by:

$$u = v = \frac{\partial p}{\partial z} = 0 \quad (3.37)$$

3.2.7. Boundary Conditions for Structure and Footing Domains

The conditions along surfaces S_2 and S_3 in Figure 3.1 are similar to the equations stated in the previous sections (Eqs. (3.28)-(3.30) and Eqs. (3.32)-(3.35)). Moreover, at the points along structure-footing common surface (S_4 in Figure 3.1), the continuity of displacement (Eqs. (3.38)-(3.39)) and traction vector (Eq. (3.40)) are introduced (perfect bond).

$$u_{st} = u_{ft} \quad (3.38)$$

$$v_{st} = v_{ft} \quad (3.39)$$

$$\Gamma_{st} \cdot n = \Gamma_{ft} \cdot n \quad (3.40)$$

3.2.8. Instantaneous Liquefaction Analysis

Marine sediment may be slightly unsaturated due to presence of air bubbles or gas in the sediment pores. Depending on the position of the wave profile, at some instances, the total mean effective

stress of the soil may reduce to zero (*i. e.* $\bar{\sigma}' = 0$) resulting in instantaneous liquefaction [5]. This state is expressed as:

$$\begin{aligned}\bar{\sigma}' &= 0 \text{ or} \\ \bar{\sigma}'_g + \bar{\sigma}'_w &= 0\end{aligned}\tag{3.41}$$

The first term represents the geostatic mean effective stress and the second is wave-induced mean effective stress. For flat seabed in plane strain condition (without structure), the above equation takes the form as:

$$\frac{1}{3}[(\rho_t - \rho_f)gz(1 + 2K_0) + (\vartheta + 1)(\sigma'_x + \sigma'_z)] = 0\tag{3.42}$$

in which ρ_t and K_0 denote total density and coefficient of lateral earth pressure at rest, respectively. For the case of sloping seabed with no structure, geostatic stresses are computed based on elasticity solutions of infinite sloping grounds [32] given by:

$$\sigma_{g,z} = \gamma z (1 + K \sin^2 \beta)\tag{3.43}$$

$$\sigma_{g,x} = K \gamma z \cos^2 \beta\tag{3.44}$$

where

$$K = \frac{K_0 + r_u(1 - K_0)}{\cos^2 \beta - K_0 \sin^2 \beta}\tag{3.45}$$

In the above, K_0 is the coefficient of lateral earth pressure at rest for level ground, and r_u is the pore water pressure ratio ($r_u = u/\gamma z$). For the full system with the presence of the structure, the geo-stresses inside the seabed due to the weight of the structure is added to the in situ stresses

expressed above (first term in the LHS of Eq. (3.42) for horizontal and Eqs. (3.43)-(3.45) for sloping seabed, respectively). Such stress field in soil is obtained by solving a boundary value problem accounting for gravitational body force of the structure.

It is noted that the saturation level impacts the amount of phase shift of the response for wave-induced normal stress components [16, 38]. For fully saturated sediments, the stress components are completely out-of-phase resulting in their mean magnitudes becoming zero. Thus, instantaneous liquefaction may not occur in fully saturated condition.

3.2.9. Steps of Computing the Coupled Response

In this study, all equations governing the domains are coupled through the interface conditions (mentioned in the previous sections). Then, the set of equations are solved simultaneously, and at each time step all the field variables (velocity potential of fluid ϕ , sediment displacement components u and v , and pore water pressure p , displacement components for structure, u_{st} , v_{st} and footing, u_{ft} , v_{ft}) are obtained (Figure 3.2). In this way, the interaction is modeled rigorously, and the response of domains (e.g. flow regime around the structure, seepage through the seabed, settlement of structure, etc.) are influence by one another.

3.3. Numerical Results and Discussion

The above governing equations in conjunction with boundary conditions are solved using finite element method in time domain following Backward Differentiation Formulas method (BDFs). Numerical models are developed in COMSOL [33] platform by introducing all equations. The

models are verified for the case of horizontal seabed, and then the effects of soil and wave characteristics on the response and instability of seabed is investigated.

3.3.1. Model Verifications for the Case of Horizontal Seabed

Herein, for the sake of verifications, the fluid-structure-sloping seabed model is simplified to the fluid-horizontal seabed system without the structure. The response of fluid and soil sub-models for the input data shown in Table 3.1 is evaluated, and the results are compared with analytical solutions [25]. A rectangular fluid domain with the length equal to one wavelength (33.7 *m*) and rigid impermeable base is considered. The domain is discretized with rectangular elements with the size of 0.4 *m* following sensitivity analysis of the response on mesh resolution. The response distribution within the depth for maximum dynamic pressure and maximum velocity components are illustrated in Figure 3.3. As shown, the results converge the closed-form solutions [22] considerably.

In order to verify the seabed sub-model, a prescribed hydrodynamic pressure along the seabed surface is applied, and the sediment response is obtained (decoupled approach). The results are compared with analytical solutions [22]. The hydrodynamic pressure distribution along the bottom of the fluid domain is given by [25]:

$$p_f(x, t) = P_0 * \cos\left(\frac{2\pi}{L}x - \omega t\right) \quad (3.46)$$

in which

$$P_0 = \frac{\gamma_w H}{2 \cosh\left(\frac{2\pi}{L}d\right)} \quad (3.47)$$

where γ_w denotes the unit weight of fresh water. The dimensions of the modeled seabed domain are selected as 33.7 m (seabed length equal to a wavelength) by 20 m (seabed thickness D). The employed soil and wave data are indicated in Table 3.1. Rectangular elements (structured mesh) are employed for discretization of the domain with the size of 0.4 m after conducting a convergence analysis on mesh density.

The solutions of maximum pore water pressure and shear stress within the seabed depth are demonstrated in Figure 3.4, and the results are compared with analytical solutions [22]. In Figure 3.4, $z/D = 0$ denotes the seabed surface. As shown, there is a good agreement among the model results and the analytical solutions. Moreover, the boundary condition along the seabed surface (Eq. (3.31)) is satisfied appropriately.

The extent of instantaneous liquefaction for sediments with a slight unsaturation degree also computed and verified [38].

3.3.2. Sloping Seabed Response: Fully Coupled Solution

A sloping seabed with the slope magnitude of 15° is considered (as presented in Figure 3.5). As shown in Figure 3.5, the wavelength and wave height (for a given wave period $T = 7s$) varies from 76.5m (in deep water) to 61.8m (close to breaking zone limit) and from 4m to 3.65m, respectively, following Eqs. (3.18) and (3.21). The parameters used in analyses are tabulated in Table 3.2. The domain length equal to 220m (which is almost three times of L_0) is selected to avoid lateral boundary effects, and the structure is placed 40m away from the right end. The water depth above the structure is 22m.

The FE mesh density and type used for discretization of domains are not constant. Rectangular elements with the size of $0.05m$ is employed for the structure and adjacent water and seabed domains whereas triangle elements with the maximum size of $2m$ are adopted away from the structure. The mesh resolution is selected after conducting parametric studies on the effects of element size having on the response.

The distributions of pore water pressure and shear stress within the sloping seabed with the demonstration of wave profile for an arbitrary time $t = 3/4 T$ are illustrated in Figure 3.6 and Figure 3.7. All dimensions indicated in the figures are scaled. As shown in Figure 6, the pore water pressure has the highest magnitudes along the bed surface and decreases within the seabed depth. In addition, positive and negative pore pressure occur under wave crest and wave trough, respectively. It is apparent from Figure 3.7 that the shear stress inside the sloping seabed has negligible magnitudes along the bed surface. This is consistent with Eq. (3.31). However, the presence of the structure increases the shear stress at its vicinity and near the bed surface due to wave loadings acting on the structure and consequently distortional deformations of soils below the structure.

The variations of maximum pore water pressure and maximum shear stress are plotted in Figure 3.8.a along a vertical line located at $\Delta x = 0.5m$ (a quarter of width dimension of the structure) from the upstream side (oceanside) of the structure. Negligible shear stress and high magnitude of pore water pressure at seabed surface are observed that become small at seabed bottom. The trend of these graphs is consistent with the surface plots demonstrated earlier in Figs. 3.6-3.7.

It is noted from the results that the footing (shown in Figure 1) has a rigid body rotation under wave loadings. The angular distortion of the footing within a wave cycle for various soil properties is plotted in Figure 3.8.b. The impacts of shear modulus of sediments, permeability, and degree of saturation (i.e. the effects of slight unsaturation caused by gas hydrates in the sand deposit) on the magnitudes of angular distortion are studied. The analyses are performed for several cases: change in shear modulus $G = 5 \times 10^7 Pa$, $G = 1 \times 10^8 Pa$ (for permeability $k = 10^{-5} m/s$ and saturation degree $S_r = 1.00$), change in permeability $k = 10^{-5} m/s$, $k = 10^{-3} m/s$ (for $G = 5 \times 10^7$ and $S_r = 1.0$), and change in saturation level $S_r = 1.0$, $S_r = 0.95$ (for $G = 5 \times 10^7 Pa$ and $k = 10^{-5} m/s$). For the sake of better comparisons, the shear modulus is selected as a constant value within the seabed depth. As demonstrated in Figure 3.8.b, the angular distortion of the footing has positive and negative values indicating various rotation direction of the structure's foundation within a wave cycle.

For soil with larger magnitude of shear modulus, the angular distortion reduces significantly (Figure 3.8.b) since stiffer soil below the footing provides more resistance against rotation. Increasing the soil permeability by a factor of 100 causes more dissipation of additional pore water pressure (due to wave action) leading to smaller effective stresses transferred to the seabed from the fluid domain (i.e. wave loadings). This yields smaller rotation of the footing; however, such reduction is small. As illustrated in Figure 3.8.b, slight levels of unsaturation in the seabed increases the compressibility of sediments leading to higher angular distortion of the softer soil, compared to fully saturated case.

3.3.3. Effect of Slope Steepness on Seabed Response

The seabed response in terms of pore water pressure and shear stress inside the seabed with various slope steepness ($\beta = 0^\circ$, $\beta = 10^\circ$, and $\beta = 15^\circ$) is studied. These field variables are obtained along vertical lines located at $\Delta x = 0.5m$ (a quarter of width dimension of the structure) from the upstream side (oceanside) of the structure. The sediment properties in all cases are kept constant. The input data used in the analyses are summarized in Table 3.2. As shown in Figure 3.9, for steeper seabed slopes, the magnitudes of maximum pore water pressure and maximum shear stress decrease [12]. This is due to two causes: (i) shoaling effects of waves (propagating towards shallower water) in which for steeper slopes, the reduction of wavelength is significant compared to milder slope angles (for given water depths) while the change in wave height is not remarkable [38]. This process leads to generation of smaller hydrodynamic pressure along the seabed surface and hence lower pore water pressure within the seabed sediments. (ii) Flow regime adjacent to the interface of domains are different, and higher water flux into the seabed occurs for steeper slopes causing energy propagation of water domain into the seabed. In this way, the seabed acts as flexible base dissipating water energy that results in low magnitudes of dynamic pressure [38].

The distributions of hydrodynamic pressure along the seabed surface within $\Delta x = 15w$ (w denotes the width dimension of the structure) from the structure (on both upstream and downstream sides) are obtained at $t = T/2$, as presented in Figure 3.10. Three slope angles $\beta = 0^\circ$, $\beta = 10^\circ$, and $\beta = 15^\circ$ are considered, and the pressure variations are demonstrated for coupled (solid line) and decoupled (marked points) solutions. Positive and negative magnitudes of Δx indicate left and right side of the structure in Figure 3.1. As shown in Figure 3.10, for all the inclination angles, the coupled and decoupled solutions lead to the almost the same trend and magnitude of hydrodynamic

pressure response for the given soil properties, wave characteristics, and dimensions of the structure. The maximum difference among the two solutions is computed as less than 2%. Such difference is similar to that of free field condition (with no structure) [38] implying that the presence of the structure may not influence the solution difference significantly. Also, it is noted that for the seabed with steeper slopes, the maximum dynamic pressure amplitude and wavelength are smaller compared to those for flat bed.

3.3.4. Effect of Sediment Permeability and Degree of Saturation on Hydrodynamic

Pressure

The maximum dynamic pressure on the sloping seabed surface near the structure within $\Delta x = 10w$ (w being width dimension of the structure) for a wave cycle is obtained from coupled and decoupled approaches. Various levels of permeability ($k = 10^{-3}m/s$, $k = 10^{-5}m/s$, and $k = 10^{-7}m/s$) and saturation levels ($S_r = 1$, $S_r = 0.96$, and $S_r = 0.93$) of sediments within the sloping seabed with the slope angle of $\beta = 15^\circ$ are considered, and the results are demonstrated in Figure 3.11. The wave characteristics ($H_0 = 4m$ and $T = 7sec$) are kept constant during all parametric analyses. The upstream and downstream sides in Figure 3.11 refer to left side and right side of the structure shown in Figure 3.5. As presented, for the case of various permeability (where $G = 10^8 Pa$, and $S_r = 1$), an increase in permeability level leads to higher dynamic pressure difference between coupled and decoupled solutions on both sides of the structure with the pressure amplitude obtained from the coupled approach being less than that of the decoupled solution. This is mostly due to the development of water flux into the seabed that causes dissipation of water energy and consequently smaller dynamic pressure; however, such difference in

amplitude is small. The results show higher pressure difference on the upstream side of the structure compared to that of downstream side.

For the case of various saturation levels ($G = 5 \times 10^7 Pa$, and $k = 10^{-5} m/s$), as presented in Figure 3.11, slight unsaturation leads to more compressibility of the soil layer, and consequently the seabed acts as a flexible base that dissipates the water energy and reduces the dynamic pressure in comparison with the rigid base; although, such reduction is not significant. Similarly, the pressure difference within the upstream of the structure is larger than that for the downstream side.

3.3.5. Effect of Structure's Dimensions on Hydrodynamic Pressure Distribution

The dimension of the structure built on the sloping seabed may alter the flow regime in its vicinity and consequently dynamic pressure distributions on the bed surface. To evaluate such behavior, a structure mounted on the sloping seabed with inclination angle $\beta = 15^\circ$ is considered, and the dynamic pressure on the bed surface is studied for a set of structure's dimensions: $H_s/w = 1, H_s/w = 2, H_s/w = 3$ where H_s and $w = 2m$ denote the height and width of the structure. The employed soil and wave characteristics are presented in Table 3.2.

The distributions of dynamic pressure on the bed surface near the structure are demonstrated in Figure 3.12.a at $t = T$. Positive and negative pressure magnitudes denote compression and tension forces on sediments. As shown in Figure 3.12.a, by increasing the structure's height, the dynamic pressure on the bed surface changes around 20% (increase and decrease in pressure for the right and left sides of the structure, respectively). Such difference becomes smaller further away from

the structure implying that the presence of structure is the major cause of such phenomenon that alters the flow regime in its vicinity.

The maximum dynamic pressure on the bed surface right beside the structure within a wave cycle is plotted in Figure 3.12.b. As shown, by increasing the ratio H_s/w , the maximum pressure grows higher on both sides of the structure. Moreover, as illustrated in Figure 3.12.b, such difference is almost the same for both sides (approximately by 20%).

3.3.6. Evaluation of Wave-Induced Instantaneous Liquefaction

Herein, the extents of zones of instantaneous liquefaction around the structure and near the seabed surface are evaluated. Liquefaction zones are identified by contours of zero total mean effective stresses inside the seabed. Parametric analyses are carried out to investigate the impacts of wave height and seabed slope angle on such extents, and the results are discussed.

3.3.7. Effects of Wave Height on Liquefaction Zones

A sloping seabed is considered with the slope angle of $\beta = 15^\circ$ consisting of slightly unsaturated sand ($S_r = 0.95$). Liquefaction extents are assessed for two wave conditions described as $H_0 = 4m$ and $H_0 = 2m$ (H_0 denotes wave height in deep water) for a given wave period of $T = 7sec$. Liquefaction contours within 30 m (equal to 15 times width dimension of the structure, $15w$) around the structure are calculated at different time steps of a wave cycle, and the results are summarized in Figure 3.13.

As shown in Figure 3.13, the locations of liquefied zones move through time as they occur below the wave trough that propagate from deep to shallower water. At the immediate vicinity of the structure, no liquefaction occurs since the weight of the structure increase the effective confining stresses inside the soil that suppresses liquefaction. The nearest location of liquefaction extents to the structure is computed as 1.62 m ($\cong 0.8w$) on the upstream side. As indicated in Figure 3.13, the width and depth of liquefaction for stronger waves are larger compared to waves with smaller wave heights (e.g. liquefaction depths $d_1 = 0.56\text{ m}$ and $d_2 = 0.32\text{ m}$ for $H_{0,1} = 4\text{ m}$ and $H_{0,2} = 2\text{ m}$, respectively at $t = 0.75T$). This is because that stronger waves with larger wave heights induce greater stresses and pores pressure inside the sediments. Also, liquefaction extents become nearer to the structure to some extent for waves with higher steepness.

3.3.8. Effects of Seabed Slope Angle on Liquefaction Extent

The effects of seabed inclination angle on the extent of instantaneously liquefied zones (caused by wave action) around the structure is investigated herein. Three slope angles $\beta = 0^\circ$, $\beta = 10^\circ$, and $\beta = 15^\circ$ are considered in which the wave and sediments characteristics are kept constant in all the cases. Sand deposit are assumed as slightly unsaturated with $S_r = 0.95$ uniformly though the seabed depth.

As described in Table 3.3, the liquefaction properties including width, depth, and area of liquefied zones and their distance with the structure are calculated. These predictions are for time steps in which wave trough (wave position that causes soil liquefaction) occurs right above the structure within a wave cycle. As shown in Table 3.3, in general, liquefaction extents reduce for steeper seabed slopes since wave-induced stresses in sediments become smaller for higher slope angles as

discussed in section 3.4. Thus, in comparison with the horizontal case, at lower depth from the seabed surface the induced mean effective stress will be equal to geostatic mean stress of the seabed. Consequently, the depth of instantaneous liquefaction (for instance) reduces for steeper slopes. This also impacts the location of instantaneous liquefaction zones with the structure where it becomes further away from the structure for larger slope angles of the seabed.

3.4. Conclusions

Response of sloping seabed around the foundation of marine structure subjected to storm waves are investigated in this paper, and the extents of instantaneous liquefaction in the sloping seabed are evaluated. Linear wave theory for the fluid, elastodynamic behavior of the structure, and hydromechanical properties of the porous seabed using poroelasticity are considered. The interaction of fluid, structure, and sloping seabed is studied through almost fully coupled analyses by performing numerical modeling implemented in COMSOL. The study of interaction of the three domains and its impacts on instantaneous liquefaction inside the seabed with finite steepness are the major contributions of this work.

The following conclusions are drawn based on the results described in this study:

1. The presence of structure mounted on the sloping seabed increases the magnitudes of shear stress at its vicinity compared to locations away from the structure. The reason is that wave loadings acting on the structure cause distortional deformations in sediments around the footing.

2. For the given wave and soil characteristics, the footing has rigid body rotation under cyclic wave loadings, and the related angular distortion of the footing is function of soil properties. Slight unsaturation of soil materials ($S_r = 0.95$) may increase the angular distortion of the footing by 20% since the soil compressibility increases. Consequently, the softer soil has less resistance to the footing rotation compared to fully saturated soil.
3. The amplitudes of maximum shear stress and maximum pore pressure inside the sediments decrease for higher slope angles of the seabed. This is caused by generation of weaker wave profiles above the seabed with steeper slopes and development of larger water flux into the seabed.
4. The distribution of hydrodynamic pressure along the bed surface near the structure are almost the same (less than 2% difference) for coupled and decoupled solutions for the given wave and soil characteristics. Such difference is similar to that of free field condition implying that the presence of the structure may not influence the solution difference significantly (coupled vs. decoupled).
5. Wave-induced instantaneous liquefaction may not occur in the immediate vicinity of the structure as the weight of the structure increase the effective confining stresses inside the soil that suppresses liquefaction. The width and depth dimensions of liquefied zone around the structure grow higher for steeper waves, and liquefaction may occur at shorter distances to the structure.

6. The extents of liquefied zones (width, depth, and area) and distance to the structure reduce for steeper slopes of the seabed. This is mostly due to the propagation of weaker surface waves and development of larger water flux into the steeper sloping seabed that yields to smaller magnitudes of wave loadings on the bed surface.

The analysis approach introduced in this study can be employed to investigate the interaction of nonlinear waves with structure and supporting sloping seabed. The accumulation of deformations and pore pressure within the granular soils may also alter the difference among coupled and decoupled solutions that needs to be evaluated in future studies.

Tables

Table 3.1.

Input data for horizontal seabed condition

Parameter	Description	Value
$L(m)$	Wavelength	33.7
$T(s)$	Wave Period	5
$H(m)$	Wave Height	1.5
$d(m)$	Water Depth	7
$D(m)$	Seabed Thickness	20
$k(m/s)$	Sediment Permeability	0.0001
n	Sediment Porosity	0.35
$K_w(Pa)$	Water Bulk Modulus	2×10^9
$\rho_f(Kg/m^3)$	Fluid Phase Density of Soil	1000
$\rho_s(Kg/m^3)$	Solid Phase Density of Soil	2600
ϑ	Poisson's Ratio	0.3
$G (Pa)$	Shear Modulus	10^7

Table 3.2.

Input data for sloping seabed condition

Parameter	Description	Value
$T(s)$	Wave Period	7
$k(m/s)$	Sediment Permeability	0.00001
n	Sediment Porosity	0.35
$K_w(Pa)$	Water Bulk Modulus	2×10^9
$\rho_f(Kg/m^3)$	Fluid Phase Density of Soil	1000
$\rho_s(Kg/m^3)$	Solid Phase Density of Soil	2600
$\rho_{st}(Kg/m^3)$	Material Density of Structure	2000
$\rho_{ft}(Kg/m^3)$	Material Density of footing	2400
ϑ	Poisson's Ratio	0.3
$G_{st}(Pa)$	Shear Modulus of Structure	7.7×10^9
$G_{ft}(Pa)$	Shear Modulus of Footing	2.1×10^{10}

Table 3.3.
Effect of slope on wave-induced instantaneous liquefaction around the structure

Seabed Slope (<i>deg</i>)	Width (<i>m</i>) (Left)	Width (<i>m</i>) (Right)	Depth (<i>m</i>) (Left)	Depth (<i>m</i>) (Right)	Area (<i>m</i> ²) (Total)	Min. Dist. With Structure (<i>m</i>)
$\beta = 0$	17.01	17.88	0.80	0.82	17.93	1.05
$\beta = 10$	21.72	9.40	0.60	0.31	10.30	1.21
$\beta = 15$	15.61	15.28	0.42	0.28	6.71	1.62

Figures

Figure 3.1. Conceptual image of the reference problem

Figure 3.2. Flowchart of the coupled analyses

Figure 3.3. Verification of fluid model: response comparison of the FE analysis vs. analytical solutions

Figure 3.4. Verification of soil model: response comparison of the FE analysis vs. analytical solutions

Figure 3.7. Variations of shear stress inside the seabed under surface waves
at $t = 0.75T$

Figure 3.8. Sloping seabed response: (a) maximum pore pressure and shear stress with depth at 0.5m from the left side of the structure, (b) impacts of change in shear modulus, permeability, and saturation degree on angular distortion of the footing within one wave cycle

Figure 3.9. Effect of slope on seabed response

Figure 3.10. Dynamic pressure distribution on the bed surface: Coupled (solid line) vs. Decoupled (marked points) solutions

Figure 3.11. Effects of permeability and saturation degree on hydrodynamic pressure magnitudes based on coupled and decoupled solutions

Figure 3.12. Dynamic pressure on the bed surface for various H_s/w : (a) pressure distribution at, $t = T$ (b) maximum pressure at the immediate vicinity of the structure within a wave cycle

Figure 3.13. Liquefaction zones for various wave conditions: $H_0 = 2m$ and $H_0 = 4m$

CHAPTER 4. WAVE-INDUCED INSTABILITY OF SEABED BENEATH GEOTEXTILE SAND CONTAINERS

Some of the contents of this chapter have been published in the proceedings of Geosynthetic Conference.

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Abstract

The use of geotextile tubes for shoreline protection has widely spread. In many cases, geotextile containers consist of geotextile layers that are filled hydraulically with dredge soil. The failure of these structures (as coastal protective systems) during storm events has been reported, as for example during Hurricane Ike. While numerous studies have investigated the internal stability of geotextile container, few are done on the stability due to wave action. The applied wave loading on geotextile containers are mostly obtained from simplified analytical solutions. In this paper, the external stability of geotextile container is studied with a focus on instability of the foundation soil (seabed) due to wave-induced instantaneous liquefaction. The characteristics of liquefaction around the structure is investigated through a parametric study. The loading distribution acting on the structure is evaluated. The full coupling between domains (wave, geotextile container, and seabed) is considered. The results show the development of liquefied zone around the geotextile container leading to instability of the structure under wave loading.

4.1. Introduction

The protection of shorelines from storm waves and sediment erosion is a significant challenge in coastal engineering. Among various coastal structures such as breakwaters and seawalls, geotextile sand containers (GSCs) have attracted attentions recently as they potentially offer an environmentally friendly cost-effective solution [45]. GSCs are comprised of a geotextile bag filled with a mixture of soil and water. Although GSCs are desirable beach protection system, numerous failure cases caused by wave action have been reported given the complexity of the loading scheme due to wave action and lack of knowledge in design procedures [46, 47, 11].

The instability of GSCs is classified into internal and external failure mechanisms [48]. The internal failure is mostly related to the tearing of geotextile, loss of the strength of fill material (due to increase in excess pore fluid pressure and consequent liquefaction) and internal movement of the confined soil. The external instability refers to sliding and overturning of the structure, loss of bearing capacity of foundation soil, and scour around the GSCs. Recio [45] in an experimental and numerical study examined the effects of deformation of GSC-structures on their hydraulic stability and concluded that deformation reduces contact areas and resisting forces between the containers resulting in instability of the structure. Neves [48] studied the wave-induced scour due to sediment transport near the geotextile encapsulated-sand systems. Based on laboratory measurements, Neves demonstrated that the maximum scour depth decreases with an increase in the water depth at the structure, and the scour depth is larger when waves are breaking near the toe of the structure [48].

Among recent studies in stability analysis of GSCs, there are only a few studies focusing on instability of foundation soil near the structure. The surface waves in shallower water (common deployment locations of GSCs) induces a significant amount of pore fluid pressure inside the seabed causing the reduction of effective stress in sediments. This, in turn, will decrease the shear strength of sediments and cause liquefaction in severe cases. Liquefaction-induced scour due to instability of seabed has been identified to be a major threat to coastal structures [4].

The extents of wave-induced instantaneous liquefaction have been investigated around the marine structures considering sediments as poroelastic materials [16, 38]. The wave-induced response and instability of seabed near the GSC with a focus on instantaneous liquefaction is studied. The full interactive process between media (fluid, GSC, and seabed) is included in the analysis; however, the effect of presence of the structure on the wave profile change is not considered herein. The deployment location of a single submerged GSC is assumed to be prior to wave breaking zone in shallow water where linear wave theory is applicable. Biot's poroelastic theory for the sediment, u-p approximation [23], linear waves and potential flow for the fluid [25], and elastodynamic equations [50] for the structure domains are solved simultaneously in a coupled finite element model. The distribution of wave loading acting on the structure as well as the extent of liquefied zone are presented.

4.2. Governing Equations

The equations governing each domain, fluid, geotextile, confined soil, and seabed are presented herein. The analyses are carried out for two-dimensional plane strain condition. The general sketch of the reference problem for a single submerged GSC is indicated in Figure 1.

4.2.1. Fluid Domain

GSCs may be installed in different locations. While, some of the GSCs are mounted near the beach line in which a portion of the structure may be above the mean water level, some are completely submerged but are still in shallow water (e.g. submerged sill). In shallow water various wave theories are applicable depending on wave characteristics [25]. In this study, it is assumed that the GSC is submerged and placed in shallow water prior to wave breaking zone. Also, the fluid is assumed to be inviscid and incompressible with irrotational flow; thus, linear wave theory is employed for modeling the waves. Although in storm condition, a more realistic wave loading is obtained based on nonlinear waves, linear waves are adopted as the first approximation in this work. The governing equation is expressed as:

$$\nabla^2 \phi = 0 \quad (4.1)$$

where ϕ denotes velocity potential. The velocity (v_f) and hydrodynamic pressure (p_f) corresponding to velocity potential are written as:

$$v_f = -\nabla \phi \quad (4.2)$$

$$p_f = \rho_f \dot{\phi} \quad (4.3)$$

in which ρ_f denotes water density.

4.2.2. GSCs Domain

The permeability of geotextile is normally within the range of sand fill [49]. In this study, however, since the external stability of the structure with respect to liquefaction in seabed is considered, the hydraulic properties of geotextile and confined soil are neglected, and the structure is assumed as elastic body defined by the following elastodynamic equations in plane strain condition:

$$G_{st} \nabla^2 u_{st} + \frac{G_{st}}{1-2\vartheta} \frac{\partial \varepsilon_{v,st}}{\partial x} + F_{B,x} = \rho_{st} \frac{\partial^2 u_{st}}{\partial t^2} \quad (4.4)$$

$$G_{st} \nabla^2 v_{st} + \frac{G_{st}}{1-2\vartheta} \frac{\partial \varepsilon_{v,st}}{\partial z} + F_{B,y} = \rho_{st} \frac{\partial^2 v_{st}}{\partial t^2} \quad (4.5)$$

in which G_{st} , ϑ , and ρ_{st} are shear modulus, Poisson's ratio, and density of the structure domain, respectively, u_{st} , v_{st} are displacement components, $\varepsilon_{v,st}$ is volumetric strain of the structure element, and F_B denotes the body force vector per unit volume. It is common to assume GSCs as impermeable media for the purpose of conducting external stability analyses (e.g. [11]).

4.2.3. Seabed Domain

The sediments are defined by Biot's poroelasticity theory in partly dynamic form, u-p approximation [23]. The acceleration of pore water is neglected in this formulation and it is assumed that the pore fluid flow obeys Darcy's law. For a homogenous isotropic medium the equations in plane strain condition take the form as:

$$G\nabla^2 u + \frac{G}{1-2\nu} \frac{\partial \varepsilon_v}{\partial x} = \frac{\partial p}{\partial x} + \rho \frac{\partial^2 u}{\partial t^2} \quad (4.6)$$

$$G\nabla^2 v + \frac{G}{1-2\nu} \frac{\partial \varepsilon_v}{\partial z} = \frac{\partial p}{\partial z} + \rho \frac{\partial^2 v}{\partial t^2} \quad (4.7)$$

$$k\nabla^2 p - \gamma_w n \beta \dot{p} + k \rho_f \frac{\partial^2 \varepsilon_v}{\partial t^2} = \gamma_w \frac{\partial \varepsilon_v}{\partial t} \quad (4.8)$$

where p is pore water pressure, and u and v are the soil skeleton displacements in horizontal and vertical directions, respectively, G shear modulus, ε_v volumetric strain of the soil element, γ_w water unit weight, k and n permeability and porosity of sediment. The total density of seabed sediment (ρ) is given by:

$$\rho = \rho_f n + \rho_s (1 - n) \quad (4.9)$$

in which ρ_s denotes the density of solid particles. The effects of partial saturation in sediments are captured by modifying the sediment compressibility (β) applicable for small degree of unsaturation written as:

$$\beta = \frac{1}{K_w} + \frac{1 - S_r}{p_{w_0}} \quad (4.10)$$

where K_w , S_r and p_{w_0} represent bulk modulus, degree of saturation, and absolute water pressure, respectively.

4.3. Boundary Conditions

At water surface, regular linear waves are applied, and the identical dynamic pressure is implemented (as a known boundary condition) written as:

$$p_f = \frac{\rho_f g H}{2} * \cos (Kx - \omega t) \quad (4.11)$$

where H is wave height, $K = 2\pi/L$ wave number (L as wave length), and $\omega = 2\pi/T$ wave circular frequency (T as wave period), g gravitational acceleration, x spatial variable in horizontal direction, and t denotes time.

The length of truncated computational domain is chosen equal to a coefficient of wavelength and periodic boundary conditions are implemented along lateral boundaries. Continuity of flux and pressure (hydrodynamic pressure in fluid domain with pore fluid pressure inside the sediments) at the interface of fluid and seabed domains (S_1 in Figure 1) are satisfied. Along the interface of fluid and GSC domains (S_2 in Figure 1), normal velocity of fluid particles ($\frac{\partial \phi}{\partial n}$) is equal to that of geotextile. Also, the dynamic pressure is identical to normal traction of structure.

The strong interaction and perfect bond between the geotextile layer and confined soil is assumed. Therefore, the continuity of displacement and normal traction are satisfied. No flow boundary condition as well as zero displacement of soil skeleton is assumed along the rigid base of seabed

with finite thickness. At the interface of GSC and seabed, a perfect bond is assumed in which no relative displacement between domains occur. Also, no flow boundary condition for pore fluid flow is implemented, and the normal component of total stress tensor of sediments is equal to the normal traction of the structure domain.

4.4. Numerical Results and Discussion

The above governing equations in conjunction with boundary conditions are solved using finite element method in time domain. All equations are introduced into COMSOL [33], and at each time step the output results of field variables ($\phi, u_{st}, v_{st}, u, v, p$) are obtained. A parametric study evaluating the effect of wave height on the extent of instantaneous liquefaction is carried out. The influence of soil and other wave characteristics on the response was not a part of this study.

4.4.1. Wave-Induced Seabed Response

The interaction problem of a single GSC mounted on horizontal seabed with finite thickness subjected to waves is studied (Figure 2), and the parameters used in analyses are tabulated in Table 4.1. The geotextile layer is assumed to have a 5 mm thickness with the density of 400 Kg/m^3 and Poisson's ratio $\nu=0.4$. The cross section of GSCs used in the modeling is part of an elliptical shape [11]; however, in this study, for simplicity and to obtain the normal component of stress vector for geotextile layer, circular cross section for GSC is selected with dimensions (height and width) similar to those of GSC structure deployed near Galveston shorelines in Texas.

The wave breaking threshold for progressive waves is chosen as $H/d=0.78$, and the criterion for water depth ratio of shallow water is $d/L \leq 0.05$ [25]. In order to simulate a storm condition, the

adopted values of wave steepness and water depth ratio are 0.75 and 0.06, respectively (the latter is slightly larger than $d/L = 0.05$). The domains are discretized by unstructured triangle mesh with various element size. Following a parametric study, the highest mesh resolution with the maximum element size of 2 mm is adopted for the thin geotextile layer and maximum element size at locations away from the structure is limited to 0.3 m.

The variations of wave-induced maximum pore water pressure and shear stress within the seabed for the points located on a vertical line at $x=0.5$ m from right side of the structure are shown in Figure 3 (for the wave height $H = 3$ m). Negligible shear stress and high pore water pressure magnitude at seabed surface are observed near the seabed surface.

4.4.2. Wave-induced Instantaneous Liquefaction

The waves generate hydrodynamic pressure within the fluid domain. This pressure is equal to pore fluid pressure along the seabed surface. The presence of excess pore water pressure in sediments reduces the effective stress and consequently the shear strength of seabed. Under certain wave conditions, the mean effective stress may reach zero values (instantaneous liquefaction) expressed as:

$$\sigma'_m = 0 \quad \text{or}$$

$$\bar{\sigma}'_g + \bar{\sigma}'_w = 0 \quad (4.12)$$

In the above equation, $\bar{\sigma}'_g$ and $\bar{\sigma}'_w$ denote mean effective stress for geostatic and wave condition, respectively. The geostatic term is obtained accounting for the submerged weight of GSC and seabed.

The extent of instantaneous liquefaction near the structure is shown in Figure 4 for two wave heights $H = 3\text{ m}$ and $H = 1.5\text{ m}$ at the same time $t=T$ assuming that sediments are partly saturated with a uniform degree of saturation $S_r = 0.98$ within the seabed. The presence of gaseous sediments within the seabed is not uncommon [16]. The slight unsaturation of the seabed is mostly due to gas bubbles occluded in the pore water. However, partly-saturated sediments are expected to be near the bed surface as the pressure at larger depth will force the air into the pore water and increases the saturation level. The dimensions of liquefied zones are scaled with respect to dimensions of the GSC.

As shown in Figure 4, instantaneous liquefaction occurs below the wave trough since in those locations an upward seepage force is developed in the seabed resisting the downward gravitational forces (at each time step instantaneous liquefaction occurs in the seabed depending on the wave profile, and in Figure 4 it is shown for $t=T$). Stronger waves induce greater stresses inside the sediments resulting in larger liquefied zones. The weight of structure suppresses liquefaction to some extent and, as shown, as the maximum liquefaction depth occurs at locations away from the structure. The sediments within the liquefied zones may lose their shear strength resulting in sediment transport and scour around the structure. In such condition, and due to reduction of soil confinement, bearing capacity issues of the foundation soil are probable leading to sinking of the GSC into the seabed and differential settlements (depending on the location of liquefaction zones). Similar scenario occurred during Hurricane Ike as shown in the earlier study by Khalilzad and Gabr [11].

It is worth mentioning that no instantaneous liquefaction within the seabed has been observed for fully saturated sediments. Such observation is consistent with analytical solutions [22]. The gradient of wave-induced pore water pressure within the gas-laden sediment near the seabed surface is higher than that for the saturated sediment. Thus, the upward seepage force resulting in instantaneous liquefaction may be larger for partly-saturated sediment. Another form of liquefaction due to progressive buildup of pore water pressure (progressive liquefaction) has been observed in loose to medium-dense non-cohesive fully saturated sediments [14].

4.4.3. Wave Loading on GSC

The distribution of wave loading acting on GSC for different times are illustrated in Figure 5. The loading values are related to the position of wave profile. At $t=T$ and $t=T/2$, right above the centerline of GSC wave crest and wave trough are occurring, respectively leading to positive and negative values of dynamic pressure, correspondingly, with symmetric distributions. Similarly, at $t=T/4$ and $t=3T/4$ the wave crest generates at the surface on the right and left side of the structure, respectively. In these time steps, non-symmetric loading distribution induced by the wave is acting on the structure. The non-symmetric loading distribution may lead to differential settlement of the GSC, with an impact on the stability of the GSC-structures, especially if these were constructed in stacked configuration.

4.5. Conclusion

In this study, response and instability of foundation soil around the GSC under progressive surface waves are analyzed. The distribution of loading on the structure under wave action is also obtained. The related governing equation and boundary conditions are programmed into a numerical model

in COMSOL, for coupled analyses of the wave, geotextile container, and seabed domains. The following conclusions are obtained based on the results presented herein:

1. In shallow water, the pore water pressure within the seabed sediments caused by waves is significant and results in instantaneous liquefaction around the GSC. Such liquefaction can lead to scour and loss of soil confinement around the structure.
2. Even for smaller wave heights, instantaneous liquefaction appears to be probable. However, the weight of the structure suppresses such liquefaction to some extent.

It is noted that the values and distribution of wave loading on the GSC is a function of wave profile. At a certain time with respect to non-symmetrical loading distribution, differential settlement can occur given the difference in loading magnitude along the structure cross section. Such deformation can lead to affect the serviceability and instability of the geotextile sand container-structures.

Tables

Table 4.1.

Input data

Wave Characteristics			
Wave Period (<i>s</i>)	10	Wave Height (<i>m</i>)	1.5,3
Seabed and GSC Characteristics			
Permeability (<i>m/s</i>)	1×10^{-4}	Porosity	0.35
Water Bulk Modulus (<i>Pa</i>)	2×10^9	Fluid Phase Density (<i>Kg/m³</i>)	1000
Solid Phase Density (<i>Kg/m³</i>)	2650	Poisson's Ratio	0.3
Shear Modulus of Sediment (<i>Pa</i>)	5×10^7	Shear Modulus of Sand Fill (<i>Pa</i>)	5×10^7
Shear Modulus of Geotextile (<i>Pa</i>)	2×10^9		

Figures

Figure 4.1. Schematic sketch of the reference problem

Figure 4.2. Model dimensions and wave characteristics

Figure 4.3. Distribution of pore water pressure and shear stress within seabed depth for points at 1.9 m distance from the centerline of the GSC

Figure 4.4. Extent of wave-induced Instantaneous liquefaction (dark blue) near the GSC at $t = T$ for different wave heights: a) $H=3$ m, b) $H=1.5$ m

Figure 4.5. Distribution of wave loading on the GSC: a) for different times ($t=T/4$, $T/2$, $t=3T/4$, $t=T$), b) free body diagram at $t=3T/4$

CHAPTER 5. WAVE-INDUCED INSTABILITY OF SLOPING SEABED: GLOBAL SHEAR FAILURE

Some of the contents of this chapter have been presented in Engineering Mechanics Institute Conference.

Citation:

Rafiei A., Rahman M.S., Gabr M.A., A. Ortiz, “Coupled Analysis of Wave-Sloping Seabed Interaction: Global Shear Failure”, Engineering Mechanics Institute Conference at CalTech, Pasadena, California, 2019.

Abstract

Instability of sloping seabed is a vital factor in foundation design of marine structures and geohazard studies of submarine landslides. Water surface waves may cause local (e.g. liquefaction-induced scour) and global instability due to shear failure (over a large zone) of sediments within the sloping seabed. The wave-induced sliding of sloping seabed was reported after Hurricane Carla damaged South East of US. The related studies of global failure evaluation of sloping seabed have been focused mostly on slightly inclined ($< 5^\circ$) bathymetry and utilized decoupled analysis approach. However, some of the marine structures are founded on (or anchored to) seabed with steeper slopes. In this study, the global shear failure of the sloping seabed due to wave and geostatic forces is evaluated using finite element with strength reduction method. The coupling of response for fluid and porous soil domains (i.e. enforcing continuity of pressure and water flux) is considered. The results indicate that the wave action reduces the global stability of the sloping seabed, and such instability is more pronounced for slopes consisting of sediments with slight unsaturation. In addition, the waves may alter the shape of potential slip surfaces inside the seabed. Parametric analyses are carried out to evaluate the effects of seabed slope angle and wave characteristics on the global shear failure of seabed, and the results are discussed.

5.1. Introduction

Waves may cause instability in submarine slopes resulting in landslides with large displacements. The wave-induced sliding of seabed slopes was first reported by [51] after Hurricane Carla damaging South East of US in 1961 [52]. The related excessive deformations in submarine landslides may cause crack of pipelines and mooring lines [53]. Therefore, analysis of submarine slides is a significant aspect in a safe design of marine structures as well as marine geological disaster studies [54].

For shallower water depths, wave-induced instability of sediments within the seabed can stem from: (a) shear stress (due to wave loading on the bed surface) exceeding the shear strength of soil, and (b) reduction of shear strength caused by increase of pore water pressure as waves propagate along the water surface. Under cyclic wave loadings, some of the changes in excess pore pressure are recoverable meaning that by removal of the loading, the magnitudes of pore pressure converge to hydrostatic distribution. However, the buildup of pore pressure can be progressive caused by tendency of fully saturated soil elements for volume reduction under cyclic wave loadings. This yields to perpetual softening of sediments with excessive deformations accumulated at the end of each cycle [9, 42, 43, 44]. For computing the wave-induced stress and pore pressure within the sediments, the focus in this study is on recoverable responses.

The instability of slopes has been identified as a fundamental problem in geotechnical engineering which has been studied mostly in the last decades using limit equilibrium methods [10]. Among them, the ordinary method of slides [56], Bishop's Modified Methods [57], force equilibrium methods [58], Janbu's general procedure of slices [59], and Spencer's method [60]

can be mentioned. Strength Reduction Finite Element method (SRFE) as an alternative to traditional approaches has been introduced initially by Zienkewics [24]. In this technique, the shear strength of soil is reduced until the equilibrium is no longer preserved within the slope, and significant part of the slope fails showing excessive deformations [10]. Then, the factor of safety (FOS) for the slope (representing the stability condition) is defined as ratio of original shear strength parameters to the reduced values.

Griffiths and Lane [10] discussed the advantages of the strength reduction technique in slope stability analyses using numerical elastoplastic finite element models and compare the computed FOS of slopes with the one obtained from the traditional methods. The strength reduction technique using finite difference method is applied by Dawson et al. [61] and the stability number for slopes with various soil properties is compared with upper bound limit analysis solution based on log-spiral failure mechanism [62]. A good agreement between approaches is obtained concluding the solutions diverge slightly for steeper slopes with higher friction angles of soil. The applicability of strength reduction method has been expanded to other geotechnical stability problems, as well, such as cantilever wall, braced excavation and retaining walls [63].

Submarine slope stability analyses date back to 1970s when Henkel [64] studied the instability caused by gravity and waves assuming mild steepness of a finite submarine slope with circular slip surface using traditional limit equilibrium methods [64]. Rahman [14] in an analytical study investigated the instability of infinite gently sloping seabed using the limit equilibrium method and presented a correlation between wave characteristics and instability of the slope. In this study, the effects of instantaneous and progressive buildup of pore water pressure due to wave action in shear

strength of sediments are considered. Recently, in a similar approach, Wang and Liu [66] performed numerical analyses of storm wave-induced instability of silty seabed using more advanced constitutive models for sediments [67]. They demonstrated various mechanisms of slope instability such as scour, liquefaction, seepage, and shear slides.

A few studies have been carried out in submarine slope instability using strength reduction technique. Cao et al. [54] studied the submarine landslides using SRFE under gravitational loading only (no wave action). They considered submerged unit weight of sediments and used elasto-plastic constitutive model with Mohr-Coulomb yield criterion and nonassociated flow rule to describe the soil behavior. They observed wider sliding shear zone than that of slopes in dry condition. Application of SRFE accounting for surface waves for gently sloping shores has been evaluated by Nian et al. [52] demonstrating that the presence of waves reduces that stability number of the slope.

As mentioned earlier, previous studies mostly considered analysis of seabed with mild steepness and included the wave-induced stresses and pore pressure within the seabed with some simplification following decoupled approach. However, some of the submarine slopes may have substantial slope angles, e.g. deployment location of MHKs at North Carolina coast with steep bed angle and strong currents (high potential for water energy conversion to electricity). Therefore, a more robust analysis of instability of sloping seabed is needed accounting for greater slope angles and interactive processes among the fluid and soil media.

In the work presented herein, a coupled analysis of interaction between fluid and porous sloping seabed is considered. This accounts for: (a) the effect of fluid motion on the response of seabed, (b) the effect of seabed response on the motion in the fluid domain (with the wave profile kept unchanged). To this end, all the interface conditions among the domains (i.e. continuity of pressure and flux) are enforced.

In this study, the effects of waves on global shear failure instability of sloping seabed are investigated. This paper is the continuation of our recent work on coupled analysis for response and instability of sloping seabed under wave action [38], concentrating on global shear failure (in large scales) of the sloping seabed. The main aspects of this investigation are: a) submarine slopes with significant steepness is studied; and b) wave-seabed interaction is simulated using a coupled approach resulting in stress and pore fluid pressure fields in sediments needed for slope instability evaluations. To this aim, first, the stresses in the sloping seabed due to gravity is obtained. Second, the stresses and pore pressure within the seabed caused by waves is computed by solving equations governing the fluid (potential flow theory [25]) and seabed (u-p approximation of Biot's equations [24] in which the acceleration of pore fluid is neglected [24]) media following coupled approach. All the equations are solved simultaneously using finite element method. Third, the instability of sloping seabed is evaluated accounting for combination of gravity and wave forces using strength reduction technique based on finite element method. In this phase, the seabed is assumed as elastic-perfectly plastic soil with Mohr-Coulomb yield criterion and associated flow rule.

The responses of soil and fluid sub-models under propagation of waves are verified by available closed-form solutions for the case of horizontal seabed [25, 22]. The distributions of shear stress

and pore pressure inside the sloping seabed is computed (from coupled analyses). For the instability assessments of the sloping seabed, first, the numerical models are developed and verified with closed-form solutions [55] against geostatic forces. Then, the effects of waves on the stability of the slope is studied by evaluating effective stress paths of soil elements. The shapes of potential slip surfaces inside the seabed as a function of wave profile are examined. Parametric analyses to study the effects of sediment properties, wave characteristics, and seabed slope angle on FOS of the seabed are performed, and the results are discussed. This study for the first time presents a detailed analysis on the variations of stability condition for finitely steep seabed slopes including the interaction of waves and the sloping seabed.

5.2. Problem Description and Mathematical Model

The idealizations, governing equations, and boundary conditions for the problem consisting of fluid and sloping seabed domains are presented herein for two-dimensional plane strain coupled analyses. The conceptual image of the reference problem is illustrated in Figure 5.1.

5.2.1. Wave-Seabed Interaction: Almost Fully Coupled Response

The interaction of fluid flow (caused by surface waves) and sloping seabed is considered with inclusion of all the interface conditions. However, it is assumed that the seabed response does not impact the wave profile at surface significantly, and the wave profile is kept unchanged. Such almost fully coupled method includes the direct interactive processes among domains rigorously.

5.2.2. Equations Governing the Fluid Flow

Intermediate zone of water depth and non-breaking wave condition are considered in this study, and potential flow theory with linear waves is used assuming incompressible and inviscid water with irrotational flow expressed by [25]:

$$\nabla^2 \phi = 0 \quad (5.1)$$

where ϕ is velocity potential. The corresponding hydrodynamic pressure (p_f) and velocity (v_f) are given by:

$$v_f = -\nabla \phi \quad (5.2)$$

$$p_f = \rho_f \frac{\partial \phi}{\partial t} \quad (5.3)$$

in which ρ_f denotes water density. Although for severe storm conditions nonlinear wave idealizations will be more realistic [26], linear waves are adopted as the first approximation in this study.

5.2.3. Governing Equations for Seabed

The seabed is assumed homogenous and isotropic, and the response is described by Biot's poroelasticity equations in partly dynamic form (u-p approximation as presented by Biot [23] and Zienkiewicz et al. [24]). The flow of pore fluid inside the sediment obeys Darcy's law, and in this form, the acceleration of pore water is neglected while the acceleration of solid particles is included

in calculations. The combination of equilibrium and conservation of mass equations for pore water in a soil element can be written as [24]:

$$k \nabla^2 p - \gamma_w n \beta \frac{\partial p}{\partial t} + k \rho_f \frac{\partial^2 \varepsilon_v}{\partial t^2} = \gamma_w \frac{\partial \varepsilon_v}{\partial t} \quad (5.4)$$

where k , n are permeability and porosity of sediment, respectively and β , γ_w , and p denote compressibility, unit weight, and pressure of pore water, and ε_v is volumetric strain of the soil skeleton element. Respectively, for partly saturated sediment, the compressibility takes the form as:

$$\beta = \frac{1}{K_w} + \frac{1 - S_r}{p_{w_0}} \quad (5.5)$$

in which K_w , S_r and p_{w_0} represent bulk modulus, degree of saturation, and absolute water pressure, respectively. Overall equilibrium of the soil element when the total stress is decomposed into effective stress and pore water pressure is given by:

$$\frac{\partial \sigma_x'}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} - \frac{\partial p}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2} \quad (5.6)$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma_z'}{\partial z} - \frac{\partial p}{\partial z} = \rho \frac{\partial^2 v}{\partial t^2} \quad (5.7)$$

Here, σ_x' and σ_z' indicate the components of normal effective stresses, τ_{xz} is shear stress, and u and v are the components of displacement in horizontal and vertical directions of the soil skeleton, respectively. The total density of seabed sediment (ρ) is expressed as:

$$\rho = \rho_f n + \rho_s(1 - n) \quad (5.8)$$

in which ρ_s and n denote the density of soil solids and porosity. For isotropic linear elastic soil skeleton, the normal and shear stresses in plane strain condition are written as [25]:

$$\sigma_x' = 2G \left[\frac{\partial u}{\partial x} + \frac{\vartheta}{1 - 2\vartheta} \varepsilon_v \right] \quad (5.9)$$

$$\sigma_z' = 2G \left[\frac{\partial v}{\partial z} + \frac{\vartheta}{1 - 2\vartheta} \varepsilon_v \right] \quad (5.10)$$

$$\tau_{xz} = 2G \left[\frac{\partial u}{\partial z} + \frac{\partial v}{\partial x} \right] = \tau_{zx} \quad (5.11)$$

where G and ϑ are shear modulus and Poisson's ratio. The final form of governing equations is obtained by substitution of Eqs. (5.9) – (5.11) into Eqs. (5.6) – (5.7) and is expressed as:

$$G \nabla^2 u + \frac{G}{1 - 2\vartheta} \frac{\partial \varepsilon_v}{\partial x} = \frac{\partial p}{\partial x} + \rho \frac{\partial^2 u}{\partial t^2} \quad (5.12)$$

$$G \nabla^2 v + \frac{G}{1 - 2\vartheta} \frac{\partial \varepsilon_v}{\partial z} = \frac{\partial p}{\partial z} + \rho \frac{\partial^2 v}{\partial t^2} \quad (5.13)$$

5.2.4. Boundary Conditions for Fluid Domain

The modeled fluid domain is assumed to be at the intermediate water depths prior to wave breaking zone. Progressive linear waves are assumed herein. The waves profile is computed, and the corresponding dynamic pressure (p_f) is imposed along the water surface (as prescribed boundary condition). For the case of horizontal seabed, such dynamic pressure is written as [25]:

$$p_f = \frac{\rho_f g H}{2} \cos(Kx - \omega t) \quad (5.14)$$

where $K = 2\pi/L$ is wave number (L being wavelength), $\omega = 2\pi/T$ wave circular frequency (T being wave period), and H denotes wave height.

By approaching shallower water, the wave height increases and the length decreases. For linear waves, the wavelength and period are correlated within dispersion equation given by [25]:

$$L = L_0 \tanh\left(2\pi \frac{d}{L}\right) \quad (5.15)$$

$$L_0 = \frac{gT^2}{2\pi} \quad (5.16)$$

where L_0 represents the wavelength in deep water.

A range of wave steepness in deep ocean is reported by Horikawa [28] based on field observations and is expressed as (H_0 denotes wave height in deep water):

$$0.008 \leq \frac{H_0}{L_0} \leq 0.055 \quad (5.17)$$

In shallower water based on conservation of energy, wave amplitude is computed as:

$$H = H_0 \left\{ \left[1 + \frac{4\pi d/L}{\sinh(4\pi d/L)} \right] \tanh \left(2\pi \frac{d}{L} \right) \right\}^{-1/2} \quad (5.18)$$

In lower depths near shoreline, waves tend to break after certain ratio of wave height to water depth. For the sloping seabed, the following empirical criterion for such threshold is adopted which is based on numerous experimental data [29, 30]:

$$\left(\frac{H}{d} \right)_b = \frac{b}{1 + \frac{ad}{gT^2}} \quad (5.19)$$

where

$$a = 43.75(1 - \exp(-19m)) \quad (5.20)$$

$$b = \frac{1.56}{1 + \exp(-19.5m)} \quad (5.21)$$

, and m denotes the seabed slope.

By implementing Eq. (5.15) and Eq. (5.18), for a given wave period, the wave height and wavelength are computed corresponding to each water depth.

At wave-seabed interface (S_1 in Figure 5.1), the continuity of fluid pressure and flux are imposed as [21]:

$$p_f = p \quad (5.22)$$

$$v_f = v_{pf} \quad (5.23)$$

where v_{pf} is the velocity of pore fluid inside the sediment. For partly-dynamic formulation of Biot's poroelastic theory, v_{pf} in vertical direction takes the form as [38]:

$$-\frac{\partial \phi}{\partial z} = -\frac{1}{n} \frac{k}{g} \frac{\partial^2 v}{\partial t^2} - \frac{1}{n} \frac{k}{\rho_f g} \frac{\partial p}{\partial z} + \frac{\partial v}{\partial t} \quad (5.24)$$

For a system of horizontal seabed and progressive surface waves, the computational domain can be truncated with a length equal to a coefficient of wavelength, and periodic boundary conditions can be employed on the lateral boundaries. However, this approach cannot be followed for the sloping seabed since the values of field variables will not be the same on the lateral edges of the domains. In this condition, a large domain is selected spanning from deep water to approximately breaking zone limit in shallower water, and zero hydrodynamic pressure on the two end boundaries is applied as a reasonable assumption [31, 38]. A parametric study to determine the adequacy of domain length is performed to avoid the fictitious boundary effects on the fluid and soil response.

5.2.5. Boundary Conditions for Soil Domain

Along the common interface of water and seabed (Figure 5.1), the continuity of pressure (Eq. (5.22)) and flux (Eq. (5.24)) is imposed, and vertical effective stress as well as shear stress are set to zero:

$$\sigma_z' = \tau_{xz} = 0 \quad (5.25)$$

For horizontal seabed, along the lateral sides, periodic boundary conditions are imposed. While, for the two ends of the sloping seabed, horizontally fixed and impermeable boundaries are assumed that are expressed as [38]:

$$u = \frac{\partial p}{\partial x} = 0 \quad (5.26)$$

Also, the bottom of the seabed with finite depth is assumed as rigid and impermeable given by:

$$u = v = \frac{\partial p}{\partial z} = 0 \quad (5.27)$$

5.2.6. The Strength Reduction Technique: Basics

Strength Reduction Technique is a numerical tool to evaluate instability of slopes and to obtain critical slip surfaces and deformed shape of the slope at failure. This technique has several advantages over traditional methods in slope stability analyses listed briefly as [10]:

- No assumptions are needed for shape and location of failure surface, and the critical sliding surface is the output of the analysis.
- No assumptions for side forces along slides (used in traditional methods) are needed.
- Capable of including soil nonlinear behavior, nonhomogeneity and awkward slope geometries.
- Progressive failure of the slope and deformations at failure can be demonstrated.

The common definition of the factor of safety is the ratio of available shear strength of soil to the shear strength required for equilibrium of the slope. In this technique, the soil shear strength is reduced until the equilibrium is no longer preserved within the slope, and significant part of the slope fails showing excessive deformations [10]. Then the factor of safety (FOS) for the slope (representing the stability condition) is defined as ratio of original shear strength parameters to the reduced values, and expressed as:

$$Factor\ of\ Safety = \frac{c' - \sigma'_n \tan \varphi'}{c'_r - \sigma'_n \tan \varphi'_r} \quad (5.28)$$

in which c' and c'_r are available and reduced magnitudes of soil cohesion, respectively, φ' as well as φ'_r are available and reduced magnitudes of friction angle, respectively, and σ'_n denotes normal effective stress on failure surface. The above equation can be written as:

$$FOS = \frac{c'}{c'_r} \quad \text{or}$$

$$c'_r = \frac{c'}{FOS} \quad (5.29)$$

and

$$FOS = \frac{\tan(\phi')}{\tan(\phi'_r)} \quad \text{or}$$

$$\phi'_r = \arctan\left(\frac{\tan(\phi')}{FOS}\right) \quad (5.30)$$

In the analyses, a small value for FOS is selected as the initial value, and it is increased until the slope failure is reached. Such failure may be defined as [10]: a) bulge of slope profile, b) limit of shear stresses on potential failure surface, and c) non-convergent solution. The last definition basically occurs since in the failure mode, the equilibrium in the system is no longer preserved, and the solution for the set of equations (this set of equations is obtained by domain discretization in finite element or finite difference methods) cannot be computed. Thus, slope failure and solution non-convergent happens simultaneously followed by interruption of computations.

5.2.7. Computation Steps

In this study, for the interaction analyses of waves and the porous seabed, all equations governing the domains are coupled through the interface conditions (coupled wave-seabed model). Then, the set of equations are solved simultaneously, and at each time step all the field variables (velocity

potential of fluid ϕ , sediment displacement components u and v , and pore water pressure p) are obtained (Figure 5.2). In this way, the interaction is modeled rigorously, and the response of domains (e.g. flow regime near the bed surface, sediments deformations, etc.) are influenced by one another.

In relation to global shear failure evaluations of the seabed slope, as presented in Figure 5.3, first, the effective stress fields for soil elements due to wave and geostatic forces are obtained. In this step, the sediments are assumed as poroelastic (for wave-seabed interaction analysis) and elastic-perfectly plastic (for geostatic analysis), respectively. The computed stress levels are then introduced as initial stress tensors for the instability analyses, while the deformations are set to zero. In this phase, the seabed is comprised of elastic-perfectly plastic soil with Mohr-Coulomb yield criterion and associated flow rule. It is significant to note that instability analyses are carried out only after the end of computations of wave-seabed interaction in which the sediment response at different times within one wave cycle is achieved.

5.3. Numerical Results and Discussion

The wave sloping seabed interaction is analyzed by solving the above governing equations and boundary conditions using finite element method in time domain. To march the solution through the time, Backward Differentiation Formulas method (BDFs) is employed. Numerical models are developed in COMSOL [33] platform by introducing all equations. The models of wave, seabed as well as slope instability (Figs. 5.2-5.3) are verified. The wave-induced distributions of pore pressure and shear stress inside sediments are computed, and the effects of wave action on global shear failure of the sloping seabed are investigated.

5.3.1. Model Verifications

Herein, with the aim of verification for fluid and soil sub-models subjected to surface waves, the domains' response is obtained and compared to analytical solutions [22, 25]. The analyses are performed for the case of horizontal seabed with the input data summarized in Table 5.1. A rectangular fluid domain with the length equal to one wavelength (33.7 *m*) and rigid impermeable base is considered. The domain is discretized with rectangular elements with the size of 0.4 *m* following sensitivity analysis of the response on mesh resolution. The response distribution within the depth for maximum dynamic pressure and maximum velocity components are illustrated in Figure 5.4. As shown, the results converge the closed-form solutions [25] considerably.

In order to verify the seabed sub-model, a prescribed hydrodynamic pressure along the seabed surface is applied, and the sediment response is obtained (decoupled approach). The results are compared with analytical solutions [22]. The hydrodynamic pressure distribution along the bottom of the fluid domain is given by [25]:

$$p_f(x, t) = P_0 * \cos\left(\frac{2\pi}{L}x - \omega t\right) \quad (5.31)$$

in which

$$P_0 = \frac{\gamma_w H}{2 \cosh\left(\frac{2\pi}{L}d\right)} \quad (5.32)$$

A rectangular domain with the length equal to one wavelength is modeled and the input data regarding seabed and wave characteristics are shown in Table 5.1. Rectangular elements

(structured mesh) are employed for discretization of the domain and maximum element size is selected as 0.4 m after conducting a convergence analysis on mesh density.

The comparison of solutions for maximum pore water pressure and shear stress inside the seabed depth is illustrated in Figure 5.5. The magnitude of dynamic pressure (P_0) is computed from Eq. (5.32) and $z/D = 0$ denotes the seabed surface. Based on the figure, seabed response obtained from the model converges closed from solutions considerably. Moreover, the boundary conditions along seabed surface, Eqs. (5.22) and (5.25), are satisfied appropriately.

Variations of factor of safety (FOS) for infinite slopes under geostatic stresses are plotted in Figure 5.6 for three slope angles ($\beta = 5^\circ, \beta = 10^\circ, \beta = 15^\circ$) and different sand types (North Carolina Duck Sand $\phi' = 33^\circ, c' = 0 \text{ kPa}$ [68] and Advocate Beach Sand $\phi' = 45^\circ, c' = 0 \text{ kPa}$ [69]). The FOS values are obtained using Strength Reduction Finite Element Method and compared with traditional limit-state-equilibrium solutions (LSE) [55]. As indicated in Figure 5.6, there is a good agreement among the solutions obtained from the two methods. As shown, for a given soil type, by increasing the slope angle, the slope stability decreases since the driving shear stresses become larger. In addition, for a given slope angle, the soil with higher effective friction angle has greater shear strength against sliding leading to more stability.

5.3.2. Sloping Seabed Response: Coupled Analysis

A sloping seabed with the slope magnitude of 15° is considered (as presented in Figure 5.7). As shown in Figure 5.7, the wavelength and wave height (for a given wave period $T = 7\text{s}$) varies from 76.5m (in deep water) to 61.8m (close to breaking zone limit) and from 4m to 3.65m,

respectively, following Eqs. (5.15) and (5.18). The parameters used in analyses are tabulated in Table 5.2. The domain length equal to 220m (which is almost three times of L_0) is selected to avoid lateral boundary effects [31, 38]. The FE mesh density and type used for discretization of domains are not constant. Rectangular elements (near the water-sloping seabed interface) and triangle elements away from the interface are employed with the size of 0.2m and 2m, respectively, following parametric studies on mesh fineness. The variations of maximum pore water pressure and maximum shear stress are plotted in Figure 5.8 along a vertical line located 40m away from the right boundary. Negligible shear stress and high magnitude of pore water pressure at seabed surface are observed that become small at seabed bottom.

5.3.3. Effects of Waves on Stability of the Sloping Seabed: Fully Saturated Soil

To study the effects of wave action on stability of the sloping seabed, a system consisting of fluid and sloping seabed domains under storm wave loadings is considered (Figure 5.7). The sediments within the seabed with the slope of $\beta = 15^\circ$ is assumed as fully saturated with the properties presented in Table 5.2. Two storm events are examined in which the wave characteristics are summarized in Table 5.3. Wave profiles corresponding to the storm conditions within the shallower water depths ($170\text{ m} < x < 220\text{ m}$) are plotted in Figure 5.9. The selected time steps are corresponding to wave trough and wave crest profiles above the point X located 15 m away from the right end (wave-breaking limit) and 1m below the bed surface. Selection of such point is arbitrary, and the objective is to demonstrate the impacts of waves on stress levels inside sediments. Since the point is near the bed surface in shallow water, it may undergo considerable magnitudes of wave loadings.

Effective stress paths (for point X) transitioning from geostatic to the combination of wave and geostatic stress states are plotted in Figure 5.10 for the two storms and the selected time steps. Positive and negative stress magnitudes indicate tensional and compressional stresses, respectively. In Figure 5.10, deviatoric stress q is defined as $q = (\sigma'_I - \sigma'_{III})/2$ where σ'_I and σ'_{III} are major and minor principal effective stresses, respectively, and p' denotes effective confining stress $p' = (\sigma'_I + \sigma'_{III})/2$. Point G denotes the static condition, and as shown, it is above the horizontal axes ($q > 0$) implying pre-shearing (static shear) due to the sloping ground. Depending on the position of the wave profile, the changes in stress states (caused by waves) take different paths, as presented in Figure 5.10. Under wave crest (stress paths GA and GB), the magnitudes of deviatoric stress increase since waves induce compressional stresses to the seabed sediments, whereas, deviatoric stresses decline under wave trough (stress paths GA' and GB'). While changes in deviatoric stress amplitudes are significant (as shown in Figure 5.10), confining stresses vary slightly particularly for the storm with lower wave height. This is because the normal components of wave-induced stresses in fully saturated soil elements are out-of-phase yielding to zero values of their average ($p'_w = 0$ and $p'_{total} = p'_g$) [16, 38]. Therefore, the effective stress paths are almost vertical lines. Moreover, the length of stress paths for the stronger wave condition is larger than that for the weaker wave. This implies that the impacts of wave action on stress magnitudes are more pronounced for the storm with larger wave height.

The effects of Storm waves on the global instability of the sloping seabed in terms of factor of safety FOS is shown in Table 5.4 for sandy sediments with effective friction angle $\phi' = 30^\circ$ and zero effective cohesion. It is apparent from Table 5.4 that the presence of waves reduces the FOS, and such reductions is more significant at some time steps. Since for the stronger wave, the induced

stresses are larger (Figure 5.10), the FOS reduction with respect to static condition is more significant (61.5%) in comparison to the weaker wave (13.4%). Table 5.4 implies that the sloping seabed with the given soil properties and the slope angle may not remain stable during 100-year Storm as stability number drops to 0.83 (which is less than unity). It is worth noting that the global shear failure of the sloping seabed occurs in large scale, and it is the result of exceeding driving stresses to resisting forces at various nodes of soil. In Figure 5.10, the stress levels are shown for a representative point, and such demonstration can be presented for other nodes, as well.

As mentioned earlier, sediments are brought to failure by the gradual increase of strength reduction coefficient (FOS) in which the equilibrium is no longer preserved inside the slope. This condition is accompanied by excessive deformations of the soil mass along the potential slip surfaces. The distributions of shear strain at failure for the sediments within the sloping seabed implies the slip surfaces are indicated in Figure 5.11 for 100-year Storm at $t = T/4$. As demonstrated, in spite of geostatic condition in which the slip surfaces for infinite slopes are planes parallel to the slope surface, the potential failure planes have nonplanar shapes under wave action following the wave loading distribution on the bed surface.

5.3.4. Effects of Waves on Stability of the Sloping Seabed: Slightly Unsaturated Soil

To investigate the impacts of saturation level on stress states of sediments, effective stress paths are computed for various saturations degrees of soil deposit inside the sloping seabed with the steepness $\beta = 15^\circ$. The soil properties are similar to the previous sections (Table 5.2), and the seabed response is evaluated under 100-year storm loadings at different times. The selected time steps are corresponding to wave trough and wave crest profiles above the point X located 15 m

away from the right end (wave-breaking limit) and $1m$ below the bed surface. Effective stress paths for fully saturated ($Sr = 1.00$) and slightly unsaturated ($Sr = 0.95$) sediments are depicted in Figure 5.12. Partial saturation in marine sediments may be caused by presence of gas hydrates within the seabed [16]. As shown in Figure 5.12, variations of effective confining stress (p') magnitude from geostatic to the combination of wave and geostatic is negligible for the saturated soil leading to vertical effective stress path. However, slight unsaturation results in phase shift of wave-induced normal stress components that causes non-zero magnitudes of p'_w (caused by wave action) and consequently distinct p'_t values with remarkable differences from p'_g (as shown in Figure 5.12). Also, slight unsaturation increases the compressibility of marine sediments resulting in larger induced stresses being transferred to the seabed [38]. Consequently, the length of effective stress paths for soils with slightly unsaturation degree is significantly larger than that for fully saturated case.

Variations of FOS for the sloping seabed at different times within a wave cycle is presented in Table 5.5 for two saturation levels $Sr = 1.00$ and $Sr = 0.95$. The strong 100-year storm event is selected for the evaluations. As shown, stability of the seabed with partly saturated soil is remarkably lower than that for saturated case. This is because the stress levels inside sediments with $Sr = 0.95$ increases substantially destabilizing the entire slope. As demonstrated in Table 5.5, such sloping seabed with the given characteristics may not remain stable during the storm event as $FOS < 1$.

5.3.5. Effects of Slope Steepness on Stability of the Seabed

With the aim of evaluating the effects of seabed slope angle on stability of the slope considering simultaneous actions of wave and geostatic stresses, three slope angles $\beta = 5^\circ$, $\beta = 10^\circ$, and $\beta = 15^\circ$ are examined. The properties of the saturated sediments are summarized in Table 5.2, and the wave characteristic related to Hurricane Matthew are employed (Table 5.3). As FOS varies with time, the minimum values (minimum FOS_{gw}) within a wave cycle are computed herein and demonstrated in Figure 5.13. As shown in Figure 5.13, in general, the seabed stability reduces for higher slope angles. However, the reduction rate is not constant and decreases for steeper slopes. Moreover, the changes in FOS (caused by surface waves) with respect to the FOS considering geostresses only (FOS_g) are computed using Eq. (5.33) which is expressed as:

$$\Delta FOS = \frac{FOS_{gw} - FOS_g}{FOS_g} \times 100 \quad (5.33)$$

Distributions of ΔFOS with seabed slope angle, as plotted in Figure 5.13, implies the impacts of wave action on the slope stability for each slope angle. As shown, the amplitudes of ΔFOS are negative indicating waves destabilize the seabed slopes. Such impact is more pronounced for milder slopes (Figure 5.13). This is due to the reason that for the given wave characteristics in deep water, the wavelength at a given depth in shallow zone is lower for bathymetries with higher steepness [12, 38]. This results in lower dynamic pressure on the bed surface and consequently lower stress levels within the sediments. Therefore, it can be concluded from the graphs in Figure 5.13 that, although, the wave-induced stresses become smaller for steeper slopes, the stability of the sloping seabed reduces for higher slope angles due to the resultant effects of wave and geostatic stresses.

5.4. Conclusion

Effects of storm waves on global shear failure of the sloping seabed with finite steepness are evaluated in this study. Interaction of waves with the sloping seabed is analyzed using almost fully coupled finite element method in which the hydromechanical properties of sediments are included in computations. Linear wave and Biot's poroelastic theories represent the fluid and soil domains, respectively. Having obtained wave-induced stress fields within the seabed, the instability of the entire slope due to the combination of wave and geostatic loadings is assessed using strength reduction finite element method.

Based on the presented findings, the following conclusions are made:

1. The distribution of wave-induced stresses inside seabed is a function of wave intensity and position of the wave profile at the surface (e.g. wave crest, wave trough).
2. Effective stress paths from geostatic to combination of wave and geostatic stresses are almost vertical lines for fully saturated sediments following phase shift of wave-induced normal stress components in soil elements. Slight unsaturation alters the shapes of effective stress paths significantly.
3. Surface waves reduce the global stability of the sloping seabed. For an extreme condition (100-year storm) the seabed stability number may decline by 61% with respect to hydrostatic condition.
4. The potential failure surfaces of the slope have a nonplanar shape following the wave loading distribution on the bed surface.

5. Slight unsaturation leads to higher compressibility of sediments and larger induced stresses comparing to fully saturated condition. This results in smaller values of FOS (78% reduction with respect to static water condition).
6. Stability of the seabed slopes reduces for higher slope angles, although the contributions of wave action on destabilizing the slope decline for steeper slope angles.

Tables

Table 5.1.
Input data for horizontal seabed condition

Parameter	Description	Value
$L(m)$	Wavelength	33.7
$T(s)$	Wave Period	5
$H(m)$	Wave Height	1.5
$d(m)$	Water Depth	7
$D(m)$	Seabed Thickness	20
$k(m/s)$	Sediment Permeability	0.0001
n	Sediment Porosity	0.35
$K_w(Pa)$	Water Bulk Modulus	2×10^9
$\rho_f(Kg/m^3)$	Fluid Phase Density of Soil	1000
$\rho_s(Kg/m^3)$	Solid Phase Density of Soil	2600
ϑ	Poisson's Ratio	0.3
$G(Pa)$	Shear Modulus	10^7

Table 5.2.
Input data for sloping seabed condition

Parameter	Description	Value
$T(s)$	Wave Period	7
$k(m/s)$	Sediment Permeability	0.0001
n	Sediment Porosity	0.35
$K_w(Pa)$	Water Bulk Modulus	2×10^9
$\rho_f(Kg/m^3)$	Fluid Phase Density	1000
$\rho_s(Kg/m^3)$	Solid Phase Density	2600
ν	Poisson's Ratio	0.3
$G(Pa)$	Shear Modulus of Sediments	5×10^7

Table 5.3.
Wave characteristics of various storms

Storm Event	Wave Height in Deep Water H_0 (m)	Wave Period T (s)
Hurricane Matthew	4	7
100-year Storm	10	17

Table 5.4.
Variations of the sloping seabed stability (FOS) within a wave cycle for different storm events

Storm Event	$t=T$	$t=0.25T$	$t=0.5T$	$t=0.75T$	Static Water
Hurricane Matthew	2.14	1.87	2.08	2.11	2.16
100-year Storm	1.16	1.19	1.16	0.83	2.16

Table 5.5.
Variations of the sloping seabed stability (FOS) within a wave cycle for different saturation levels (100-year Storm)

Degree of Saturation	$t=T$	$t=0.25T$	$t=0.5T$	$t=0.75T$	Static Water
$S_r = 1.00$	1.16	1.19	1.16	0.83	2.16
$S_r = 0.95$	0.46	0.49	0.51	0.52	2.16

Figures

Figure 5.1. Schematic sketch of the reference problem

Figure 5.2. Computation chart of the coupled wave-seabed model

Figure 5.3. Computation chart for instability analyses of seabed slopes

Figure 5.4. Verification of fluid model: response comparison of the FE analysis vs. analytical solutions

Figure 5.5. Verification of soil model: response comparison of the FE analysis vs. analytical solutions

Figure 5.6. Verification of slope instability model for various slope angles and effective friction angle: This study (Strength Reduction Finite Element Method, SRFEM) vs. Analytical solutions (Limit State Equilibrium Method)

Figure 5.7. Model setup for sloping seabed ($\beta = 15^\circ$)

Figure 5.8. Distributions of maximum pore pressure and max shear stress along a vertical line 40 m away from the right boundary of the seabed

Figure 5.9. Wave profiles for two storm conditions (Hurricane Matthew $H_0 = 4m$ and 100-year Storm $H_0 = 10m$) at times corresponding to wave trough and wave crest above the point of interest

Figure 5.10. Effective stress paths (for point X) related to the wave crest and wave trough for various wave heights: Hurricane Matthew $H_0 = 4m$ and 100-year Storm $H_0 = 10m$

Figure 5.11. Demonstration of wave-induced potential failure planes of the sloping seabed under 100-year Storm at $t = T/4$

Figure 5.12. Effective stress paths (for point X) related to the wave crest and wave trough for various saturation levels: $S_r = 1.00$ (solid blue line) and $S_r = 0.95$ (dashed red line)

Figure 5.13. Demonstration of minimum FOS_{gw} for various slope angles and impacts of wave action in FOS reduction

**CHAPTER 6. NUMERICAL ANALYSIS OF BUILDUP OF PORE WATER PRESSURE
AND DEFORMATION OF SEDIMENTS DUE TO CYCLIC WAVE ACTION**

Abstract

Instability of the seabed due to water surface waves may cause damage to coastal/offshore structures. Liquefaction-induced scour due to wave action may impact the stability and bearing capacity of the foundation element of coastal structures. Liquefaction in marine sediments may be instantaneous in which the total mean effective stress reaches zero at a time instance, or it may occur due to buildup of pore water pressure associated with nonlinear deformations (residual liquefaction). In this study, the effects of static shear stresses on generation of pore pressure buildup are investigated under cyclic wave loadings. Moreover, the formulation to model the concurrent progressive generation of pore pressure and deformation during cycling is presented, and the elemental response (based on such formulations) for a soil sample is evaluated. The interaction of waves and sloping seabed (with significant slope angles) is assessed through coupled analyses. Potential flow theory and Biot's formulations (modified to include soil nonlinearity) represent the fluid and soil media. Experimentally derived numerical models for pore pressure and deformation buildup are employed to simulate accumulation of response at the end of loading cycles. The results indicate that increase of slope steepness results in lower ultimate pore pressure amplitudes yet with higher increasing rate at initial time of applied wave loadings.

6.1. Introduction

The stability of seabed sediment near the foundation element of marine structures is a significant challenge in the safe design and maintenance of such structures under severe storm conditions. Wave-induced liquefaction in cohesionless sediments may occur due to different mechanisms. Instantaneous liquefaction happens near the surface of seabed consisting of partly-saturated sediments. In this condition, the gradient of excess pore water pressure is significantly high causing upward seepage forces that resist in situ geostatic stress leading to zero effective stress of soil in severe storm conditions. On the hand, for fully-saturated seabed, the pore pressure builds up progressively by the passing surface waves through time. The gradual increase of pore water pressure is associated with nonlinear deformation of soils and the potential for accumulation of volumetric strain. The increase in pore pressure may be considerable resulting in vanishing the effective stress among soil particles identified as residual liquefaction.

Protective measures for stability of sediments in marine environment due to hydrostatic and wave actions should be considered [70-75]. Cyclic changes in magnitude of pore pressure may destabilize the soil elements [76, 77]. Intensive research has been conducted on soil constitutive behavior under cyclic shearing. Coupled and uncoupled schemes in the framework of effective stress analysis has been considered to investigate the cyclic plasticity of the soil [78]. These schemes refer to a) analysis of effective stress and pore pressure generation separately, and b) consideration of pore fluid flow and soil deformation simultaneously using Biot's equations [23], respectively [78]. In addition to the advance plasticity approach, a number of simplified models describing progressive generation of pore water pressure caused by cyclic loading have been developed based on experimental data (e.g. from cyclic simple shear test, cyclic torsional shear

tests, cyclic triaxial test, etc.) [8, 79]. Seed and Rahman [79] developed a simple model for pore pressure buildup based on large scale cyclic simple shear test data [80] by adding a source term in one dimensional consolidation equation. This model has been used broadly by other researchers and has become more advanced in later studies to include complicated field conditions.

An analytical solution for the residual pore pressure generation due to water waves has been obtained in which a linearized form of the source term (used in the model of Seed and Rahman [79]) is introduced [81]. The effect of source term linearization on residual pore pressure amplitude has been demonstrated to be significant for larger wave amplitudes with longer wave periods and softer soil [82]. Sumer et al. [83], employing the model [79] in two-dimensional plane strain condition, developed a closed-form solution for accumulated pore pressure, and verified the results with experimental data from wave flume and soil pit. Jeng and Zhao [82] in a numerical study used the phase-resolved (instantaneous) amplitude of cyclic shear stress at each time step in the source term (instead of maximum shear stress) and demonstrated a good agreement of the model results with experimental data [82].

All the aforementioned studies are related to the initial isotropic geostatic conditions. While, the presence of static shear stress (e.g. in sloping ground, under foundation of structures, etc.) has been shown to alter the soil cyclic behavior significantly [84]. The impact of this factor (defined as static shear stress ratio SSR) has been considered initially by multiplying some correction factors in cyclic resistance ratio (CRR) in isotropic condition [84, 85]. Pan and Yang studied the effect of SSR on residual deformation and pore pressure of loose and dense sand in more details following

undrained cyclic triaxial test [8, 86]. They proposed a model for buildup of pore water pressure accounting for a range of SSR.

In this study, the effects of static shear stress on residual pore pressure and deformations of saturated sand within the sloping seabed are evaluated using simplified analyses. Two approaches are followed to obtain solutions. (i) In the approach one, residual pore pressure is calculated without considering the related soil nonlinear deformations. This is carried out by using experimentally derived models [8, 82]. (ii) In the second approach, considering coupled flow and deformations processes, Biot's formulations [23] are employed with additional source terms and update of strength parameters through cycling. The related modifications to Biot's equations are based on Bouckovalas' model [9]. In this way, the residual pore pressure in conjunction with residual deformation are obtained automatically by the model. In relation to the first approach, up approximation of Biot's equations [24] for seabed are solved considering coupling effects of fluid and soil media, and the maximum shear stress is obtained within the seabed (linear waves theory is used to model the fluid domain). Then, the rate of residual pore pressure is obtained following the model of Pan and Yang [8] and implemented as source term in 2D plane-strain consolidation equation [82]. The basics of inclusion of SSR into the waves-sloping seabed interaction problem is elaborated in detail. The results obtained from the approach (i) are compared with experimental data [83], and the effects of seabed slope angle on residual pore pressure amplitudes are evaluated. The formulations related to approach (ii) are presented, and the progressive buildup of pore pressure for undrained cyclic triaxial test is computed and compared with other studies [9].

6.2. Mathematical Model

The basic mechanics and governing equations for the problem consisting of fluid and sloping seabed domains are presented herein for two-dimensional plane strain coupled analyses. The schematic sketch of the reference problem is illustrated in Figure 6.1.

6.2.1. Basic Mechanics

Waves in shallow water may induce significant magnitudes of pore pressure, deformation, and stress inside the marine sediments. Depending on the wave loading amplitudes, the soil response may have two components: oscillatory (recoverable) and residual (non-recoverable). With the waves-generated loading-unloading cycles on soil deposit within the seabed, saturated sandy soil in loose to medium dense condition (as the case study for this paper) contracts at the end of each cycle. This leads to the buildup of positive pore water pressure. For a high magnitude of wave loadings, in which sand experiences considerable strains, irreversible deformations and pore water pressure may occur during unloading and accumulate progressively with increasing number of cycles. This in turn leads to a reduction of effective stress (and shear strength of soil), and zero mean effective stress may be reached causing residual liquefaction, in extreme conditions. Temporal variations of pore pressure under cyclic wave loadings are indicated in Figure 6.2.

In order to investigate the above phenomenon of progressive buildup of pore water pressure in the analysis of seabed response, the following approaches (with varying level of simplification) are possible:

Approach-I: The oscillatory response (stresses and pore water pressure) are solved initially with coupled flow and deformation model (represented by equilibrium and mass balance equations). Additionally, the progressively build-up pressure is solved by solving the mass balance equation again with a source term representing the generation of pore water pressure due to cyclic shearing. This source term is defined through an empirical equation derived on the experimental results.

Approach-II: In the above approach the effect of residual pore water pressure on deformation and stiffness of the soil element is not accounted. In order to achieve a better coupling of flow and deformation, a new source term may be included in general equilibrium equation that represents the equivalent nodal forces due to residual strains. Then, the general equilibrium and mass balance equations are solved and total pore water pressure (oscillatory and residual components) and displacement may be computed simultaneously.

Approach-III: The best approach will be to use a nonlinear constitutive model for the deformation of soil in the overall model for the flow and deformation retaining the Biot's formulation. This way, the pore water pressure will be progressively generated by the model itself which is capable of providing the progressively developing inelastic volumetric strains. Some preliminary work has been done using this approach [87].

In this paper, Approach-I and Approach-II are adopted in the analyses.

6.2.2. Residual Pore Pressure Calculations Using Pan and Yang's Model

The governing equation and steps required to calculate the residual pore pressure due to wave loadings are presented herein following Approach-I.

As mentioned earlier, the oscillatory response of the seabed soil due to wave action needs to be obtained as the first step. Linear wave theory [25] and u-p approximation of Biot's equations [24] are selected to model the wave and seabed domains. The governing equations for the seabed are expressed as:

$$k\nabla^2\tilde{p} - \gamma_w n \beta \dot{\tilde{p}} + k \rho_f \frac{\partial^2 \varepsilon_v}{\partial t^2} = \gamma_w \frac{\partial \varepsilon_v}{\partial t} \quad (6.1)$$

$$G\nabla^2\tilde{u} + \frac{G}{1-2\vartheta} \frac{\partial \varepsilon_v}{\partial x} = \frac{\partial \tilde{p}}{\partial x} + \rho \frac{\partial^2 \tilde{u}}{\partial t^2} \quad (6.2)$$

$$G\nabla^2\tilde{v} + \frac{G}{1-2\vartheta} \frac{\partial \varepsilon_v}{\partial y} = \frac{\partial \tilde{p}}{\partial y} + \rho \frac{\partial^2 \tilde{v}}{\partial t^2} \quad (6.3)$$

in which $\tilde{u}$, $\tilde{v}$, and $\tilde{p}$ denote oscillatory components of field variables ε_v oscillatory volumetric strain, ϑ Poisson's ratio, ρ soil density, G shear modulus, k and n are permeability and porosity of soil, respectively, β and γ_w denote compressibility and water unit weight, and ρ_f is water density.

For the evaluation of residual response, the governing equation representing the development of pore pressure is written as [82]:

$$\frac{\partial \bar{p}}{\partial t} = c_v \left(\frac{\partial^2 \bar{p}}{\partial x^2} + \frac{\partial^2 \bar{p}}{\partial z^2} \right) + f_p \quad (6.4)$$

in which $\bar{p}$ denotes the residual pore water pressure, $c_v = Gk/\gamma_w(1 - 2\theta)$ the consolidation coefficient, and f_p source term simulating accumulation of pore water pressure. Equation (6.4) is nothing but the soil consolidation relationship in 2D plane strain condition with an additional term (f_p). The residual pressure buildup for a range of relative density is given by [79, 83]:

$$f(x, z, t) = \frac{\sigma_0'}{T} \left[\frac{|\tau_{xz}|}{\alpha_r \sigma_0'} \right]^{-1/\beta_r} \quad (6.5)$$

where $\sigma_0' = (1 + 2K_0)/3 \times \gamma'z$ in situ geostatic stress, K_0 coefficient of lateral earth pressure at rest, γ' submerged specific weight, $|\tau_{xz}|$ amplitude of shear stress, $\alpha_r = 0.34 Dr + 0.084$, Dr relative density, and $\beta_r = 0.37 Dr - 0.46$. The total response (oscillatory plus residual) of seabed is obtained by solving the poroelastic equations (Eq. (6.1)-(6.3)) in conjunction with Eq. (6.4). The boundary and initial conditions related to progressive development of pore pressure are expressed as:

$$\bar{p}(z, t = 0) = 0 \quad (6.6)$$

$$\bar{p}(z = 0, t) = 0 \quad (6.7)$$

$$\frac{\partial \bar{p}(z = -h, t)}{\partial z} = 0 \quad (6.8)$$

in which h is soil thickness. It is worth mentioning that the Eq. (6.5) is obtained from large scale cyclic simple shear test under isotropic geostatic condition (zero static shear stress); therefore, the subsequent residual pore pressure will be mostly valid for level grounds.

For sloping ground or near foundation of structures anisotropic initial condition may exist with significant static shear stress magnitudes (τ_s). Depending on direction and magnitude of τ_s , the cyclic behavior of soil may be different. Figure 6.3 and Figure 6.4 demonstrates the effect of τ_s on pore pressure buildup ($\bar{p}$) and residual axial strain (ε_a), respectively, following undrained cyclic triaxial test results [8]. As shown in Figure 6.3, for $\tau_s = 0$ after a number of loading cycles, pore water pressure increases gradually and reaches initial mean effective stress ($P'_0 = 100 \text{ KPa}$) resulting in progressive liquefaction. Such gradual pore pressure increase leads to reduction of soil stiffness step by step. This behavior is known as *cyclic mobility* and mostly occurs in dense sand with zero static shear stress.

The presence of τ_s causes a sudden jump in pore pressure (in comparison to $\tau_s = 0$) at initial cycles and then is followed by a limiting magnitude of pore pressure ($u_{lim} = 55 \text{ KPa}$) less than initial mean effective stress; therefore, liquefaction does not occur in this case. Moreover, Figure 6.4 illustrates the variation of axial strain with loading cycle. As shown, prior to initial liquefaction for zero static shear stress condition, negligible accumulative deformation is generated, and larger magnitudes are obtained only after soil failure. However, for $\tau_s = 50 \text{ KPa}$, the soil deformation occurs progressively and can reach significant values. This behavior is identified as *residual deformation accumulation* happening mostly in dense sand with no shear stress reversal, and may cause landslides due to excessive deformations [8].

Figure 6.5 indicates the variation of limiting pore pressure ratio ($u_{r,lim} = u_{lim}/\sigma'_0$) with static shear stress ratio (SSR) [8]. As shown, by increasing the steepness of the ground (change in SSR magnitude), the buildup pore water pressure ratio decreases significantly. As noted earlier, for the level ground ($SSR = 0$), $u_{r,lim}$ is equal to unity.

In order to develop a numerical model for pore water pressure generation, a curve is fitted to the cyclic triaxial data in pore pressure ratio-cyclic number space (Figure 6.6) and a mathematical equation is proposed as [8]:

$$u_{r,n} = \frac{2}{\pi} \arcsin N_n^{aN_n^{2b}+b} \quad (6.9)$$

where $u_{r,n} = u_r/u_{r,lim}$ is normalized pore pressure ration, a and b are coefficients function of the slope angle of ground, $N_n = N/N_{lim}$ normalized number of loading cycle, N_{lim} number of cycles required to reach liquefaction or state of constant pore pressure.

The values of coefficients a and b are tabulated in Table 6.1 shown below [8], and their distribution with SSR are plotted (Figure 6.7). Also, the limiting number of cycles N_{lim} varies with the level of applied cyclic stress ratio and static shear stress, Figure 6.8 [8]. Obviously, under the larger CSR magnitudes, ultimate state (liquefaction or constant pore pressure state) reaches faster resulting in smaller required number of cycles and vice versa.

The applied cyclic shear stress takes various values within the ground/seabed depth leading to a corresponding distribution of N_{lim} at different elevations within the soil layer. Thus, it is practical

to derive a correlation between CSR and N_{lim} for a range of SSR . This has been carried out by defining a mathematical function including a number of unknown coefficients and computing the related coefficients by fitting a curve to the data (obtained from Figure 6.8) using the least square method. The proposed equation takes the form as:

$$CSR = a_1 * N_{lim}^{b_1} \quad (6.10)$$

where a_1 and b_1 are the coefficients function of SSR magnitudes. It is noted that this single equation includes all the graphs shown in Figure 6.8. The computation results for the values of these coefficient are illustrated in Figure 6.9. Moreover, following curve fitting method, the coefficients a_1 and b_1 can be described by a third order polynomial equation (g) (useful for interpolation of values of a_1 and b_1 for SSR magnitudes not indicated in Figure 6.8) written as:

$$g(SSR) = p_1 SSR^3 + p_2 SSR^2 + p_3 SSR + p_4 \quad (6.11)$$

in which p_1 to p_4 are coefficients needed to be computed. The values of these coefficients are calculated and given in Table 5.2. The fitted curves (g) for a_1 and b_1 are also plotted in Figure 6.10.

The next step of computing residual pore pressure in sloping ground accounting for static shear stress magnitude is to correlate ground steepness (β) with SSR using elasticity solutions for inclined soil layers given by [32]:

$$\sigma_{g,z} = \gamma z (1 + K \sin^2 \beta) \quad (6.12)$$

$$\sigma_{g,x} = K \gamma z \cos^2 \beta \quad (6.13)$$

where

$$K = \frac{K_0 + r_u(1 - K_0)}{\cos^2 \beta - K_0 \sin^2 \beta} \quad (6.14)$$

In the above, K_0 is the coefficient of lateral earth pressure at rest for level ground, and r_u is the pore water pressure ratio ($r_u = u/\gamma z$). The correlation results are plotted in Figure 6.11. As shown, by increasing the inclination angle, SSR values become greater. The results also indicate that the values of SSR are constant through the soil depth (assuming infinite slope condition).

Finally, the rate of residual pore water pressure should be obtained derived from the proposed model (Eq. (6.11)). This equation can be written in the following form as:

$$\sin\left(\frac{r_u}{r_{u,lim}} \frac{\pi}{2}\right) = N_n^{aN_n^{2b}+b} \quad (6.15)$$

introducing $N = t/T$ results in:

$$\sin\left(\frac{r_u}{r_{u,lim}} \frac{\pi}{2}\right) = \left(\frac{t}{TN_{lim}}\right)^{a\left(\frac{t}{TN_{lim}}\right)^{2b}+b} \quad (6.16)$$

Taking time derivative of both sides of Eq. (6.16) leads to the desired residual pore pressure rate expressed as:

$$\frac{\partial u_g}{\partial t} = \frac{2 r_{u,lim} \sigma'_{0}}{\pi \cos\left(\frac{u_g}{\sigma'_{0} r_{u,lim}} \frac{\pi}{2}\right)} \frac{\left(\frac{t}{c}\right)^{\left[b+a\left(\frac{t}{c}\right)^{2b}\right]} \times \left[b + a \left(\frac{t}{c}\right)^{2b} + 2ab \log\left(\frac{t}{c}\right) \left(\frac{t}{c}\right)^{2b}\right]}{t} \quad (6.17)$$

where $c = TN_{lim}$.

The steps to obtain residual pore pressure magnitude within the sloping seabed due to wave action are summarized below:

1. Solve the wave-seabed boundary value problem and obtain the amplitudes of oscillatory shear stress within one wave cycle at each point inside the sloping seabed.
2. Select *SSR* value based on seabed slope steepness following Figure 6.11.
3. Determine the coefficients a and b based on the selected *SSR* following Figure 6.7.
4. Determine the coefficients $a1$ and $b1$ based on the selected *SSR* following Eq. (6.11) or Figure 6.10.
5. Determine the $r_{u,lim}$ based on the selected *SSR* following Figure 6.5.

6. Obtain N_{lim} as a function of CSR from step 4 and incorporate it in Eq. (6.17).
7. Solve Eq. (6.4) with f_p being $\partial u_g / \partial t$ computed from Eq. (6.17).

6.2.3. Calculations of Residual Pore Pressure and Residual Deformations Using

Bouckovalas' Model

Herein, with the use of more advanced model of Bouckovalas [9], the progressive generation of pore water pressure and inelastic deformations within the seabed due to cyclic wave loadings are studied (Approach-II). The governing equations are presented in the framework poro-elasticity with additional terms to account for soil nonlinear behavior. The hydro-mechanical properties of soil elements are included in the computations, and the empirical additional terms are obtained from numerous cyclic triaxial tests on sand [9, 42]. The model being used herein, to include soil nonlinearity, has been developed by Bouckovalas et al. [9, 42]. The analyses include the shear strength degradation of the soil elements and consequently potential large deformations of sandy soil.

As mentioned earlier, the soil response has two components corresponding to oscillatory and residual behavior. The total strain including the irreversible deformations is written as :

$$\varepsilon = \varepsilon^\sigma + \varepsilon^c \quad (6.18)$$

where ε^σ and ε^c denote strain due to change in effective stress and cyclic (or residual) strain stems from inelastic behavior, respectively. The effective stress of soil elements is given by:

$$\sigma' = C^t(\varepsilon - \varepsilon^c) \quad (6.19)$$

Coupled flow and deformation equation of a soil element in u-p form (**u**, displacement, **p**, pore water pressure as field variables) with inclusion of residual deformations, are as follows:

$$G^t \nabla^2 u + \frac{G^t}{1 - 2\vartheta} \frac{\partial \varepsilon_v}{\partial x} = \frac{\partial p}{\partial x} + \rho \frac{\partial^2 u}{\partial t^2} + C^t \frac{\partial \varepsilon^c}{\partial x} \quad (6.20)$$

$$G^t \nabla^2 v + \frac{G^t}{1 - 2\vartheta} \frac{\partial \varepsilon_v}{\partial z} = \frac{\partial p}{\partial z} + \rho \frac{\partial^2 v}{\partial t^2} + C^t \frac{\partial \varepsilon^c}{\partial z} \quad (6.21)$$

$$k \nabla^2 p - \gamma_w n \beta \frac{\partial p}{\partial t} + k \rho_f \frac{\partial^2 \varepsilon_v}{\partial t^2} = \gamma_w \frac{\partial \varepsilon_v}{\partial t} \quad (6.22)$$

where G^t is tangential shear modulus that decreases during cycling. Herein, the additional terms of Eqs. (6.20) and (6.21) ($C^t \frac{\partial \varepsilon^c}{\partial x}$, $C^t \frac{\partial \varepsilon^c}{\partial z}$) account for soil nonlinear behavior that transform to equivalent forces acting on nodes in the FEM discretization.

Considering an analogy between creep under static loading and deformation due to cyclic loading of soils, a simple creep model is applied to predict the permanent strain (ε^c) accumulation resulting from repeated loading. This model is represented by the following equations [9]:

$$d\varepsilon_{vol} = \frac{d\sigma'_{oct}}{K^t} + d\varepsilon_{vol}^0 \quad (6.23)$$

$$de_{ij} = \frac{dS_{ij}}{G^t} + de_{ij}^0 \quad (6.24)$$

in which e_{ij} = deviatoric strain, S_{ij} = deviatoric stress, i (j) = principal stress (strain) direction, K^t = tangential bulk modulus, $d\varepsilon_{vol}^0$ and de_{ij}^0 parameters obtained from curve fitting methods based on cyclic laboratory results, and are non-linear functions of the average effective stresses. Also, since the wave period is large, and depending on drainage condition, the dissipation of pore water pressure due to flow of water might occur. The effect of such dissipation is incorporated in this model as well. The above model is used in computation of permanent strains and is incorporated as a source term in general equilibrium equations of porous seabed.

For undrained cyclic triaxial test in which the soil is isotopically consolidated, Eq. (6.23) takes the following form where du denotes the change of residual pore water pressure due to cycling:

$$du = K^t d\varepsilon_{vol}^0 \quad (6.25)$$

In the above equation, $d\varepsilon_{vol}^0$ is the change of nominal volumetric strain obtained from numerous lab data which is given by [9]:

$$d\varepsilon_{vol}^0 = D_1(1 - Q^{c1})\gamma_{cyc}^{c3} N^{c4-1} dN \quad (6.26)$$

The coefficients D_1 , $c1$, $c3$, and $c4$ are obtained from drained cyclic triaxle tests and are unique for each sand type; in this case, Q denotes the static shear stress ratio, N number of cycle, and γ_{cyc} is

cyclic shear strain corresponding to the given cycle number (N). The magnitudes of γ_{cyc} are obtained using a hyperbolic model based on effective cyclic stress path of the soil element [9].

6.3. Numerical Results and Discussion

In this section, the governing equations are programmed in COMSOL platform, and the results are presented. To verify the model, the results obtained from implementation of Approach-I (residual pore pressure model) are computed and verified with experimental data [83]. The effects of seabed slope angle on the residual pore pressure is evaluated using Pan and Yang's model. Moreover, temporal variations of pore pressure in undrained cyclic triaxial test is obtained using Bouckovalas' model and compared with experimental data [9] (Approach-II).

6.3.1. Model Verification

The development of residual pore pressure through time based on Eq. (6.4) (Approach-I) is computed, and the results are compared with experimental data [83]. First, the poroelastic equations (Eqs. (6.1)-(6.3)) are solved using finite element method, and the amplitude of shear stress (when steady state phase of the response is reached) at each point of the soil domain is computed. Then, the equation governing the residual pore water pressure (Eq. (6.4)) is solved using the source term attained from the step 1. The input data for wave and soil characteristics employed in the simulation are shown in Table 6.3.

As indicated in Figure 6.12, a reasonable agreement between the results of this study and those of experiments is obtained [83]. The Figure 6.12.a illustrates the variation of residual pore water pressure with time for a generic point at $z=8.5$ cm from the soil surface. Also, the distribution of

total pore water pressure with time from the wave flume test results [83] is plotted. In Figure 6.12.b, the distribution of $\bar{p}$ with respect to depth at $t = T$ is depicted. The residual pressure increases within the soil depth. The discrepancy among the graphs may be related to the difference in boundary conditions between the numerical model and actual boundaries for the lab test. For instance, the experiment is carried out on a small soil box; however, the introduced governing equations are defined in plane strain condition.

The above method is a simplified approach based on empirical coefficients obtained from cyclic simple shear test. It can predict the buildup of pore pressure relatively well prior to liquefaction of sediments. The effect of pore fluid pressure increase (due to cyclic loading) on strain softening of a soil element cannot be captured by this method. The progressive deformation of soil and post liquefaction behavior need to be obtained by employing more rigorous models.

6.3.2. Effect of Seabed Slope on Residual Pore Pressure

Herein, the effect of seabed slope angle (causing non-zero static shear stress) on the residual pore pressure is investigated using Pan and Yang's model [8] (Approach-I). Two seabed slopes with inclination angles $\beta = 3^\circ$, and $\beta = 6^\circ$ are considered. The thickness of seabed at the left side (deep water) is selected as 10 m for all cases, and the wave characteristics in deep zone are: wave height $H=4\text{ m}$, wave period $T=7\text{ s}$. The properties of fully saturated sediments with relative density $Dr = 0.63$ are summarized in Table 6.3.

The temporal variation of pressure response for different slope angles is demonstrated in Figure 6.13 for a generic point at the middle of the seabed domain 2 m below the bed surface. As shown,

increase in seabed steepness results in higher slope of the graph at initial times, and the final pressure amplitudes at higher time steps expected to be greater for the milder slope ($\beta = 3^\circ$) following the slope of graphs at $t=500 T$. This finding is consistent with the results of earlier studies, as for instance Figure 6.3 related to Pan and Yang's work [8]. Since the ultimate pressure amplitudes take smaller values for higher slope angles, the potential of residual liquefaction may be lower for fully saturated sloping seabed compared to flat bed.

6.3.3. Residual Pore Pressure Calculation in Cyclic Undrained Triaxial Test

The temporal variation of residual pore water pressure (u) for "Leighton-Buzzard 120/200" sand in undrained cyclic triaxle test is computed based on Bouckovalas' Model (Approach-II), and the results are plotted in Figure 6.14. The initial effective confinement stress and applied cyclic stress are 300 kPa and 120 kPa, respectively. The void ratio of sand is $e = 0.74$, and the coefficients in Eq. (6.26) are obtained as: $D_1 = 2.8 \times 10^{-3}$, $c_1 = 11$, $c_3 = 1.26$, and $c_4 = 0.4$.

In Figure 6.14, N_l is the number of cycles required to bring the soil element to the state of liquefaction (i.e. zero mean effective stress) and is computed as $N_l = 21$. As shown, the general trend for the predicted results obtained from calculations herein agrees with that of Bouckovalas' computations [9]. The discrepancies among the graphs, however, are mostly related to differences in magnitude of cyclic shear strain obtained in each calculation. In future work, the focus will be on minimizing such differences. Also, it is noted that predicting results in general do not converge completely to the lab results implying non-homogeneity of the soil sample and inherent errors in performing lab experiments.

6.4. Conclusion

In this study, the residual pore pressure response under the action of surface waves is investigated. Numerical models (based on lab data) for pore pressure buildup with various levels of complications are employed. Using the Pan and Yang's model, the equation for time rate of residual pore pressure buildup is rewritten (used as source term in calculations), and the model is incorporated in 2D plane strain analysis of wave-sloping seabed interaction. The model is able to include the effects of pre-shearing in the analyses. Moreover, a more advanced numerical model (based on lab data) capable of concurrent simulation of residual pore pressure and deformation is used. This model is employed to capture nonlinear response of soil sample in undrained cyclic triaxial testing. All the related equations and boundary conditions are programmed in COMSOL, and the response is obtained using finite element method. The following conclusions are obtained based on the results presented herein:

1. The buildup of pore water pressure under cyclic wave loading prior to the liquefaction of sediments is obtained appropriately indicating the adequate accuracy of the applied simplified approach.
2. For higher slope angles of the seabed, the generation rate of pore pressure is greater at initial times of cyclic loading, and it reduces through the time.
3. The ultimate pore pressure amplitude for steeper slopes is smaller than that of milder steepness implying that residual liquefaction potential may not be significant for sloping seabed.

4. A good convergence is achieved among the computation results of this study and those of Bouckovalas' work (related to temporal variation of residual pore pressure in undrained cyclic triaxial test).

Tables

Table 6.1.
Coefficient values with respect to SSR

	<i>SSR = -0.1</i>	<i>SSR = 0</i>	<i>SSR = 0.25</i>	<i>SSR = 0.4</i>
<i>a</i>	2.1	0.2	-0.21	-0.12
<i>b</i>	0.75	0.4	0.21	0.12

Table 6.2.
Coefficient values for the function $g(SSR)$

	<i>p₁</i>	<i>p₂</i>	<i>p₃</i>	<i>p₄</i>
<i>a₁</i>	-0.87	0.81	0.54	0.28
<i>b₁</i>	0.53	-0.25	0.049	-0.10

Table 6.3.
Input data for model verification

Parameter	Description	Value
$L(m)$	Wavelength	3.45
$T(s)$	Wave Period	1.6
$H(m)$	Wave Height	0.18
$d(m)$	Water Depth	0.55
$D(m)$	Seabed Thickness	0.4
$k(m/s)$	Sediment Permeability	1.5×10^5
n	Sediment Porosity	0.51
$K_w(Pa)$	Water Bulk Modulus	2×10^9
$\rho_f(Kg/m^3)$	Fluid Phase Density	1000
$\rho_s(Kg/m^3)$	Solid Phase Density	2650
ν	Poisson's Ratio	0.29
$G(Pa)$	Shear Modulus of Sediments	1.92×10^6

Figures

Figure 6.1. Schematic sketch of the reference problem

Figure 6.2. Components of wave-induced pore water pressure

(a)

(b)

Figure 6.3. Effect of static shear stress on residual pore water pressure, $P'_0 = 100 \text{ KPa}$: a) without static shear stress, b) with static shear stress $\tau_s = 50 \text{ KPa}$ (After Pan and Yang 2018)

(a)

(b)

Figure 6.4. Effect of static shear stress on residual axial strain, $P'_0 = 100 \text{ KPa}$: a) without static shear stress, b) with static shear stress $\tau_s = 50 \text{ KPa}$ (After Pan and Yang 2018)

Figure 6.5. Variations of limiting pore pressure ratio with static shear stress ratio (After Pan and Yang 2018)

Figure 6.6. Normalized pore pressure ratio as a function of normalized cycle number for a) isotopically consolidated sand, b) various SSR magnitudes (After Pan and Yang 2018)

Figure 6.7. Distribution of coefficients a and b with respect to SSR

Figure 6.8. Variations of N_{lim} with the applied CSR for a range of SSR (After Pan and Yang 2018)

Figure 6.9. Variations of coefficients a_1 and b_1 with SSR

Figure 6.10. Variation of coefficients a_1 and b_1 with SSR : predicted trend vs. known values

Figure 6.11. Variations of static shear stress ratio with ground steepness

Figure 6. 12. Comparison of pore water pressure with experimental results (Sumer et al. 2012):
a) Variation of total (light blue color) and residual pore pressure (red color) with time for a generic point at $z = 8.5$ cm from soil surface, b) Distribution of residual pore pressure in depth at $t = T$

Figure 6.13. Effect of seabed steepness on temporal variation of residual pore pressure

Figure 6.14. Temporal variations of residual pore water pressure in undrained cyclic triaxial test

CHAPTER 7. SUMMARY, CONCLUSIONS, CONTRIBUTIONS, AND FUTURE WORKS

7.1. SUMMARY

In this work, the response and instability of the sloping seabed supporting a marine structure under combination of wave and geostatic loadings are evaluated. Finite Element analyses are performed by MATLAB scripting and programming the governing equations into COMSOL platform. The following presents the summary of work done for each chapter:

1. In chapter 2, the response of finitely steep sloping seabed under surface water waves is investigated, and the development of instantaneous liquefaction is evaluated for free field condition. An almost fully coupled approach is employed for fluid-soil interaction that captures the coupling process among the media rigorously. Subsequently, the impacts of such coupling scheme on sediments response are assessed. The extents of instantaneous liquefaction near bed surface as a function of seabed slope angle are investigated.
2. In chapter 3, with the inclusion of a structure mounted on the sloping seabed, the wave-induced response of sediments around the foundation element is evaluated, and the extent of instantaneous liquefaction is computed (full system). The impacts of soil properties on angular distortion of the footing are examined. The effects of seabed slope angle on pore pressure amplitudes, stress levels, and extents of instantaneous liquefaction inside the seabed are evaluated. The impacts of various coupling schemes on the response and instability of the seabed are demonstrated.
3. In chapter 4, the response and instability of foundation soil around a single Geotextile Sand Containers (GSC) under progressive surface waves are analyzed. The distribution of

loading on the structure under the wave action is obtained. The extent of liquefaction around the GSC is calculated under various wave characteristics.

4. In chapter 5, effects of storm waves on global shear failure of the sloping seabed with finite steepness are evaluated using strength reduction finite method. The variations of slope stability number (factor of safety) caused by wave action are elaborated through demonstration of effective stress paths within a wave cycle. The impacts of wave characteristics, soil properties, and seabed slope angle on the seabed stability are investigated, and the results are discussed.
5. In chapter 6, the residual pore pressure response under cyclic wave loadings is investigated using experimentally derived models. The impacts of seabed slope angle on temporal variation of pore water pressure is studied by incorporating Pan and Yang's model into coupled wave-sloping seabed analyses. The progressive buildup of pore water pressure during undrained cyclic triaxial test (elemental response) is calculated employing more advanced Bouckovalas' model capable of concurrent modeling of residual pore pressure and deformation of soil elements. To this end, a MATLAB code is generated that simulates the progressive pore pressure buildup as well as strength degradation of the soil element through the time. In future studies, the developed MATLAB program will be integrated with COMSOL to analyze the general boundary value problem of wave-seabed interaction.

7.2. Conclusions

The conclusions for each aspects of this work have been presented in details in the previous chapters. Here in the following only the general conclusions are being summarized

1. A general framework for modeling and computing of wave-structure-soil is developed and applied to these marine geotechnical problems: (a) wave-sloping seabed interaction (poroelastic response), (b) wave-MHK-sloping seabed interaction (poroelastic response), (c) wave-GSC-sloping seabed interaction (poroelastic response), (d) wave-sloping seabed interaction (global shear failure), and (e) wave-sloping seabed interaction (residual response). All the interactions among domains (fluid, solid, and soil) are considered, and the response is obtained using finite element analysis.
2. Chapter 2-3: For the steeper slopes, the wave-induced pore pressure and stress amplitudes decrease. This is caused by development of larger water flux into the seabed and weaker wave profiles on the water surface. Consequently, the size of instantaneously liquefied zones diminishes for higher slope angles.
3. Chapter 2-3: The difference between coupled and decoupled solutions has found to be negligible for the given wave and soil characteristics.
4. Chapter 3-4: Instantaneous liquefaction may not occur in the immediate vicinity of the structure as the weight of the structure increase the effective confining stresses inside the soil.

5. Chapter 5: Surface waves reduce the global stability of the sloping seabed. Large wave height and slightly unsaturation level of sediments results in significant stability reduction of the seabed slopes.
6. Chapter 5: The potential failure surfaces of the slope have a nonplanar shape following the wave loading distribution on the bed surface.
7. Chapter 6: The ultimate pore pressure amplitude for steeper slopes is smaller than that of milder steepness implying that residual liquefaction potential may not be significant for sloping seabed.

7.3. Contributions

The major contributions of this study are listed as below:

1. Generating a modeling framework to analyze interactions of wave-solid-soil and employing such framework to study few marine geotechnical problems.
2. Coupled analysis of wave-sloping seabed interaction and evaluation of response and instability of sediments (Free Field)
3. Coupled analysis of wave-structure-sloping seabed interaction and evaluation of response and instability of sediments in the vicinity of structures such as: a) MHK, b) GSC (Full System).
4. Study of global shear failure of submarine slopes due to wave action and gravitational forces based on strength reduction finite element method (SRFEM).
5. Analysis of wave induced progressive buildup of pore water pressure in sloping seabed following the full interactions of wave-seabed.

7.4. Suggested Future Works

Even though the research presented herein attempted to elaborate upon and accommodate the essential elements of analysis for wave-structure-sloping seabed interaction at most, there may be worthy topics, which can fill the gaps in this research as below.

With the aim of computing the wave-induced response in marine sediments, linear wave theory has been employed to model the storm conditions as the first approximation. However, the wave steepness in extreme conditions is high, and nonlinear idealizations of waves yield to more realistic results that need to be included in future research.

The difference among results obtained from various solving approaches (coupled vs. decoupled) has been found to be negligible given the wave and soil characteristics. With the inclusion of soil nonlinearity (due to accumulation of pore pressure and deformation during cycling) and simulations of nonlinear waves, such difference may be more significant. This factor needs to be evaluated in future studies.

The extent of instantaneous liquefaction in this study has been obtained assuming poroelastic response soil elements. A time dependent analysis is required to investigate the impacts of soil nonlinearity on the extents of instantaneous liquefaction. Such analysis needs to be coupled with the response of fluid domain that includes the interactive process among wave and soil media. Also, at each time step the strength parameters of sediments should be updated based on the solution of the previous step.

The analyses presented herein for wave-GSC-seabed interaction considers a single GSC. However, in field condition, stacks of GSCs are used for better protection of shorelines against surging waves. Therefore, there is a need to model a group of GSCs and to investigate the soil response and instability around the structure with new geometry.

Effects of presence of structure built on sloping seabed on global shear failure of the entire slope needs to be evaluated using more advanced soil nonlinear models. Since it has been shown that structures alter the wave-induced stress distributions in their vicinity, the shape of potential slip surfaces around the foundation element may change compared to free field condition that needs to be considered. Moreover, in this work, a strong interaction condition (perfect bond) among soil and footing has been assumed. For heavier structures under significant wave loading amplitudes, it is common to have relative displacements between soil particles and the structure. Such phenomenon may be modeled by introducing interface element at the common interface of domains. This needs to be studied in detail in future works.

The developed framework to model the nonlinear response of soil under cyclic wave loadings (using Bouckovalas' model) needs to be expanded and incorporated into the analysis of general boundary value problem of wave-structure-sloping seabed interaction. Also, the impacts of rotation of principal stresses on the soil response during cyclic shearing need to be investigated thoroughly.

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